Effects of Elliptical Ring Electrodes on Shear Vibrations of Quartz Crystal Plates

Rongxing Wu
Piezoelectric Device Laboratory, School of Mechanical Engineering and Mechanics, Ningbo University, Ningbo, Zhejiang 315211, China.
Department of Architectural Engineering, Ningbo Polytechnic, Ningbo, Zhejiang 315800, China.

Ji Wang and Jianke Du
Piezoelectric Device Laboratory, School of Mechanical Engineering and Mechanics, Ningbo University, Ningbo, Zhejiang 315211, China.

Jiashi Yang
Piezoelectric Device Laboratory, School of Mechanical Engineering and Mechanics, Ningbo University, Ningbo, Zhejiang 315211, China.
Department of Mechanical and Materials Engineering, the University of Nebraska-Lincoln, Lincoln, NE 68588-0526, U.S.A.

(Received 10 July 2017; accepted 6 November 2017)

A theoretical analysis is performed on the thickness-shear vibrations of an AT-cut quartz piezoelectric crystal plate with elliptical ring electrodes. The scalar differential equation by Tiersten and Smythe is used. An analytical solution is obtained. Numerical results from the solution show that the thickness-shear mode of interest may be trapped by the ring electrodes and can have a convex, concave, or nearly flat vibration distribution near the plate center, which is fundamentally important when the plate is used as an acoustic wave mass sensor. The vibration distribution is found to be sensitive to both the geometric and physical parameters of the electrodes. Therefore, a careful design is needed to realize the desired trapped mode with suitable center convexity for sensor application.

1. INTRODUCTION

Crystal plates can be used to make acoustic wave resonators as frequency standards for both timekeeping and frequency operations with broad applications. They have been under sustained study experimentally, numerically and theoretically (see the review1 and, more recently2–14). Many crystal resonators operate with the thickness-shear (TSh) mode of an AT-cut quartz plate.15, 16 The resonant frequencies of a crystal plate can be affected by many effects such as a temperature change, stress or stain, a surface mass layer, or contact with a fluid, etc. Therefore, the detection of frequency shifts in a crystal plate can be used as the basis for making various acoustic wave sensors.17, 18

This paper is concerned with mass sensors based on quartz plates vibrating in TSh modes. They are called quartz crystal microbalances (QCMs). In QCMs, the inertia of an additional thin mass layer lowers the resonant frequencies of the plate. The sensitivity of QCMs was given by the well-known Sauerbrey equation, which was based on pure TSh modes without in-plane mode variations.17 Pure TSh modes can only exist in an unbounded and uniform plate with an additional mass layer that is also uniform and unbounded. However, in real QCMs of finite plates, the operating modes in fact have slow in-plane variations due to the plate boundaries. These in-plane mode variations cause deviation from the frequency prediction by the Sauerbrey equation. In-plane mode variations also arise when the additional mass layers or electrodes cover the crystal plates partially in the central region. Typically, the vibration of the operating mode in a QCM has a bell-shape distribution similar to the normal distribution in statistics, large near the plate center where the additional mass layer or the electrode is, and decays away from the mass layer or electrode edge.

An additional mass layer on a plate can be viewed as a continuous distribution of mass points. In the ideal case of an unbounded crystal plate in which the TSh mode is uniform in the plane of the plate, the frequency response of the plate to a point mass is independent of the location of the mass. However, in a finite plate with in-plane mode variations, the frequency response of the plate to a point mass depends on the location of the mass. Near the plate center where the vibration amplitude is relatively large, so is the frequency shift or sensitivity caused by the point mass there. Qualitatively, the sensitivity distribution roughly follows the vibration amplitude distribution, large near the plate center and small near the plate edges. Ideally, for mass sensor application, a nearly uniform vibration distribution with little in-plane mode variation is desired. In QCMs, researchers have made use of circular and rectangular ring electrodes to achieve nearly flat or uniform vibration distribution in resonators.20–25

It has been known that, because of the in-plane material anisotropy of a quartz plate, the optimal shape of an AT-cut quartz plate device or its electrodes is not exactly circular and instead is almost elliptical,26 with the major axis exceeding the minor axis by approximately 26%. Existing theoretical re-