1. INTRODUCTION

The continuous emergence of high-rise and super high-rise buildings prompted the development of high-trip, high-speed elevators. In the course of operation, elevators inevitably experience a variety of vibration phenomena, with a large part of vibration being related to the elevator lifting system.

An elevator lift system is mainly composed of hoisting ropes with a certain axial movement length, a counterweight at one end, and a car at the other end. Hoisting ropes exhibit certain time-varying characteristics. As the lifting part of the elevator, hoisting ropes are elongated and shortened during elevator operation. Thus, parameters, such as stiffness and hoisting rope damping, constantly change. Furthermore, time-varying characteristics are apparent at high speed, thereby seriously affecting the comfort and safety of elevators. With the change in rope length, natural frequencies and vibration energy also change. When the rope is shortened, free vibration energy increases exponentially, and dynamic instability easily occurs, thereby seriously affecting the safety of passengers. The effect of longitudinal vibration on the lifting system is markedly stronger than that of lateral vibration under consistent initial conditions. Therefore, the effect of the time-varying characteristics of hoisting ropes, in the longitudinal vibration of the elevator lifting system, should be studied.

Numerous studies on the longitudinal vibration of traction elevator hoisting systems have been conducted. The basis of these studies is to establish discrete models based on distributed parameters, which more closely simulate the actual operating state of the elevator. The advantage of this type of model is that it is easy to understand and solve. However, this type of model disregards the continuity of wire rope flexibility and thus cannot effectively reflect the dynamics of elevator lifting systems. Therefore, the continuous model based on a distributed parameter, which can describe the flexible characteristics of traction ropes better, is gradually applied.

The distributed parameter continuous system model generally simplifies the lifting system to an axially moving chord with a certain mass attached at one end, and the kinetic behavior of the system is described by establishing either a functional differential control or a partial differential control equation. The parameters of the model are continuously distributed in time and space; thus, the model possesses infinite degrees of freedom and can satisfactorily describe the flexible continuous characteristics of the elevator lifting system. Zhang established a differential equation of motion and energy equations representing the longitudinal vibration of an arbitrary variable-length flexible lifting system by using the Hamilton principle and validated the accuracy of continuous system modeling and energy methods. Bao established the equations governing flexible lifting wire ropes by using the Hamilton principle, studied the nonlinear vibration of a flexible lifting wire rope with time-varying length, and verified the theoretical model by experiment. In addition, considering the interaction between the rigid and the deformation motions of the hoisting rope, Bao established a differential equation that represented the longitudinal vibration of the hoisting rope of a variable-length lifting system, and a numerical example is given to analyze the model. These studies considered the high-speed elevator lifting system as the research object. However, the weight of the
compensation rope and the tensile force provided by the tensioning device are not considered when the model is developed. The weight of the compensation rope and tensile force are the existing objectives in high-speed elevators. Therefore, the established model is reasonable when the weight of the compensating rope and the tensile force provided by the tensioning device are considered.

The exact solution for the time-varying model of the longitudinal vibration of a high-speed elevator hoisting system cannot be easily obtained; thus, a numerical solution is usually applied. In literature, the Runge–Kutta method was used to solve the model, but the algorithm poses a computational stability problem and requires a small step size. Therefore, the number of steps and the amount of calculation increases, which can in turn result in error accumulation. Precise integration is an effective numerical calculation method proposed by Prof. Zhong. This method offers the most important features of high precision, show its absolute stability, and is widely used in solving linear stationary systems. At present, literature successfully applied a fine integral method to evaluate the transient response of time-varying systems.

We established the control equation corresponding to the longitudinal vibration of the hoisting rope in a high-speed traction elevator lift system. This approach was based on the Hamilton principle and considered the weight of the compensating rope and the tensile force provided by the tensioning device. The Galerkin weighted residual method was used for discretization. The time-domain curves of the longitudinal vibration response of the hoisting rope were simulated by the fine integral method. The dynamic characteristics of the elevator, influence of car mass and hoisting rope mass, and tension on the longitudinal vibration of the elevator are analysed during the upward and downward process.

2. NONLINEAR TIME-VARYING MODEL OF THE LONGITUDINAL VIBRATION OF HOISTING ROPES IN A HIGH SPEED TRACTION ELEVATOR SYSTEM

To study the time-varying characteristics of the longitudinal vibration of traction rope conveniently, the modeling and solving of this paper were based on the following three assumptions:

1. Hoisting ropes are continuous and uniform, with constant cross-sectional area $A$ and elastic modulus $E$ during movement;
2. The influence of lateral vibration from the hoisting ropes is ignored, and elastic deformation caused by the vertical vibration is neglectable with respect to the length of the ropes.
3. The influence of bending rigidity on hoisting ropes, friction force, and airflow are ignored.

Figure 1 shows a diagram of a hoisting system for a high-speed traction elevator. Considering the influence of the time variation of the traction rope, the compensating rope and the tensioning device on the longitudinal vibration, the elevator vibration system was regarded as consisting of a traction rope, a car and a compensation rope. Thus, Fig. 1 was simplified into a longitudinal vibration time-varying model of the traction rope in an elevator hoisting system, as shown in Fig. 2.
By using the finite deformation theory of continuum, the displacement vector and velocity vector of \( v(t) \) in the \( x \)-axis were as follows:

\[
\mathbf{r} = [x(t) + y(x(t), t)]j; \quad (1)
\]

\[
\mathbf{v} = [v(t) + y_i(x(t), t)]j; \quad (2)
\]

where \( j \) was the unit vector in the \( x \)-axis direction, \( y_i(x(t), t) \) is the derivative of \( y(x(t), t) \) with respect to \( t \), and \( y_i \) represented \( y(x(t), t) \) and \( y_i(x(t), t) \), respectively.

Similarly, the displacement vector and velocity vector of the car in the \( x \)-axis direction were as follows:

\[
\mathbf{r}_c = [l(t) + y]j; \quad (3)
\]

\[
\mathbf{v}_c = [v(t) + y]j. \quad (4)
\]

The kinetic energy of the system was expressed as follows:

\[
E_k = \frac{1}{2}mV^2\bigg|_{x=l(t)} + \frac{1}{2}P_1\int_0^{l(t)} V^2\,ds. \quad (5)
\]

The elastic potential of the system was:

\[
E_s = \int_0^{l(t)} \left(P_y x + \frac{1}{2}EA_y \right)\,ds; \quad (6)
\]

where \( y_s(x(t), t) \) was the derivative of \( y(x(t), t) \) with respect to \( x \), and \( y_s \) represented \( y_s(x(t), t) \). \( P \) was the tensile force of the rope during a static balance of tension. With its own gravity and that of the car, the hoisting rope was subjected to the tension of the tensioned rope and the tensile force applied by the tensioning device. Thus, tension in the static equilibrium was expressed as:

\[
P = \left[ m + \rho_1 (l(t) - x) + \rho_2 (l_0 - l(t)) \right] g + f. \quad (7)
\]

The gravitational potential energy of the system was expressed as:

\[
E_g = -\int_0^{l(t)} \rho_1 g dt - mg |_{x=l(t)}. \quad (8)
\]

According to the Hamilton principle:

\[
I = \int_{t_1}^{t_2} [\delta E_k - \delta E_s - \delta E_g] \,dt = 0.
\]

The use of time and geometric boundary conditions yielded the following:

\[
\delta y(0, t) = \delta y(x, t_1) = \delta y(x, t_2) = 0.
\]

The derivation law of the parametric integral and partial integration method was applied as follows:

\[
\frac{\partial}{\partial t} \int_0^{l(t)} \rho (v + y') \delta y \,dx = \int_0^{l(t)} \rho_1 (v + y') \delta y \,dx + \int_0^{l(t)} \frac{\partial}{\partial t} \rho_1 (v + y') \delta y \,dx + \rho_1 (v + y') \delta y \bigg|_{x=l(t)}; \quad (9)
\]

and deducted from the following:

\[
\int_{t_1}^{t_2} \int_0^{l(t)} \left[ \rho_1 (y_{tt} + a) - P_x - \rho_1 g - EA_y xy \right] \delta y \,dx \,dt + \int_{t_1}^{t_2} \left[ m(a + y_{tt}) + \rho_1 v(v + y) - mg + P + EA_y x \right] \delta y \bigg|_{x=l(t)} \,dt = 0. \quad (10)
\]

The longitudinal vibration of the high-speed traction elevator lifting system was obtained using the dynamic equation as follows:

\[
\rho_1 \left( y_{tt} + a \right) - P_x - \rho_1 g - EA_y xy = 0, \quad 0 < x < l(t); \quad (11)
\]

\[
m(a + y_{tt}) + \rho_1 v(v + y) + EA_y x - mg = 0, \quad x = l(t). \quad (12)
\]

Equation (12) is the boundary condition of the cord at \( x = l(t) \).

### 2.2. Galerkin Discretization of the Time-Varying Partial Differential Equation for the Longitudinal Vibration of the Lift System

The parameters of the partial differential control Eq. (11) with infinite degrees of freedom present strong time-varying characteristics. As a result, it was difficult to get their analytical solution. The coefficient matrix of the algebraic equations obtained by the Galerkin discrete method was symmetric and the approximation accuracy was higher than those of the other methods. Therefore, the Galerkin method was used to discretize the partial differential control equation. Then, differential equations with infinite degrees of freedom were discretized into ordinary differential equations with finite degrees of freedom and solved by numerical methods. For facilitating the discrete method, a dimensionless parameter \( \xi \) was introduced and the original variables were normalized, which was \( \xi = x/l(t) \). The time domain of \( x \) became the fixed domain \([0, 1] \) of \( \xi \). Assuming that distribution function \( y \) (with infinite degrees of freedom) can be used to represent the solution of Eq. (11):

\[
y(x, t) = \sum_{i=1}^{n} \varphi_i(\xi)q_i(t) = \sum_{i=1}^{n} \varphi_i \left( \frac{x}{l(t)} \right) q_i(t). \quad (13)
\]

\( \varphi_i(\xi) \) was the trial function, and \( q_i(t) \) was the time-dependent generalized coordinates

\[
\varphi_i(\xi) = \sqrt{2}\sin \left( \frac{2i-1}{2} \pi \xi \right), \quad (i = 1, 2, \ldots, n). \quad (14)
\]

Then

\[
y_x = \frac{1}{l(t)} \sum_{i=1}^{n} \varphi_i'(\xi) q_i(t); \quad (15)
\]

\[
y_{xx} = \frac{1}{l(t)} \sum_{i=1}^{n} \varphi_i''(\xi) q_i(t); \quad (16)
\]

\[
y_t = \sum_{i=1}^{n} \varphi_i(\xi) q_i(t) - \frac{\xi v}{l(t)} \sum_{i=1}^{n} \varphi_i'(\xi) q_i(t); \quad (17)
\]

\[
y_{tt} = \sum_{i=1}^{n} \varphi_i(\xi) q_i(t) - \frac{2\xi v^2}{l(t)} \sum_{i=1}^{n} \varphi_i'(\xi) q_i(t) + \frac{\xi^2 v^2}{l(t)} \sum_{i=1}^{n} \varphi_i''(\xi) q_i(t) + \frac{\xi^2 v^2}{l(t)} \sum_{i=1}^{n} \varphi_i'\left( \frac{x}{l(t)} \right) q_i(t). \quad (18)
\]

Equation (15) was substituted into the kinetic Eq. (12). Both sides were multiplied by \( \varphi_i(\xi) \), and \( \xi \) was integrated in the
where \( q_i = [q_1(t), q_2(t), \ldots, q_n(t)] \) was the generalized coordinate vector,
\[
M = \rho_0 \delta_{ij} + \frac{m}{l^2} \varphi_i(1) \varphi_j(1);
\]
\[
C = -\frac{2\rho_1 v}{l} \int_0^l \xi \varphi'_i \varphi'_j d\xi + \rho_0 v \frac{1}{l} \varphi_i(1) \varphi_j(1);
\]
\[
K = \frac{m v^2}{l^3} \varphi''_i(1) \varphi_j(1) - \rho_0 a \frac{v^2}{l} \int_0^l \xi \varphi'_i \varphi'_j d\xi - \rho_1 v \frac{1}{l^2} \int_0^l \xi \varphi'_i \varphi'_j d\xi - \frac{\rho_1 v^2}{l} \int_0^l \xi \varphi'_i \varphi'_j d\xi - E A \int_0^l \varphi''_i \varphi''_j d\xi;
\]
\[
F = -\rho_0 a \int_0^l \varphi_j d\xi - \frac{m a}{l} \varphi_i(1) - \rho_1 a \frac{v}{l} \varphi_j(1) - \rho_2 g \left( l_0 - l \right) + f(t) \varphi_j(1).
\]

3. PRECISE INTEGRATION FOR THE TIME VARYING MODEL OF LONGITUDINAL VIBRATION OF THE LIFT SYSTEM

For the solution of the \( n \)-dimensional nonlinear time-varying differential equation shown in Eq. (16), the general numerical method cannot achieve high accuracy because of its time-varying characteristics. Precise integration was used to solve the problem and achieve highly accurate results.

First, follow the introduction of the dual variable of Hamiltonian system. Make
\[
p = M \dot{z} + C(t) \dot{z} / 2 \quad \text{or} \quad \dot{z} = M^{-1} p - M^{-1} C(t) x / 2.
\]
By substituting the above formula into the dynamic equation, the following equation can be obtained:
\[
\dot{p} = (C(t) M^{-1} C(t) / A - K(t)) x - C(t) M^{-1} p / 2 + f(t).
\]

The above equations were written in the general form of a linear system
\[
\begin{align*}
\dot{z} &= A x + C p + r_x \\
\dot{p} &= B x + D p + r_p
\end{align*}
\]
where:
\[
A = M^{-1} C(t) / 2, \quad B = C(t) M^{-1} C(t) / A - K(t), \quad C = -C(t) M^{-1} / 2, \quad D = M^{-1}, \quad r_x = 0, \quad r_z = f(t).
\]
Therefore,
\[
\dot{z} = H z + \phi(t);
\]
where:
\[
z = \begin{bmatrix} x \\ p \end{bmatrix}, \quad H = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \quad \phi(t) = \begin{bmatrix} r_x \\ r_p \end{bmatrix}.
\]
Given that the non-homogeneous term is linear in the time step \((t_k, t_{k+1})\), the equation was
\[
\dot{z} = H z + \phi_k + \phi_k (t - t_k).
\]
Then, the solution at \( t_{k+1} \) moment was written as
\[
z_{k+1} = T_k \left[ z_k + H_k^{-1} \left( \phi_k + H_k^{-1} \phi_k \right) \right] - H_k^{-1} \left[ \phi_k + H_k^{-1} \phi_k + \phi_k (t_{k+1} - t_k) \right];
\]
\[
\text{Table 1. Elevator movement profile regions.}
\]
<table>
<thead>
<tr>
<th>Stage</th>
<th>Time required</th>
<th>Stage description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( t_1 )</td>
<td>Acceleration increases to ( a = -a_{\text{max}} )</td>
</tr>
<tr>
<td>2</td>
<td>( t_2 )</td>
<td>Acceleration remains ( -a_{\text{max}} ) constant</td>
</tr>
<tr>
<td>3</td>
<td>( t_3 )</td>
<td>Acceleration decreases to zero, ( v = -v_{\text{max}} )</td>
</tr>
<tr>
<td>4</td>
<td>( t_4 )</td>
<td>Speed ( v_{\text{max}} ) is unchanged</td>
</tr>
<tr>
<td>5</td>
<td>( t_5 )</td>
<td>Acceleration increases to ( a = a_{\text{max}} )</td>
</tr>
<tr>
<td>6</td>
<td>( t_6 )</td>
<td>Acceleration remains ( a_{\text{max}} ) constant</td>
</tr>
<tr>
<td>7</td>
<td>( t_7 )</td>
<td>Acceleration decreases to zero, ( v = 0 )</td>
</tr>
</tbody>
</table>

where \( T_k = e^{H_k (t_{k+1} - t_k) \lambda_0} \). According to the given initial conditions of \( z_0 \) and with a constant step size \( t_{k+1} - t_k \), the steps were gradually performed to obtain \( z_1, z_2, \ldots, z_k \) by Eq. (22), which is a typical “self-starting” algorithm.

4. DESCRIPTION OF THE PARAMETERS FOR THE LONGITUDINAL VIBRATION DYNAMIC EQUATION

The longitudinal vibration control Eq. (16) of the lift system of high-speed elevators showed that the structural parameters (hoisting weight, hoisting rope density, drag rope elastic modulus, hoisting rope cross-sectional area, and compensating rope density) and the operation state parameters (travel, speed, acceleration, and acceleration) of the elevator in the simulation and analysis of the dynamic equation of the high-speed elevator hoisting system were necessary inputs. In theoretical analysis, parameter values cannot be determined arbitrarily. Proper parameter selection was the premise of the dynamic behavior analysis of the lifting system and the key to reflect the vibration characteristics of the lift system.

4.1. Elastic Modulus of the Hoisting Rope

The values of elastic modulus given in current literature were somewhat different. Feyree experimentally measured the elastic modulus of a hoisting rope and recorded \( 8.3 \sim 12.5 \times 10^{10} \text{N/m}^2 \). The elevator lifting system included a rope spring and a car bottom vibration rubber, and the stiffness of these parts was less than that of the rope. Therefore, the elastic modulus of the hoisting rope was calculated in this study as \( E = 8 \times 10^{10} \text{N/m}^2 \).

4.2. High-Speed Elevator Operating Status Parameters

Because this article studied high-speed passenger elevator, high-speed and super high-speed elevators often only run in two specific floors. There was no stopping in the whole operation process. Therefore, the high-speed elevator operating state parameters here were the whole process of operation parameters during the upward and downward process.

The elevator uplink process was divided into seven stages according to the ideal operating curve of the elevator. Table 1 describes the curve phases of the lift operation and the operating curve of the elevator in each stage by fitting the ideal operating state into a quintic polynomial Eq. (23)
\[
l_i(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 + C_4 t^4 + C_5 t^5
\]
5.1. Dynamic Characteristics of the Longitudinal Vibration During Upward and Downward Motion of the Elevator

Given the hoisting quality of a single hoisting rope of \( m = 400 \text{ kg} \) and tensile force of \( F = 300 \text{ N} \), the dynamic characteristics of the elevator upward and downward processes are analysed.

Figure 4 shows the dynamic characteristics of the elevator during the uplink process. The length of elevator hoisting rope gradually decreases as the elevator rises. However, the acceleration peak of the longitudinal vibration response gradually increases. The response displacement of the longitudinal vibration of the elevator repeatedly mutates during the upward process. It can be found from Fig. 4 that, during the upward process, the longitudinal vibration displacement of the high-speed elevator changes suddenly when the elevator runs 0 \(- 1.3 \text{ s, 6.6 \(- 7.9 \text{ s, 29.9 \(- 31.2 \text{ s, 36.7 \(- 38.8 \text{ s}} \). Comparing with the upward acceleration curve of the high-speed elevator in Fig. 3, it is found that the abrupt change in time for the elevator vibration displacement coincides with the change in time for the elevator acceleration. Based on this, it can be concluded that the longitudinal vibration is closely related to the acceleration of operation. Therefore, in the installation process of high-speed elevator, under the condition of meeting the design requirements, the elevator comfort can be improved by reducing the tension of the elevator.

Figure 5 shows the dynamic characteristics of the elevator during the downward process. The length of the elevator hoisting rope gradually increases with the downward movement of the elevator. The acceleration peak of longitudinal vibration response gradually decreases and the elevator longitudinal vibration gradually weakens. As shown in Fig. 5, during the downward process, the longitudinal vibration displacement of the high-speed elevator changes abruptly when the elevator runs 0 \(- 1.3 \text{ s, 6.6 \(- 7.9 \text{ s, 29.9 \(- 31.2 \text{ s and 36.7 \(- 38.8 \text{ s}} \). Compared with the downward acceleration curve of the high-speed elevator in Fig. 3, it can be determined that the longitudinal vibration of the elevator is closely related to the change of its running acceleration.

5.2. Effect of Tensile Force \( F \) on the Longitudinal Vibration of the Elevator

With the hoisting of a single hoisting rope of \( m = 400 \text{ kg} \), the influence of tensile force \( F \) on the longitudinal vibrations of the elevator is analysed.

Figures 6(a), 6(b), and 6(c) show the acceleration responses of longitudinal vibration during the elevator upward and downward processes with tensile forces of \( F = 300, 400, \) and \( 500 \text{ N} \). The response of longitudinal vibration acceleration of the upstream and downstream processes in Figs. 6(a), 6(b), and 6(c) is analysed. With the increasing tensile force of the tensioning system, the amplitude of the acceleration response increases.
5.3. Influence of the Lifting Quality (Including Car, Car Frame, and Rated Load) of a Single Hoisting Rope on the Longitudinal Vibration of the Elevator

A tensile force of \( F = 300 \) N of the tension system is considered; then, the dynamic characteristics of longitudinal vibration with the lifting quality values of a single hoisting rope of 350, 400, and 450 kg are analysed.

Figures 7(a), 7(b), and 7(c) show the responses of the longitudinal vibration acceleration during the upward and the downward process with \( m \) of 350, 400, and 450 kg, respectively. Contrary to the vibration acceleration response in the uplink and downlink processes, the amplitude of the acceleration response of the longitudinal vibration becomes increasingly small regardless of the direction of elevator movement. In other words, with a large mass of elevator car, the longitudinal vibration of the elevator is weak. Therefore, in the tensile
Figure 6. Effect of tensile force on longitudinal vibration of the elevator.
Figure 7. Effect of the lifting quality of a single rope on the longitudinal vibration of the elevator.
range of the traction rope, the longitudinal vibration of the elevator can be reduced by appropriately increasing the weight of the car during elevator design.

6. CONCLUSIONS

1. Aiming at the nonlinear time-varying characteristics of the traction rope of the high-speed traction elevator hoisting system, considering the weight of the compensation rope and the tension force of the tensioning device, through studying the longitudinal vibration of the lifting system of the high-speed traction elevator, the time-varying dynamic model of the longitudinal vibration of high-speed traction elevator systems with variable mass, variable damping, and variable stiffness is established by means of the energy method and Hamilton principle.

2. The ideal operating state of the elevator, as fitted into the quintic polynomial, was utilized as an input motion parameter for simulating the longitudinal vibration time-varying dynamic response of the lifting system. Following this, the simulation analysis of the nonlinear time-varying dynamic equation of the high-speed elevator lifting system was conducted using the precise integration method. It is observed that, during operation, as the length of the hoist rope increases, the acceleration of the longitudinal vibration of the elevator increases too. As the tensile force, provided by the tensioning device, increases, the acceleration decreases.

3. This study establishes a time-varying dynamic model of longitudinal vibration in a high-speed elevator and analyses the dynamic characteristics. This work further improves the longitudinal vibration of high-speed traction elevator systems with arbitrarily varying length, longitudinal vibration and energetics on flexible hoisting rope with time-varying length, dynamic modeling and vibration control of high-speed traction elevator hoisting system, considering the weight of the compensation rope and the tension force of the tensioning device, through studying the longitudinal vibration of the lifting system of the high-speed traction elevator, the time-varying dynamic model of the longitudinal vibration of high-speed traction elevator systems with variable mass, variable damping, and variable stiffness is established by means of the energy method and Hamilton principle.

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