Response of Duffing’s Oscillator to Harmonic Base Excitation and Significance of First Order Term

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(Received 9 November 2019; accepted 1 April 2020)

Responses of systems with nonlinear stiffness subjected to base harmonic excitation are determined. An expression to estimate the amplitude in the fundamental frequency of oscillation is derived from first principles using Lindstedt’s method. It is observed that the amplitude determined using the zero order approximation is in error at low frequencies. Therefore, an expression for the first order approximation of the amplitude of response at the fundamental frequency is derived. Zero order and first order approximation terms together form the response. Characteristics showing the variation of the amplitude with the excitation frequency for various nonlinear spring parameters are presented. The issue at low frequencies is resolved by the incorporation of the first order term. An expression for the phase difference and the expression of the asymptote where the responses converge are also derived.

NOMENCLATURE

- \( k \) : stiffness
- \( m \) : mass
- \( c \) : coefficient of viscous damping
- \( f_{spring} \) : spring restoring force
- \( x \) : absolute displacement of the system
- \( \dot{x} \) : velocity of the system
- \( \ddot{x} \) : acceleration of the system
- \( \epsilon \) : small parameter, \( \epsilon << 1 \)
- \( \alpha, \beta \) : parameters related to the nonlinear spring
- \( \epsilon \zeta \) : damping factor
- \( \omega \) : parameter related to the natural frequency
- \( y \) : base displacement
- \( \epsilon Y \) : amplitude of the base excitation
- \( \Omega \) : frequency of excitation
- \( t \) : time
- \( T \) : time period of oscillation
- \( \tau \) : modified time variable
- \( \varphi \) : phase angle
- \( \omega_0 \) : linearised natural frequency
- \( x_i \) : \( i^{th} \) order term of the displacement, \( i = 0, 1, 2 \ldots \)
- \( \varphi_i \) : \( i^{th} \) order term of the phase angle, \( i = 0, 1, 2 \ldots \)
- \( A_i \) : \( i^{th} \) order term of the amplitude of the displacement, \( i = 0, 1, 2 \ldots \)

1. INTRODUCTION

Many systems encountered in practice exhibit nonlinear behavior due to the change in their stiffness with deformation. This may be because of its geometrical construction or the characteristics of the material itself. Bi-linear and cubic are a couple of simple models that are used for representing such stiffness characteristics. A single degree-of-freedom (SDF) system with a spring having cubic non-linearity is called a Duffing’s oscillator. Many equipment, machines, buildings etc. are provided with isolators to reduce their vibrations or the vibrations transmitted. Behavior of all such isolators can be represented by this model. Vaiana et. al., Sudhir Kaul, Losanno et. al. and Madera Sierra et. al. used such models to represent the behavior of isolators, which show that such models are generally used for representing the nonlinear stiffness of isolators. These models find place even in the analysis of non-linear dampers and other devices.

Determining the response of a Duffing’s oscillator is a classic example of a nonlinear problem and a large amount of work is carried out on this subject. The text book by Meirotvich gives a good insight into this problem. Considering the non-linearity to be weak, perturbation techniques are widely used for arriving at the solution. Lindstedt’s method, method of averaging, harmonic balance method, method of multiple scales are the most commonly used perturbation techniques.

Using the above mentioned techniques, the free vibration characteristics and the response to harmonic force excitation are studied extensively. These studies have provided expressions for determining the frequency of the free vibration of the Duffing’s oscillator. Expressions were also presented for the amplitude of the response when excited by a harmonic force. The responses show ‘jump phenomenon’ when they are subjected to force excitation. There are several works that discuss on the jump phenomenon. Malatkar and Nayfeh obtained the minimum force that needs to be applied to make the system to have jumps in their responses. Brennan et.al. obtained expressions for determining the frequencies at which the jumps in the responses occur. Works are also reported in determining the parameters of the nonlinear system from its response characteristics using methods like nonlinear subspace identification etc. All these works are related to force excitation.