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A bio-inspired swing vibrator applied in fluid driving has the characteristics of a large deformation and rigid-flexible structure. Currently, there is a shortage of influential theories and methods to analyze its vibration. This research established a dynamic model for studying the bionic rigid-flexible swing vibrator based on pseudo-rigid-body theory. The weak coupling relationship between vibration arm DOF and flexible caudal-fin DOF was proposed by comparing the diagonal elements with the characteristic matrix’s anti-diagonal elements. Analysis results showed that the presence of a flexible caudal-fin had minimal impact on the vibration modes of the vibration arm. The flexible caudal-fin vibration could be treated as a dynamic response that uses the vibration arm’s end as a foundation. Experimental studies were conducted, and a laser Doppler vibrometer was used to measure the vibrator’s vibration modes. The experimental results indicated that whether the vibrator works in water or in air, the presence of a flexible caudal-fin had little impact on the resonance frequency of the vibration arm. Finally, experimental results were discussed based on theoretical analysis and conclusions. This paper provides an academic tool for modeling, optimization, and a working mechanism analysis on the rigid-flexible swing vibrator.

1. INTRODUCTION

Valve-less pumps have a complete range of applications in biology, medicine, chemistry, chemical engineering, and micro-electromechanical systems for their advantages in fluid delivery such as simple structure, easy miniaturization, and without wear pollution from contacting components, etc. The diffuser/nozzle fluid pump proposed by Stemme¹ is the typical structure for valveless pumping. The valveless pump derived from this principle has been put forward, such as multi-section mixing valve-less pump,² double-export valve-less pump,³ and so on. However, its weakness, such as low efficiency, large backflow, and heavy flow pulsation, inevitably limits its further applications. As the bionic swing-driven valve-less pumps can eliminate the inherent weakness of diffuser/nozzle valveless pumps, they are drawing more and more attention from researchers around the world. In taxonomy, pumps are divided into two categories in principle, one is a volume change type pump, which mainly includes the plunger pump, piston pump, diaphragm pump, and the diffuser/nozzle pump. The other is the rotary pump, which mainly includes the screw pump, vane pump, and gear pump. The proposed pump is neither a rotating type nor volumetric type. Therefore, it is a new branch of pump taxonomy, we call it the swing drive pump.

The swing drive principle was initially applied to agitate air to form a fan, and piezoelectric fans have become an area of research.⁴–⁶ Until 2008, PIRESA developed a miniature water pump with a piezoelectric bimorph and applied it to the heat dissipation of LED lights. However, the performance of the pump was insufficient, and the flow rate recorded in the literature only reached 114cc/min.⁷ The follow-up study found that the flexible structure at the end of the vibrator can effectively improve the pumping performance.⁸,⁹ The previous research provided an analytical method for the swing vibrator¹⁰ and optimized the pump’s design.¹¹ It was found that when the vibration arm’s structural parameters are constant, both the vibration amplitude and the pump flow rate increase first and then decrease with the increase of the length of the flexible caudal-fin.¹² The size of the vibration arm had the same relationship with the pump flow rate.¹³

Considering the modeling problem of the rigid-flexible structure, Li simulated the dynamic behavior of compliant parallel guided mechanisms based on pseudo rigid body theory in ADMS software.¹⁴ Gao employed the particle swarm optimization (PSO) algorithm to predict the large deflection of cantilever beams, and the determination of the parameters is
complicated.\textsuperscript{15} Nesic presented a methodology for the dynamical model of a compliant mechanism consisting of thin elastic beam segments based on the transfer matrix method.\textsuperscript{16} These references give us a reference to solve the vibration problem of a rigid-flexible coupled oscillator, but they cannot explain the experimental phenomenon completely.

To sum up, the existing theoretical analysis and numerical simulation methods cannot accurately describe the swing behavior of rigid-flexible vibrators. And this shortage is mainly displayed in the following two aspects: 1) The analysis results based on the traditional theories have significant errors compared with the experiment; and, 2) the supposed differences between the rigid-flexible vibrator and the constant cross-section vibrator are not shown both in the theoretical analysis and numerical simulation results.

Our research group’s previous results showed that the pumping effect of the swing vibrator is conditional, that is, the tail-fin must be very thin, and its length has the optimal solution. If it is too long or too short, it will cause the pumping effect to be small or even disappear. The high efficient mechanism of the bionic vibrator with these features, as yet, has not been entirely explained. The dynamic model of this bionic vibrator is the key for researchers to optimize the structure. The vibration characteristics of the rigid-flexible swing vibrator should be further studied. For these challenges, this project was carried out based on the pseudo-rigid-body theory and laser vibration measurement experiments. The purpose of this paper is to establish the pseudo-rigid-body model theoretically to derive the coupling relationship between two parts of a rigid-flexible structure. The results are used to explain the experimental phenomena and provide a theoretical tool for the structural optimization of the vibrator. The research objectives were to:

1.) Develop a dynamic model for the rigid-flexible vibrator, and provide a theoretical basis for the optimization design of the bionic valve-less pump;
2.) Analyze the coupling relationship between the vibration arm and the flexible caudal-fin; and,
3.) Explain the experimental results using results from this model analysis.

2. THE WORKING PRINCIPLE AND STRUCTURE OF THE PUMP

By flexing their bodies and caudal-fins, fish get the power to swim forward in the water. If the fish’s head is fixed, then the swing of the caudal-fin could push the water to flow in the opposite direction of the fish’s swimming direction, which is the inspiration source for the pump vibrator design. The structural features of a fish body can be described as: (1) the whole body is like a spindle; (2) the mass is mainly distributed in the body’s front part; and (3) the caudal-fin holds a little weight and has a broad swing range. The fish’s swing and deflection during swimming are shown in Fig. 1 (a). To resemble a biological fish body, the swing vibrator was also designed into a variable cross-sectional shape accordingly. Particularly, the caudal-fin should have a flexible structure with smaller weight and stiffness, as shown in Fig. 1 (b).

The pump comprises a pump base, a pump lid, a piezoelectric stack, an isolation block, a steel ball, flexible hinge 1, flexible hinge 2, and flexible caudal-fin. The vibrator arm and the flexible caudal-fin form a swing vibrator. As the excitation source, the piezoelectric stack generates longitudinal vibration under the excitation of harmonic voltage. The function of the steel ball and the isolation block is to ensure that the piezoelectric stack is subjected to the action of normal force, thereby preventing the piezoelectric stack from being damaged due to non-uniform stress. An amplifying mechanism composed of a lever/flexible hinge mechanism I and lever/flexible hinge mechanism II is used to increase the amplitude of point D. The waterproof glue isolates the piezoelectric stack from the water in the pump chamber, avoiding short circuit damage to the piezoelectric stack. A flexible blade is pasted at the end of the vibrator arm to form an abrupt section similar to a caudal peduncle of a biological fish body at a D point. The overall structure of the pump is shown in Fig. 1 (c). The working site photo of the pump is shown in Fig. 1 (d).

Figure 1. Working principle and structure of the pump.\textsuperscript{8}
The flow rate of the pump can be expressed as follows:

\[ Q = 8f^2 \pi \theta_0 \sqrt{Aw \cdot \frac{R^3}{3}} \]

\[ \times \int_0^{\pi} \sqrt{\left( \sin(\theta_0 \sin 2\pi ft) - \sin^3(\theta_0 \sin 2\pi ft) \right)} \, dt; \quad (1) \]

where \( l \) is the working frequency, \( w \) is the width of the vibrator, \( \theta_0 \) is angular amplitude, \( A \) is the sectional area of conduit, \( R \) is the effective swing radius.

3. VIBRATION ANALYSIS OF VIBRATOR

The vibrator model in Fig. 1 is simplified to obtain an analytical expression, as shown in Fig. 2. The \( x \)-direction coordinates of each suddenly changed cross-section of the vibrator can be marked as \( l_1 \), \( l_2 \), and \( l_3 \), respectively. The vibrator is decomposed into several parts, and each of them has an equal cross-section. The differential equation of the section \( i \) is:

\[ \frac{d^4 \varphi_i(x)}{dx^4} = \varphi_i(x) \beta^4, \quad i = 1, 2, 3; \quad (2) \]

where, \( \varphi_i(x) \), \( (EI) \) and \( m_i \) are the modal shape, bending stiffness, and mass per unit length.

Based on the continuity boundary conditions between the segments, the characteristic equation is obtained as follows:

\[ J_{12 \times 12} P_{12 \times 1} = 0; \quad (3) \]

where, \( J \) is the characteristic matrix, and \( P \) the characteristic vector, that is, the coefficient vector of the mode shape.

\[ P = \begin{bmatrix} A_1, B_1, C_1, D_1, A_2, B_2, C_2, D_2, A_3, B_3, C_3, D_3 \end{bmatrix}^T_{1 \times 12}; \quad (4) \]

where, \( A_i, B_i, C_i, D_i \) are the undetermined coefficients of the mode function of segment \( i \).

The characteristic vector \( P \) has non-zero solutions, so the determinant of \( J \) is zero:

\[ |J_{12 \times 12}| = 0. \quad (5) \]

By solving Eq. (5), the analytical expression of \( \omega \) can be solved.

However, the problem with this model is that the hypothetical premise is small deformation, which is inconsistent with the large swing at the end of the vibrator; hence the model cannot truly reflect the dynamics of the vibrator.

The results of the FEM also have a large deviation from the experiment. Figure 3 shows the variation of the second-order resonance frequency with the length of the flexible caudal-fin, which shows that with the increase of the caudal-fin length, the second-order resonance frequency becomes smaller and smaller, and the optimal working frequency should also become smaller and smaller. However, the experimental results showed that the optimum operating frequency of the vibrator with different caudal-fin lengths fluctuates near the frequency of the vibrator arm.

Pseudo-rigid-body theory is an effective method to solve rigid-flexible structures with strong nonlinearity and large deformation.

4. PSEUDO-RIGID-BODY MODEL FOR THE VIBRATOR

The pseudo-rigid-body model uses rigid components, which have equivalent relationships between “force and deformation” to simulate the flexible portion of the vibrator. This method builds a theoretical connection between a rigid body and flexible components. Hence it is of great significance in the analysis and design of rigid-flexible structure.

The whole swing vibrator can be divided into a vibrating arm, flexible hinge, and flexible caudal-fin according to the rigidity distribution. The vibration arm structure determines that its main movement is a rotation around the centerline of the flexible hinge, and the bending vibration of the arm could be neglected. Relative to the vibration arm, the flexible hinge can be regarded as a flexible segment. The flexible caudal-fin is typically a flexible component coming with large deformation. This mechanism, composed of both rigid and flexible components, modeling with the pseudo-rigid-body theory, is expected to receive better results.

The established pseudo-rigid-body model for the swing vibrator is shown in Fig. 4. The flexible hinge is replaced with a torsion spring, which has a torsional stiffness \( k_1 \). The swing center is \( O' \). As the bending vibration is very weak, the vibration arm could be equivalent to a rigid body oscillating around \( O' \). Its moment inertia around \( O' \) is \( J_1 \), and the length of the vibration arm is \( L_{arm} \). The restoring moment from torsion spring is \( M_1 \). The flexible caudal-fin, characterized by the small thickness and large deformation, is a typical flexible component. It also can be simplified as a combination of a rigid body and a torsion spring, and its center of rotation is \( O'' \). The pseudo-rigid body’s moment of inertia to \( O'' \), the restoring moment from torsion spring, torsion spring’s stiffness, the length
of the flexible caudal-fin, pseudo-rigid-body plate’s characteristic radius factor, and the characteristic radius, are marked as $J_2$, $M_2$, $k_2$, $L_{tail}$, $\xi$, $\xi_{tail}$, respectively. While the lever-flexible hinge structure I stimulates the vibration arm on point B, it also imposes a constraint on the vibration arm. Thus, a spring is adopted to represent this constraint, and its stiffness is $k_3$. Generalized coordinates of the pseudo-rigid-body system are represented by $\theta_1$ and $\theta_2$. In this way, the complicated rigid-flexible system, which has variable cross-section and infinite degree of freeform, can be simplified as a 2 DOF system that is easy to be solved.

The differential equation of the free-swinging vibration arm around the axle is:

$$J_1\ddot{\theta}_1 + (K_1 + K_2 + K_3L_3)\theta_1 - K_2\theta_2 = 0.$$  \hfill (6)

The vibration arm does a micro-amplitude vibration, leading to a tiny displacement of $O''$ in the vertical direction; hence, the dynamic behavior of the flexible caudal-fin can be approximately considered a swing vibration that takes $O''$ as an axis. The differential equation can be written as:

$$J_2\ddot{\theta}_2 + K_2(\theta_2 - \theta_1) = 0.$$  \hfill (7)

Equations (6) and (7) can be expressed using a matrix:

$$\begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_3 + k_3L_3 \\ -k_2 \\ -k_2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = 0.$$  \hfill (8)

Equation (8) is the simplified differential equation for the vibrator with 2 DOF.

### 5. PARAMETERS FOR VIBRATION EQUATION

1) The flexible hinge’s torsion stiffness $k_1$

$$k_1 = \frac{(2Eb^2)}{(9\pi r^2)};$$  \hfill (9)

where $E$ is the material’s elasticity modulus, $b$ is the width of the flexible hinge structure, $t$ is the minimum thickness of the hinge, and $r$ is the cutting radius.

2) Vibration arm’s moment inertia $J_1$

$J_1$ is the moment inertia of the vibration arm with respect to $O'$. For the convenience of calculation, this part is divided into three zones, namely areas I, II, and III, as shown in Fig. 6. Their moment inertias are $J_{11}$, $J_{12}$, and $J_{13}$, respectively.

3) Formula for $k_2$

The equivalent model of the pseudo-rigid-body structure established for the flexible beam is shown in Fig. 7. During deformation, the track of the endpoint of the flexible beam is not a concentric arc. To approximately express this track, the beam is divided into two sections, and both are assumed to be rigid. Between them, movement is transmitted by the hinge, and force is transmitted by the torsion spring. Different characteristic radius coefficients $\xi$ correspond to different deformation trajectories of the end of the pseudo-rigid-body model. For uniform section beam, generally $\xi = 0.8517$.\hfill (17)

Transorming $E$ into a dimensionless lateral load index $\alpha^2$:

$$\left(\alpha^2\right)_t = \frac{F_tL_{tail}^2}{E_3I}.$$  \hfill (10)
where, \( m_\gamma \) is the mass of the pseudo-rigid pole.

5) Formula for \( k_3 \)

\( k_3 \) represents the constraint effect due to the contact of the two vibrating arms. Since the piezoelectric stack shown in Fig. 1 requires a certain amount of pre-pressure, the initial position of the vibrating arm shown in Fig. 3 is below the equilibrium position, that is, the angle \( \theta \) is negative. And \( k_3 \) could be modeled using a Sigmoid function:

\[
    k_3 = k_C \text{Sig} \left( \frac{y^*}{y_s}, \kappa \right) .
\]

In Eq. (17), \( k_C \) is the contact stiffness, which is expressed as:\n
\[
    k_C = \frac{4}{3} \left( \frac{R_1 R_2}{R_1 + R_2} \right)^{\frac{1}{2}} \frac{E_1 E_2}{E_1(1 - \mu_1^2) + E_2(1 - \mu_2^2)} = \frac{2}{3} E;
\]

where, \( R_1 \) and \( R_2 \) are the equivalent radii of the two vibration arms at the contact point; \( E_1 \) and \( E_2 \) are the elastic modulus of the contact material, respectively; \( \mu_1 \) and \( \mu_2 \) are the Poisson’s ratio of the contact material, respectively.

And the Sigmoid function \( \text{Sig} \) is defined as follows:

\[
    \text{Sig} \left( \frac{y^*}{y_s}, \kappa \right) = \frac{1}{1 + \exp \left( -\kappa \left( \frac{y^*}{y_s} - 1 \right) \right)} .
\]

In Eq. (19), \( y_s \) is the distance from point B to the collision point when the arm is in equilibrium. \( y^* \) is the amplitude at contact point B, and \( \kappa \) represents smoothness during vibration. \( \kappa = 0 \) means that the equivalent spring \( k_3 \) is a pure linear spring, and as \( \kappa \) tends to infinity, the spring becomes a piecewise linear spring.


Dimensions of the vibrator in Fig. 6 are given in Table 1. The vibration arm’s material is 65Mn, and the flexible caudal-fin’s material is QSn. Properties of the used materials are shown in Table 2. The calculated model parameters are as follows:

\[
    J_1 = 3.3517 \times 10^{-6}, \quad k_1 = 39.702,
\]
\[
    J_2 = 1.4667 \times 10^{-10}, \quad k_2 = 2.1460 \times 10^{-5},
\]
\[
    L_3 = 8 \times 10^{-3}, \quad k_3 = 4.0134 \times 10^4 .
\]
The Eq. (8) is written as:

$$\begin{bmatrix} k_1 + k_2 + k_3 L_3 - \lambda J_1 & -k_2 \\ -k_2 & k_2 - \lambda J_2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = 0. \quad (21)$$

It can be seen that the anti-diagonal elements of the characteristic matrix are much smaller than the diagonal elements. Thus, the two DOFs $\theta_1$ and $\theta_2$ are weakly coupled. An essential feature of this weakly coupled system is that the vibration modes are close to those of the uncoupled system. It indicates that the presence of a flexible caudal-fin has a little influence on the dynamic behavior of the vibrating arm, and the motion of the flexible caudal-fin can be seen as a vibration response based on the end of the vibration arm. This conclusion provides a theoretical tool for mechanism analysis and optimization of the swing vibrator.

To verify this approach, a laser Doppler vibrometer is used to investigate the influence of the flexible caudal-fin on the vibrator arm. Experimental procedures were as follows.

1. Take a vibrator without a flexible caudal-fin and place it in the air for vibration measurement scanning. The scanning area is shown in Fig. 8 (a).

2. Take a vibrator with a flexible caudal-fin and place it in the air for vibration measurement scanning. The scanning area is shown in Fig. 8 (b).

3. Submerge the vibrator in water and repeat the above test steps.

Fig. 8 and Fig. 9 are the frequency response curves obtained by scanning. It shows that in the air, the resonance frequency of the non-tail fin vibrator was 1,888 Hz. After the tail fin was attached to the end of the vibrator, its resonance frequency was 1,884 Hz, which is almost unchanged compared with the previous experiment. Some tiny peaks in Fig. 9, such as at 1100 Hz, 2600 Hz and 3700 Hz, are interference modes, which were ignored and not further studied.

Similar experimental results were obtained in water, as shown in Fig. 9. The resonance frequency of the non-tail fin vibrator is 1,533 Hz. After the flexible tail fin is attached, the resonance frequency is 1,455 Hz. The difference is only 5%, which shows that the effect of the flexible tail fin on the vibration performance of the vibrating arm is also very weak. This experimental result agreed well with the weak coupling theory expressed in Eq. (21). It should be noted that the focus of this paper is the influence of the presence of flexible caudal-fin on the vibrator arm, so the scanning region is the vibrator arm part. As for the relationship between pump performance and tip amplitude, it has been discussed in detail in reference.10

7. DISCUSSION

Existing studies13,21 had shown that when the size of the vibrating arm was fixed, as the length $L_{\text{tail}}$ of the caudal-fin increased, the amplitude of the caudal-tail fin and pumping capacity displayed a trend of increasing at first and then decreasing. Similarly, when the size of the caudal-tail $L_{\text{tail}}$ was set and the length of the vibrating arm changed, the pumping capacity and the amplitude of the tail fin had the same results. However, according to the linear vibration theory of the beam, as the length of the vibrating arm or caudal-fin increases,
the ending amplitude of the vibrator will increase. Obviously, the dynamic performance of the swing vibrator cannot be explained by the linear vibration theory of the beam. The weak coupling relationship between the vibration arm and flexible caudal-fin stated above provided a reasonable explanation for this conflict.

As the $\theta_1$ and $\theta_2$ are weak coupling, the flexible caudal-fin motion can be approximated as a vibration response based on the vibrating arm end. Try to decouple the Eq. (21) as follows:

\[
(k_1 + k_2 + k_3L_3 - \lambda J_1)\theta_1 = 0
\]
\[
(k_2 - \lambda J_2)\theta_2 = 0.
\]

(22)

Taking $\theta_1$ and $\theta_2$ as generalized coordinates, the inherent frequency of the vibration system is presented in Eq. (23) (see top of this page).

Substituting the data of Table 1 and Eq. (20) into Eq. (23), the resonance frequency is found to be:

\[
f_1 = \frac{\omega_1}{2\pi} = 1651 \text{ Hz};
\]

(24)

which is in good agreement with the experimental result shown in Fig. 8 with an error of 12.55%.

\[
f_2 = \frac{1}{2\pi} \omega_2;
\]

(25)

Put the Eq. (15), (16) into Eq. (25):

\[
f_2 = 1 \sqrt{\frac{\pi \xi^2 E_2 I_2}{(\xi L_{tail})^2}} = 1 \sqrt{\frac{\pi \xi^2 E_2 I_2}{\frac{1}{2} \rho_2 wt^2 (\xi L_{tail})^4}}.
\]

(26)

Substitute the data of Table 1 and Table 2 into the Eq. (26):

\[
f_2 = 869 \text{ Hz}.
\]

(27)

For $\omega_1$ the parameter $L_{03}$ represents the length of the vibration arm (Fig. 6). Eq. (23) reveals that when other structural parameters are fixed, $f_1$ is a one-variable function of $L_{03}$, and its overall function curve is declining. Similarly, Eq. (26) indicates that $f_2$ is a one-variable function of $L_{tail}$, and the function curve also has the tendency of monotonous decreasing.

If keeping the length of the vibration arm unchanged and changing the value of $L_{tail}$, is a constant, but the $f_2$ is a descending curve. When these two curves intersect at point P (as shown in Fig. 10 (a), the natural frequency of the vibrating arm and the caudal-fin is the same, and resonance occurs. Hence at point P, the vibration of the flexible caudal-fin reaches the most severe condition. According to Eq. (1), the best pumping performance is obtained.\(^{10}\)

If keeping $L_{tail}$ fixed and varying the value of $L_{03}$, $\omega_2$ is a constant but the $\omega_1$ intensive vibration, and the pressure head of the pump reaches the maximum value at the intersection point M, as shown in Fig. 10 (b). The analysis result was entirely explained by the experimental phenomenon described in the literatures.\(^{13,21}\)

8. CONCLUSIONS

A method to analyze the dynamic model of the rigid-flexible structure was proposed in this paper, which was verified by experiments.

1) A pseudo-rigid-body model for the rigid-flexible structure was established. The complicated rigid-flexible system with a variable cross-section and infinite degree freedom was simplified into a two DOF system based on this model.

2) By comparing the diagonal elements with the anti-diagonal elements of the simplified model characteristic matrix, the coupling relationship between DOF $\theta_1$ of the vibration arm and DOF $\theta_2$ of the flexible caudal-fin was investigated. It was shown that the presence of a flexible caudal-fin had minimal impact on the modes of the vibration arm, and the vibration of the flexible caudal-fin could be seen as a vibration response that took the vibration arm end as a foundation.

3) Through the laser vibration measurement system, the influence of the existence of the flexible caudal-fin on the vibration characteristics of the vibrating arm was investigated, and the experimental results were in good agreement with the theoretical analysis.

4) This research provided a powerful tool for the opti-
mization design and driving mechanism analysis of the rigid-flexible structure swing vibrator.

5) Due to the advantages of strong pumping capacity, no contact wear, easy miniaturization, and so on, this type of micro pump will have a wide range of applications in agricultural drip irrigation, liquid cooling of electronic components and other fields. But the accurate control of its flow needs further study.

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