Experimental Investigation on the Occurrence of Internal Resonances in a Clamped-Clamped Beam

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The geometrically non-linear vibration of an aluminium beam clamped at both ends was investigated experimentally, with the goal of verifying the occurrence of internal resonances. The beam was excited transversely with a harmonic excitation and the deflections of the first and higher harmonics were analysed in order to detect the mode shapes involved. One to three and one to five internal resonances between the first and higher order modes and between the second and higher order modes were found.

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1. INTRODUCTION

The non-linear mode shape of a beam vibrating with large displacements and clamped at both ends changes with the amplitude of vibration because of two main reasons. The first is the fact that the longitudinal forces stiffen the beam, causing a smooth variation of the mode shape. Several authors have demonstrated this, both theoretically and experimentally.[1-4] The second reason is modal coupling. The response of a beam excited into non-linear vibration at a certain frequency involves the frequency of excitation and some of its harmonics. Because the natural frequencies change with the vibration amplitude, they can become commensurate (i.e., related by $m_1\omega_1 + m_2\omega_2 + \ldots + m_n\omega_n \cong 0$, where m_i are integers) and energy can be interchanged between different modes of vibration. If this phenomenon, known as internal resonance, occurs, then the motion is defined by the coupled modes and the non-linear mode shape changes considerably during the period of vibration.

Internal resonances between the first mode and higher order modes have been detected in numerical and analytical analyses of the geometrically non-linear vibration of beams.[1,5,6] The multi-modal vibrations and strong variation of the mode shape during the period of vibration, which result from internal resonance, have also been clearly demonstrated by analytical and numerical methods.[1] In this work, the geometrically non-linear vibration of an aluminium beam clamped at both ends was investigated experimentally, with the goal of verifying the occurrence of internal resonances.

2. MEASUREMENTS AND ANALYSIS

Experimental work was carried out on a slender beam of dimensions $585 \times 30 \times 3$ mm, made of an aluminium alloy. This beam was clamped at its ends in two steel blocks, which were screwed to a heavy steel table. An electromagnetic exciter was attached to the beam by means of a drive rod and of a force transducer. The signal sent to the shaker was generated by the analyser and amplified by a power amplifier. Fig. 1 represents the experimental set-up.

A slow frequency sweep was performed to identify the non-linear phenomena of interest. In order to avoid transients, some time was allowed before collecting data after each increment in frequency. The beam was divided into seventeen points (Fig. 2). However, due to the symmetry of the system, most often measurements were only taken along half of the span (points 1 to 9). The first four linear modes and the seventeen points where the accelerometer was fixed are represented in Fig. 3.

The resonance frequencies change with the amplitude of vibration, and sub-harmonic and super-harmonic resonances may be important. Therefore, it may be misleading to carry out an investigation on which modes are involved in the response by analysing only the power spectra. By measuring the shapes associated with each harmonic, the modes involved in the response are found with more certainty.

In order to detect modal coupling between the first and higher order modes, the beam was excited transversely at its mid span with sinusoidal force excitations. Three excitation force amplitudes were applied: 0.25, 0.5 and 0.8 N. These values should be considered as approximate ones, because there was no closed-loop system to guarantee that the force applied by the shaker had a constant amplitude during the frequency sweep and because of the force transducer's inertia.

Figures 4-6 represent the displacement amplitudes of the first, third and fifth harmonics of the response, when the excitation force amplitude is respectively 0.25, 0.5 and 0.8 N. The displacements are shown for point 9, which is located at the middle of the beam, and for point 5, where the amplitudes of the second and third mode shapes are large (Fig. 3). Here h represents the thickness of the beam.

Analytical and numerical analyses of geometrically non-linear beam vibration, under the action of a transverse harmonic force, only predict odd harmonics in the response. Thus we will concentrate on these. However, it should be pointed out that even harmonics were actually found in the response. Probably, their excitation occurs because the applied force is not purely harmonic and not applied exactly in the transverse direction. Small deviations from exact harmonic and transverse excitation cause the appearance of even harmonics.[7]