Acoustic Energy Concept for the Design of a Flow Meter

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An acoustic energy source concept is used to develop simple theoretical expressions for the optimisation of the dimensions for the design of a flow meter. Basic equations are first generated for the prediction of vortex shedding frequency due to a bluff body placed inside a pipe or a channel flow. The Strouhal number predicted using these expressions is compared with limited experimental data for a 6-inch commercial vortex flow meter operating at different Reynolds numbers. The results show good agreement between theory and experiment.

NOMENCLATURE

\(a\) — speed of sound
\(A\) — a non-dimensional parameter
\(C\) — characteristic length of a bluff body shedding vortices
\(d\) — diameter of a circular cylinder
\(D\) — diameter of pipe or channel width
\(f\) — vortex shedding frequency
\(N\) — an integer
\(P\) — perimeter of the vortex shedding body
\(S_R\) — Strouhal number (= \(fd/U_c\))
\(U_f\) — velocity associated with vortex street
\(U_\infty\) — free stream velocity
\(X\) — spacing between two consecutive vortices of the same row
\(Y\) — spacing between two parallel rows of vortices
\(K\) — circulation of vortices

2. BASIC CONCEPTS

The problem of vortex shedding can be viewed as an acoustic energy effect in the wake.\(^9,11\) The basic assumption behind this consideration is that the velocity fluctuations in a turbulent wake and the fluctuating surface pressure stresses at the body surface act as acoustic sources. The sound intensity of these sources varies to the eight power of the relative fluid velocity. For alternate vortex shedding such as on a circular cylinder, three centres of disturbances can be identified: two strong areas at the separation points and a weak one at the stagnation point. Acoustic pressure pulse sources can then be considered concentrated at these centres. The frequency of the vortex pair can be assumed to be given by:\(^12\)

\[ f = \frac{3}{2\pi} (U_\infty - U_f) \]  

(1)

No practical method was suggested in reference\(^12\) to determine \(U_f\). Consequently, this expression has rarely been used.

Using well known hydrodynamic principles, however, a theoretical expression of \(U_f\) can be obtained for two parallel rows of vortices of the same spacing, \(X\), but of opposite circulation \(K\). The vortices are so arranged that each vortex of the upper row is directly above the mid point of the line joining two vortices of the lower row and are separated by a distance \(Y\). Under such conditions, the velocity of the vortex street, \(U_f\), is given by:\(^13\)

\[ U_f = \frac{K}{2X} \tan \left( \frac{\pi Y}{X} \right). \]

(2)

When the bluff body producing alternate vortices is placed inside a pipe, the velocity contribution due to the wall effect becomes \(K Y / X D\), where \(D\) is the diameter of the pipe or width of the channel. The general stream in the distant wake assumes a velocity equal to \((U_\infty K Y / X D)\). Consequently, the frequency of vortex shedding can be written as:\(^14\)

\[ f = \frac{1}{X} \left( U_\infty + \frac{K Y}{X D} - U_f \right). \]

(3)

From Eqs. (2) and (3):

\[ f = \frac{U_\infty}{X} \left[ 1 - \frac{U_f}{U_\infty} (1 - G) \right], \]

(4)