# **Propagation of High Frequency Waves in Slender Engineering Structures\***

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A one-dimensional wave-guide based on a Cosserat-type primary structure with secondary substructures of oscillator-type attached is developed. Propagating longitudinal waves are studied under simplifying assumptions: e.g., straight axis, frictionally damped principal rod-structure with variable cross-section, unloaded mantle, i.e., excitation is solely concentrated at the foundation of the building model, etc. Using harmonic linearisation, the resulting integral equation is solved by taking the logarithmic derivative and filtering out negligible contributions. The main results, crucial in tall building acoustics, are critical frequencies and the separation for spatially increasing or decreasing amplitude frequency response functions, and the input-independent upper limit for the strain amplitude. Two main counteracting effects on the propagating wave are identified: energy absorption (due to friction in the primary structure and due to energy absorption in local resonance of the secondary systems) and amplification by decreasing cross-sectional area.

\* In memory of Academician Alexander Yu. Ishlinsky, who passed away on February 7, 2003

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#### **1. INTRODUCTION**

High-frequency vibration in complex engineering structures is challenging problem. Attempts to describe all of the details of real structures fail for many reasons. First, the existence of many inherently uncontrolled factors plays an essential role. In structural dynamics, uncertainties arise from stiffness, mass and damping fluctuations caused by variations in material properties as well as by variations resulting from manufacturing and assembly. The boundary conditions for each structural member also are not known precisely since the high frequency dynamic properties of joints between structural members are especially uncertain. Second, the essential heterogeneity and presence of secondary systems have to be taken into account. Third, the interpretation of the solution of an "exact" boundary-value problem presents great difficulty. The field of vibration of a complex structure (for instance, under a broad-band excitation) is a very complicated function of time and spatial coordinates because a great many modes are excited in the structure. Hence, new approaches are needed for an adequate modelling of structures at high frequencies since with the conventional methods, results become unreliable and can hardly be interpreted.

There exist a great number of approaches, among which the Statistical Energy Analysis<sup>1</sup> is the more celebrated one. However, it belongs to a class of discrete approaches, in which field is averaged within each structural member. Thus this approach is not applicable to problems of wave propagation. An alternative is the so-called continuous approach, the approach of high-frequency structural dynamics<sup>2,3</sup> being utilised in the present paper. The basic substantiation for this approach is that the structures at high frequencies behave like a mechanical system with a continuous spectrum of natural frequencies.<sup>4</sup> This result naturally leads to the representation of complex structures in the form of an elastic carrier structure, in which the primary structure is modelled with oscillators attached to it. The oscillators are introduced into the model to describe numerous secondary systems which comprise the major portion of the structural members of a unit. A number of investigations have been concerned with such a representation. One pioneering work<sup>5</sup> dealt with a one-dimensional representation, whilst other papers<sup>6,7</sup> were devoted to the three-dimensional analysis of isotropic and anisotropic carrier media, respectively.

The present paper is devoted to the problem of propagation of high frequency vibration in such structures, which allows the application of one-dimensional models.<sup>8</sup> It is shown that closed-form solutions are obtained for a rather general case of a structure with varying cross-sectional area, nonlinear energy absorption and secondary systems. The vibration field is shown to have an upper limit which does not depend on the intensity of external excitation.