Modal Analysis of Mistuned Turbine Blade Packet Due to Combined Blade and Lacing Wire Damage

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The turbine disk blade system is a cyclic symmetric structure, initially tuned with all its blades perfectly identical in geometry and material properties; similarly interconnecting lacing wires are of equal stiffness. The cyclic symmetry of the bladed disks gets destroyed due to small differences in material properties or geometric variation between individual blades or lacing wires causing mistuning. Although mistuning is typically small, it can have a drastic effect on the dynamic response of the system. In particular, mistuning can also cause vibration localization for a few blades and the associated concentration of vibration energy can lead to an increase in blade amplitude and stress levels. Numerical simulations are performed with the characteristic equations of the simplified continuum model. Two different damage severity indices are included in the model to study the combined effect of cracked blades and damaged lacing wires on the natural frequencies of grouped blades. This study highlights the characteristic changes in the sub modal frequencies under combined damage in a stand still position. Although the major cause of mistuning is blade damage, lacing wire damage is more frequent and often acts as a precursor to blade damage and thus the present study focuses on mistuning due to combined damage.

NOMENCLATURE

- $A$: Cross sectional area of blade in m$^2$
- $E$: Modulus of elasticity in GPa
- $I$: Moment of inertia in m$^4$
- $K$: Stiffness of blade in N/m
- $L$: Length of blade in m
- $a$: Crack location from the root
- $b$: Crack location from the free end
- $f$: Natural frequency of vibration in Hz
- $k$: Stiffness of lacing wire/spring
- $x$: Translational coordinate
- $y$: Transverse displacement
- $c_f$: Local flexibility coefficient due to crack
- $\omega$: Natural frequency of vibration in rad/sec
- $\phi$: Crack Severity
- $\alpha$: Lacing wire damage severity ratio
- $\beta L$: Non dimensional angular frequency
- $\gamma$: Non dimensional crack flexibility coefficient
- $\rho$: Density of blade material in kg/m$^3$
- $\lambda$: Relative stiffness ratio of lacing wire to blade
- $\mu$: Non dimensional crack location parameter (a/L)

1. INTRODUCTION

The effect of mistuning on turbo machine blade vibration in a grouped blade-disk system has been a widely researched area for the last two to three decades. Research in this area has gained importance, essentially due to the critical applications of turbo machines in aero engines and power plants. It has been found that even a small amount of mistuning can lead to a stress build up through mode localization under forced vibration. During the operational life stages of the system, mistuning is caused due to the presence of blade to blade variation in geometry, manufacturing tolerances or the evolution of cracks in the blade and/or damage in the lacing wire.

Prohl was first to study the dynamics of blade groups using lumped parameter modeling of blades with shroud rings attached to the blade tips.\textsuperscript{1} In 1924, Campbell wrote a breakthrough paper. This is considered to be the first work to explain mode localization in bladed disk assemblies. Experimental results were used to support the phenomenon explained theoretically by the presence of travelling and standing waves.\textsuperscript{2} Weaver and Prohl used the energy method and presented the modal characteristics of a packet of blades. They observed that the blade disk system had more modes and frequencies than the single blade.\textsuperscript{3} Deak and Baird analyzed a blade packet interconnected by lacing wire, using a coupled model with two bending and one torsional mode. They also studied the effect of root flexibility and centrifugal stiffening.\textsuperscript{4} Montoya developed the equations of motion to calculate the dynamic stiffness matrix for the coupled bending and torsional vibration of a twisted blade.\textsuperscript{5} A model was formulated using the variational method by Rao, in which governing differential equations were derived from an energy integral using Hamilton’s principle.\textsuperscript{6} Huang developed a computational procedure for calculating the free vibration of rotationally cyclic structures with various types of connecting elements using the transfer matrix method.\textsuperscript{7} Ewins and Imregun used method of substructure synthesis with receptance coupling.\textsuperscript{8} Mercadal et al. studied the issues that arise in blade resonance identification using Non-contacting Stress Monitoring Systems (NSMS) when blade resonances have slight variation causing mistuning and are dynamically coupled.\textsuperscript{9} Grossi et al. used the calculus of variations to obtain the equations of motion and natural boundary conditions at the intermediate elastic constraints like lacing wire connections.\textsuperscript{10,11} Wang et al. investigated the minimum stiffness of additional support that raises the natural frequency of a beam to its upper limit for different boundary conditions.\textsuperscript{12} Petreski presented the results of the investigation of the dynamic behavior, i.e., the natural frequencies and mode shape changes for a group of two, three and five blades as a result of changes made with the lacing wire.\textsuperscript{13} In recent work, Lim et