Research on Dynamic Model of Double Helical Gear Pair Based on TCA and LTCA

Cheng Wang, Shouren Wang and Gaoqi Wang

School of Mechanical Engineering, University of Jinan, Jinan 250022, China.

(Received 27 October 2018; accepted 2 February 2018)

Numerous dynamic models of spur gears, helical gears, bevel gears, and face gears can be found in various studies. However, studies that focus on the dynamic model of a double helical gear pair are quite limited. The author proposed a model of a double helical gear pair by only considering the axial vibration. The author did not consider the friction and multiple backlashes in the proposed model. The friction force of the tooth surface and backlash are important factors that can cause complex non-linear phenomena in gear pairs. Therefore, a dynamic model of a double helical gear pair that takes into consideration the axial vibration, friction and multiple backlashes is proposed. Firstly, based on the tooth contact analysis (TCA) of a double helical gear pair, the path of contact and meshing time from engagement to disengagement are obtained. The formula for determining the sliding friction coefficient is introduced. Based on TCA and the dynamic meshing force provided by the subsequent dynamics model of double helical gear pair, the sliding friction coefficient of the tooth surface is calculated. Secondly, the stiffness excitation, gear-into impact excitation and error excitation (including the axial displacement caused by the errors of manufacture and installation under low speed) are calculated according to the existing research results. Following this, a dynamic model of a double helical gear pair that takes into consideration the axial vibration, friction and multiple backlashes is both built and solved. Finally, an example is presented to verify the corresponding results.

NOMENCLATURE

- \( j = 1, 2 \) subscript
- \( i = p, q \) subscript
- \( n = x, y, z \) subscript
- \( \zeta = h, p, q \) subscript
- \( l = 1, 2, \ldots, 12 \) subscript
- \( y_{ij} \) translational vibration displacement of two ends helical gear center point along the y-direction (Fig. 4)
- \( z_{ij} \) translational vibration displacement of two ends helical gear center point along the z-direction (Fig. 4)
- \( x_{ij} \) translational vibration displacement of two ends helical gear center point along the x-direction (Fig. 4)
- \( \theta_{ij} \) angular vibration displacement of two ends helical gear center point (Fig. 4)
- \( m \) equivalent mass of double helical gear pair
- \( b_l \) half backlash
- \( k_j \) torsional stiffness of two ends gear pair
- \( c_j \) torsional damping of two ends gear pair
- \( k_{p1x}, k_{p1y}, k_{q1x}, k_{q1y}, k_{q1z}, k_{p2x}, k_{p2y}, k_{q2x}, k_{q2y}, k_{q2z} \), Support stiffness
- \( c_{p1x}, c_{p1y}, c_{q1x}, c_{q1y}, c_{q1z}, c_{p2x}, c_{p2y}, c_{q2x}, c_{q2y}, c_{q2z} \), Support damping
- \( k_{yz} \) meshing stiffness along the tangential direction
- \( c_{yz} \) meshing damping along the tangential direction
- \( k_{jz} \) meshing stiffness along the axial direction
- \( c_{jz} \) meshing damping along the axial direction
- \( e_{jy} \) meshing error along the tangential direction
- \( e_{jz} \) meshing error along the axial direction
- \( k_{qx} \) torsional stiffness along X-direction of pinion shaft (Fig. 4)
- \( c_{qy} \) torsional damping along X-direction of pinion shaft
- \( k_{py} \) torsional stiffness along Y-direction of pinion shaft (Fig. 4)
- \( c_{py} \) torsional damping along Y-direction of pinion shaft
- \( k_{qz} \) tensile (compression) stiffness along the Z-direction of pinion shaft (Fig. 4)
Numerous analytical models on gear dynamics, including spur, helical, bevel, face gears etc., can be found in various studies. However, studies that focus on a double helical gear pair are quite limited. Jauregui proposed a single-degree dynamic model. Ajmi proposed a twelve-degree dynamic model. Sondkar built a dynamic model of a double-helical planetary gear set. Kahraman proposed a linear, time-invariant dynamic model of double helical gear pair systems including shafts and bearing supports. In the above models, the axial vibration caused by axial displacement under low speed and tooth surface friction is not considered. Although Wang proposed a dynamic model of a double helical gear pair in consideration of axial vibration, the friction force and multiple backlashes are not considered.

The friction force of the tooth surface, backlash and time-varying meshing stiffness are important factors that cause complex non-linear phenomena in gear pairs. For double helical gear pairs, the axial displacement under low speeds should also be considered. Combining TCA and loaded tooth contact analysis (LTCA), the axial displacement caused by the errors of manufacture and installation under low speeds can be obtained, and then the error excitation, stiffness excitation and impact excitation can be calculated. Therefore, the specific objectives of this study are as follows:

1. Based on TCA of double helical gear pair, the path of contact and meshing time from engagement to disengagement are obtained. The formula for determining the sliding friction coefficient is introduced. Based on TCA and the dynamic meshing force provided by the subsequent dynamics model of a double helical gear pair, the sliding friction coefficient on tooth surface is calculated out.

2. The dynamic model of a double helical gear pair that takes the axial vibration, friction and multiple backlashes into consideration is built up and solved. Finally, an example is demonstrated.

A flow diagram for the dynamic model of a double helical gear pair is shown in Fig. 1. It should be pointed out that the related research about stiffness excitation, gear-into impact excitation and error excitation can be found in reference. The TCA and LTCA of a double helical gear pair can be found in references. The related research of TCA and LTCA can be also found in references.
Elastohydrodynamic lubrication model. S. Baglioni used this formula to calculate the variable friction coefficient of the addendum modification on spur gear. The calculation of sliding friction coefficient was represented as

\[ \mu = \frac{f(SR, P_h, v_0, S)P_h^2||SR||^3v_0^2e^{v_0R^2}}{R^2}; \] (1)

\[ f(SR, P_h, v_0, S) = b_1 + b_2|SR|P_h \log_{10}(v_0) + b_3e^{\lambda P_h}; \] (2)

In this study, 75W90 gear oil (a typical gear oil) was used, \( b_i (i = 1, 2, \ldots, 9) = -8.92, 1.03, 1.04, -0.35, 2.81, -0.10, 0.75, -0.39, \) and 0.62. \( P_h = \sqrt{W/E/(2\pi R)} = Z_R/F/(IR); \) \( SR = \frac{v_r}{v_e}. \) The calculation of \( v_e \) and \( v_r \) was presented in reference. Corresponding parameters can be obtained by TCA, LTCA and references.

2.2. The Direction Judgment of Friction Force

According to the projecting velocity of driving and driven gears in the vertical line of the meshing line (Fig. 2), the friction force was the negative value in the \( PN_1 \) segment and was the positive value in the \( PN_2 \) segment. Where \( P \) was the pitch point, \( N_1 \) and \( N_2 \) were the tangent points of the meshing line and the base circle. In this case: \( \theta_1 \) was the angular velocity of gear 1, \( \omega_2 \) was the angular velocity of gear 2, \( r_{a1} \) was the radius of base circle of gear 1, \( r_{a1} \) was the radius at the \( i \) point of gear 1, \( v_{b1} \) was the velocity at the \( k \) point of gear 1, \( u_{k1} \) was the velocity component of \( u_{k1} \), \( v_{b1} \) was the velocity at the \( i \) point of gear 1, \( v_{k1} \) was the velocity component of \( v_{k1} \), \( r_{i2} \) was the radius of base circle of gear 2, \( r_{k2} \) was the radius at the \( k \) point of gear 2, \( v_{k2} \) was the velocity at the \( k \) point of gear 2, \( u_{k2} \) was the velocity component of \( u_{k2} \), \( v_{i2} \) was the velocity at the \( i \) point of gear 2, and \( u_{i2} \) was the velocity component of \( v_{i2} \).

2.3. The Calculations of Distance Between Friction Force and Gear Center

According to Fig. 3, the calculation of distance between friction force and gear center were represented as

\[
\begin{align*}
    s_1 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - \sqrt{r_{pow}^2 - r_{qow}^2} - \lambda + \theta_og - \theta_og - \theta_og - e_y \\
    s_2 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - s_1 \\
    \lambda &= y_2 + \theta_og - y_p - \theta_og - e_y
\end{align*}
\] (3)

Where, \( \lambda \) was the relative displacement between meshing points of two teeth surfaces along the meshing line. \( y_p, y_q \) were the translational vibration displacements of two ends of helical gear center points along the meshing line. \( \theta_og, \theta_og \) were the angular vibration displacements of two ends of helical gear center points, \( e_y \) was the meshing error along the meshing line. In Fig. 3, \( B \) was the width of the teeth, \( \beta \) was the helical angle.

3. THE DYNAMIC MODEL OF DOUBLE HELICAL GEAR PAIR CONSIDERING AXIAL VIBRATION, FRICTION AND MULTIPLE BACKLASHES

3.1. Dynamic Model of a Double Helical Gear Pair

The model of dynamic analysis for double helical gear pair is shown in Fig. 4 (Where, the torsional stiffness, the torsional damping, the bending stiffness and the bending damping of axis are not shown). Considering the friction force of contact tooth surface, the dynamic model of double helical gear pair was a sixteen-degree vibrating system. The sixteen degrees were represented as

\[
\delta = \begin{bmatrix} x_{p1} & y_{p1} & z_{p1} & \theta_{p1} & x_{q1} & y_{q1} & z_{q1} & \theta_{q1} & x_{p2} & y_{p2} & z_{p2} & \theta_{p2} & x_{q2} & y_{q2} & z_{q2} & \theta_{q2} \end{bmatrix}^T; 
\] (4)
3.2. Descriptions of the Backlash

The existence of backlash generated impact between engaged teeth, which influenced the stability of gear transmission. The clearance function $f$ was expressed as

$$f_{cja}(y) = \begin{cases} 
  y - b_t & y > b_t \\
  0 & y \leq |b_t| \\
  y + b_t & y < -b_t 
\end{cases} \quad (5)$$

For the left gear pair, taking the interaction between the left and right end gear pair into consideration, the dynamic meshing force along the tangential direction and the axial direction were respectively represented as

$$F_{p1y} = k_{p1} f_{h1y} \left( y_{1p} - y_{1q} - e_{y1} \right) + c_{y1} \left( \dot{y}_{1p} - \dot{y}_{1q} - \dot{r}_{y1} \right) = \cos \beta \left[ k_{p1} f_{h1y} \left( y_{1p} - y_{1q} - e_{y1} \right) + c_{y1} \left( \dot{y}_{1p} - \dot{y}_{1q} - \dot{r}_{y1} \right) \right]; \quad (6)$$

$$F_{z1} = k_{z1} \left( z_{1p} - z_{1q} - e_{z1} \right) + c_{z1} \left( \dot{z}_{1p} - \dot{z}_{1q} - \dot{r}_{z1} \right) = \sin \beta \left[ k_{z1} \left( z_{1p} - z_{1q} - e_{z1} \right) + c_{z1} \left( \dot{z}_{1p} - \dot{z}_{1q} - \dot{r}_{z1} \right) \right]; \quad (7)$$

In this case, the relation between the vibration displacement of center point $q_1$, $p_1$, $q_2$, and $p_2$ and generalized displacement of driving gear and driven gear were represented as $y_{1p}$, $y_{1q}$, $z_{1p}$, and $z_{1q}$.

For the right gear pair, considering the interaction between the left and right end gear pair, the dynamic meshing force along the tangential direction and the axial direction were respectively represented as

$$F_{p2y} = k_{p2} f_{h2y} \left( y_{2p} - y_{2q} - e_{y2} \right) + c_{y2} \left( \dot{y}_{2p} - \dot{y}_{2q} - \dot{r}_{y2} \right) = \cos \beta \left[ k_{p2} f_{h2y} \left( y_{2p} - y_{2q} - e_{y2} \right) + c_{y2} \left( \dot{y}_{2p} - \dot{y}_{2q} - \dot{r}_{y2} \right) \right]; \quad (8)$$

$$F_{z2} = k_{z2} \left( z_{2p} - z_{2q} - e_{z2} \right) + c_{z2} \left( \dot{z}_{2p} - \dot{z}_{2q} - \dot{r}_{z2} \right) = \sin \beta \left[ k_{z2} \left( z_{2p} - z_{2q} - e_{z2} \right) + c_{z2} \left( \dot{z}_{2p} - \dot{z}_{2q} - \dot{r}_{z2} \right) \right]; \quad (9)$$

The formula (5) – formula (9) came from reference.24 According to Fig. 4, the dynamic equations of this system were established by the Newton second law.

$$m_p \ddot{x}_{p1} + c_{px1} \dot{x}_{p1} + k_{px1} f_{px1} (x_{p1}) + c_{x2} (\dot{x}_{p1} - \dot{x}_{p2}) + k_{px} (x_{p1} - x_{p2}) = \chi_1 \mu_1 F_{p1y}; \quad (10)$$

$$m_p \ddot{y}_{p1} + c_{py1} \dot{y}_{p1} + k_{py1} f_{py1} (y_{p1}) + c_{y2} (\dot{y}_{p1} - \dot{y}_{p2}) + k_{py} (y_{p1} - y_{p2}) = -F_{p1y}; \quad (11)$$

$$F_{p2z} = k_{p2} f_{h2z} \left( y_{2p} - y_{2q} - e_{y2} \right) + c_{y2} \left( \dot{y}_{2p} - \dot{y}_{2q} - \dot{r}_{y2} \right) = \cos \beta \left[ k_{p2} f_{h2z} \left( y_{2p} - y_{2q} - e_{y2} \right) + c_{y2} \left( \dot{y}_{2p} - \dot{y}_{2q} - \dot{r}_{y2} \right) \right]; \quad (12)$$

$$I_{p1} \ddot{\theta}_{p1} + F_{p1y} R_{p1} + R_{p1} \left[ k_{p1} (\theta_{p1} - \theta_{p2}) + c_{p1} (\dot{\theta}_{p1} - \dot{\theta}_{p2}) \right] = -s_1 \chi_1 \mu_1 F_{p1y} + T_{p1}; \quad (13)$$

$$m_p \ddot{x}_{q1} + c_{qx1} \dot{x}_{q1} + k_{qx1} f_{qx1} (x_{q1}) + c_{x2} (\dot{x}_{q1} - \dot{x}_{q2}) + k_{qx} (x_{q1} - x_{q2}) = -F_{x2}; \quad (14)$$

$$m_q \ddot{y}_{q1} + c_{qy1} \dot{y}_{q1} + k_{qy1} f_{qy1} (y_{q1}) + c_{y2} (\dot{y}_{q1} - \dot{y}_{q2}) + k_{qy} (y_{q1} - y_{q2}) = F_{y2}; \quad (15)$$

$$m_q \ddot{z}_{q1} + c_{qz1} \dot{z}_{q1} + k_{qz1} f_{qz1} (z_{q1}) + c_{z2} (\dot{z}_{q1} - \dot{z}_{q2}) + k_{qz} (z_{q1} - z_{q2}) = F_{z2}; \quad (16)$$

$$I_{q1} \ddot{\theta}_{q1} - F_{q1y} R_{q1} - R_{q1} \left[ k_{q1} (\theta_{q1} - \theta_{q2}) + c_{q1} (\dot{\theta}_{q1} - \dot{\theta}_{q2}) \right] + s_2 \chi_1 \mu_1 F_{p1y} = T_{q1}; \quad (17)$$

$$m_p \ddot{x}_{p2} + c_{px2} \dot{x}_{p2} + k_{px2} f_{px2} (x_{p2}) + c_{x2} (\dot{x}_{p2} - \dot{x}_{p1}) + k_{px} (x_{p2} - x_{p1}) = \chi_2 \mu_2 F_{p2y}; \quad (18)$$

$$m_p \ddot{y}_{p2} + c_{py2} \dot{y}_{p2} + k_{py2} f_{py2} (y_{p2}) + c_{y2} (\dot{y}_{p2} - \dot{y}_{p1}) + k_{py} (y_{p2} - y_{p1}) = -F_{p2y}; \quad (19)$$

$$m_p \ddot{z}_{p2} + c_{pz2} \dot{z}_{p2} + k_{pz2} f_{pz2} (z_{p2}) + c_{z2} (\dot{z}_{p2} - \dot{z}_{p1}) + k_{pz} (z_{p2} + z_{p1}) = -F_{z2}; \quad (20)$$

$$I_{p2} \ddot{\theta}_{p2} + F_{p2y} R_{p2} + R_{p2} \left[ k_{p2} (\theta_{p2} - \theta_{p1}) + c_{p2} (\dot{\theta}_{p2} - \dot{\theta}_{p1}) \right] - s_2 \chi_2 \mu_2 F_{p2y} = -T_{p2}; \quad (21)$$

$$m_p \ddot{x}_{q2} + c_{px2} \dot{x}_{q2} + k_{px2} f_{px2} (x_{q2}) + c_{x2} (\dot{x}_{q2} - \dot{x}_{q1}) + k_{px} (x_{q2} - x_{q1}) = \chi_2 \mu_2 F_{p2y}; \quad (22)$$

$$m_q \ddot{y}_{q2} + c_{qy2} \dot{y}_{q2} + k_{qy2} f_{qy2} (y_{q2}) + c_{y2} (\dot{y}_{q2} - \dot{y}_{q1}) + k_{qy} (y_{q2} - y_{q1}) = F_{y2}; \quad (23)$$

$$m_q \ddot{z}_{q2} + c_{qz2} \dot{z}_{q2} + k_{qz2} f_{qz2} (z_{q2}) + c_{z2} (\dot{z}_{q2} - \dot{z}_{q1}) + k_{qz} (z_{q2} + z_{q1}) = F_{z2}; \quad (24)$$

$$I_{p2} \ddot{\theta}_{p2} - F_{p2y} R_{p2} - R_{p2} \left[ k_{p2} (\theta_{p2} - \theta_{p1}) + c_{p2} (\dot{\theta}_{p2} - \dot{\theta}_{p1}) \right] + s_4 \chi_2 \mu_2 F_{p2y} = T_{p2}. \quad (25)$$
The relative displacement $\lambda_j$ between the meshing points of two teeth surfaces along the meshing line was expressed as

$$\lambda_j = y_{qj} - y_{pj} - e_{yj}. \quad (26)$$

The phase separation of the rotation angle between two ends gear pair $\gamma_i$ was represented as

$$\gamma_i = \theta_{j1} - \theta_{j2}; \quad (27)$$

$\lambda_j$ and $\gamma_i$ were taken as new free degrees. The transformation equations of Eq. (A1)–Eq. (A20) are shown in Appendix 1.

## 4. ILLUSTRATIVE EXAMPLE

The dynamic model of a double helical gear pair was considered as an example. The parameters of a double helical gear pair are given in Table 1. In the example, through preliminary calculation, the displacement in x-direction, y-direction, z-direction and meshing line direction was very small, therefore, all support backlashes were set to 0 and the two ends’ meshing backlashes were set to 0.004 mm.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Pinion</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tooth number</td>
<td>31</td>
<td>102</td>
</tr>
<tr>
<td>Normal modulus / (mm)</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>Normal pressure angle / (°)</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Helical angle / (°)</td>
<td>28.34</td>
<td></td>
</tr>
<tr>
<td>Face width / (mm)</td>
<td>90×2</td>
<td></td>
</tr>
<tr>
<td>Helical direction</td>
<td>Right left</td>
<td>Left right</td>
</tr>
<tr>
<td>Run-out groove width / (mm)</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>Rotation speed of pinion / (r/min)</td>
<td>2881</td>
<td></td>
</tr>
<tr>
<td>Output torque / (N·m)</td>
<td>2000</td>
<td></td>
</tr>
</tbody>
</table>

Based on the results of TCA and LTCA, the meshing synthetic stiffness, meshing stiffness of single-pair teeth, the axial displacement under low rotation speeds and the gear-into impact excitation were calculated. The calculation results can be found in reference.\(^\text{13}\)

### 4.1. The Related Calculations of Friction Force on Tooth Surface

Based on TCA of double helical gear pair, Fig. 5 shows the path of contact on pinion tooth surface. In the example, the meshing time of a pair of teeth from engagement to disengagement is 0.0035271 s.

For the left gear pair, From mesh position 1 to mesh position 5, the distance between friction force and gear center were represented as

$$\begin{align*}
s_1 &= \sqrt{a^2 - (r_p b + r_q b)^2} - \sqrt{r_{qa}^2 - r_{qb}^2 + r_{pb} t} - \lambda \\
s_2 &= \sqrt{a^2 - (r_p b + r_q b)^2} - s_1 \\
\lambda &= y_q + \theta_q r_q - y_p - \theta_p r_p - e_y
\end{align*} \quad (28)$$

From mesh position 5 to mesh position 12, the distance between friction force and gear center were represented as

$$\begin{align*}
s_1 &= \sqrt{r_p^2 - r_b^2} - \lambda \\
s_2 &= \sqrt{a^2 - (r_p b + r_q b)^2} - s_1 \\
\lambda &= y_q + \theta_q r_q - y_p - \theta_p r_p - e_y
\end{align*} \quad (29)$$

From mesh position 12 to mesh position 15, the distance be-
found that the value of $\mu$ near the pitch point was almost zero, which fit the results mentioned by reference.

A double helical gear pair that takes the axial vibration, friction and multiple backlashes into consideration is built up by Newton’s second law and solved by the method of the fourth-order Runge-Kutta algorithm with variable step lengths. The dynamic model of a double helical gear pair is calculated by using the existing formula for determining the sliding friction coefficient, the related results of TCA and the dynamic meshing force provided by the subsequent dynamics model of a double helical gear pair.

The results show that the sliding friction coefficient of the tooth surface as an example, we get the vibratory responses of displacement, speed, and acceleration in time-domain spectrum shown in Fig. 7, respectively. The phase diagram, poincare mapping graph and corresponding frequency responses transformed by FFT are shown in Fig. 8. From Fig. 7 and Fig. 8, the example shows that the bending vibrations (including x-direction, y-direction) are much smaller than axial vibration and torsional vibration. The example confirms that the axial vibration should not be neglected. Lastly, the displacements in each direction of x, y, z and meshing line in the example is very small. Therefore, all support backlashes and meshing backlashes should avoid being too large.

5. CONCLUSIONS

Based on the TCA of a double helical gear pair, the contact path and meshing time from engagement to disengagement is obtained. The sliding friction coefficient of the tooth surface is calculated by using the existing formula for determining the sliding friction coefficient, the related results of TCA and the dynamic meshing force provided by the subsequent dynamics model of a double helical gear pair. The dynamic model of a double helical gear pair that takes the axial vibration, friction and multiple backlashes into consideration is built up by Newton’s second law and solved by the method of the fourth-order Runge-Kutta algorithm with variable step lengths. A pair of double helical gear pairs are taken as an example. The distances between the friction force, gear center, and sliding friction were similar results on the right tooth surface. In this example, the distance between friction force and gear center were represented as

$$
\begin{align*}
    s_1 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - \sqrt{q_{pa}^2 - r_{qb}^2 + r_{pb}w_p} - \lambda \\
    s_2 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - s_1 \\
    \lambda &= y_q + \theta_q r_q - y_p - \theta_p r_p - e_y
\end{align*}
$$

(30)

Where, $t_1$ was the time from position 5 to position 12. For the right gear pair, From mesh position 16 to mesh position 20, the distance between friction force and gear center were represented as

$$
\begin{align*}
    s_1 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - \sqrt{q_{pa}^2 - r_{qb}^2 + r_{pb}w_p} - \lambda \\
    s_2 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - s_1 \\
    \lambda &= y_q + \theta_q r_q - y_p - \theta_p r_p - e_y
\end{align*}
$$

(31)

From mesh position 20 to mesh position 27, the distance between friction force and gear center were represented as

$$
\begin{align*}
    s_1 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - \sqrt{q_{pa}^2 - r_{qb}^2 + r_{pb}w_p} - \lambda \\
    s_2 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - s_1 \\
    \lambda &= y_q + \theta_q r_q - y_p - \theta_p r_p - e_y
\end{align*}
$$

(32)

From mesh position 27 to mesh position 30, the distance between friction force and gear center were represented as

$$
\begin{align*}
    s_1 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - \sqrt{q_{pa}^2 - r_{qb}^2} \\
    s_2 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - s_1 \\
    \lambda &= y_q + \theta_q r_q - y_p - \theta_p r_p - e_y
\end{align*}
$$

(33)

Where, $t_1$ was the time from position 20 to position 27.

According to Eq. (1)–Eq. (2), the values of sliding friction coefficient on the left tooth surface are shown in Fig. 6. There were similar results on the right tooth surface. In this example, the meshing time of a pair of teeth from engagement to disengagement was 0.0035271 s. Combined Fig. 5, the meshing time near the pitch point was also shown in Fig. 6. It can be found that the value of $\mu$ near the pitch point was almost zero, which fit the results mentioned by reference.\(^{19}\)

4.2. The Solution of Dynamic Model of Double Helical Gear Pair

The equations are solved by the method of the fourth-order Runge-Kutta algorithm with varying step lengths. The gear-mesh frequency is 1489 Hz. Taking the left end gear pair as an example, we get the vibratory responses of displacement, speed, and acceleration in time-domain spectrum shown in Fig. 7, respectively. The phase diagram, poincare mapping graph and corresponding frequency responses transformed by FFT are shown in Fig. 8. From Fig. 7 and Fig. 8, the example shows that the bending vibrations (including x-direction, y-direction) are much smaller than axial vibration and torsional vibration. The example confirms that the axial vibration should not be neglected. Lastly, the displacements in each direction of x, y, z and meshing line in the example is very small. Therefore, all support backlashes and meshing backlashes should avoid being too large.

5. CONCLUSIONS

Based on the TCA of a double helical gear pair, the contact path and meshing time from engagement to disengagement is obtained. The sliding friction coefficient of the tooth surface is calculated by using the existing formula for determining the sliding friction coefficient, the related results of TCA and the dynamic meshing force provided by the subsequent dynamics model of a double helical gear pair. The dynamic model of a double helical gear pair that takes the axial vibration, friction and multiple backlashes into consideration is built up by Newton’s second law and solved by the method of the fourth-order Runge-Kutta algorithm with variable step lengths. A pair of double helical gear pairs are taken as an example. The distances between the friction force, gear center, and sliding friction were similar results on the right tooth surface. In this example, the distance between friction force and gear center were represented as

$$
\begin{align*}
    s_1 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - \sqrt{q_{pa}^2 - r_{qb}^2 + r_{pb}w_p} - \lambda \\
    s_2 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - s_1 \\
    \lambda &= y_q + \theta_q r_q - y_p - \theta_p r_p - e_y
\end{align*}
$$

(30)

Where, $t_1$ was the time from position 5 to position 12. For the right gear pair, From mesh position 16 to mesh position 20, the distance between friction force and gear center were represented as

$$
\begin{align*}
    s_1 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - \sqrt{q_{pa}^2 - r_{qb}^2 + r_{pb}w_p} - \lambda \\
    s_2 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - s_1 \\
    \lambda &= y_q + \theta_q r_q - y_p - \theta_p r_p - e_y
\end{align*}
$$

(31)

From mesh position 20 to mesh position 27, the distance between friction force and gear center were represented as

$$
\begin{align*}
    s_1 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - \sqrt{q_{pa}^2 - r_{qb}^2 + r_{pb}w_p} - \lambda \\
    s_2 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - s_1 \\
    \lambda &= y_q + \theta_q r_q - y_p - \theta_p r_p - e_y
\end{align*}
$$

(32)

From mesh position 27 to mesh position 30, the distance between friction force and gear center were represented as

$$
\begin{align*}
    s_1 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - \sqrt{q_{pa}^2 - r_{qb}^2} \\
    s_2 &= \sqrt{a^2 - (r_{pb} + r_{qb})^2} - s_1 \\
    \lambda &= y_q + \theta_q r_q - y_p - \theta_p r_p - e_y
\end{align*}
$$

(33)

Where, $t_1$ was the time from position 20 to position 27.

According to Eq. (1)–Eq. (2), the values of sliding friction coefficient on the left tooth surface are shown in Fig. 6. There were similar results on the right tooth surface. In this example, the meshing time of a pair of teeth from engagement to disengagement was 0.0035271 s. Combined Fig. 5, the meshing time near the pitch point was also shown in Fig. 6. It can be found that the value of $\mu$ near the pitch point was almost zero, which fit the results mentioned by reference.\(^{19}\)
Figure 7. The vibratory responses of displacement, speed, and acceleration in time-domain spectrum.

Figure 8. The phase diagram, poincare mapping graph and corresponding frequency responses.
tation coefficient is calculated, respectively. The effect of the sliding friction coefficient is highly consistent with the results mentioned by reference. Based on these results, the dynamic model of a double helical gear pair is calculated. The corresponding results are verified by the example of the left end gear pair. The dynamic model of a double helical gear pair laid the foundation for the dynamic design of the next step.

ACKNOWLEDGEMENTS

The authors wish to acknowledge the financial support of the National Natural Science Foundation of China (Grant No. 51475210), Taishan Scholar Project Special Funds (2016-2020) during the course of this investigation. The authors would also like to thank the editor and anonymous reviewers for their suggestions for improving this paper.

REFERENCES

The transformation equations of Eq. (10)–Eq. (25) is represented as

\[ F_{p1y} = \cos \beta \left[ k_1 f_{h1y}(\lambda_1) + c_1 \lambda_1 \right]. \]  \hspace{1cm} (A1)

The transformation equations of Eq. (7) is represented as

\[ F_{z1} = \sin \beta \left[ k_1 \left[ z_{p1} - z_{q1} - \lambda_1 \tan \beta - 2 \varepsilon_{z1} \right] + c_1 \left[ \dot{z}_{p1} - \dot{z}_{q1} - \lambda_1 \tan \beta - 2 \dot{\varepsilon}_{z1} \right] - k_1 \varepsilon_c - c_1 \varepsilon_z \right]. \]  \hspace{1cm} (A2)

The transformation equations of Eq. (8) is represented as

\[ F_{p2y} = \cos \beta \left[ k_2 f_{h2y}(\lambda_2) + c_2 \lambda_2 \right]. \]  \hspace{1cm} (A3)

The transformation equations of Eq. (9) is represented as

\[ F_{z2} = \sin \beta \left[ k_2 \left[ z_{p2} - z_{q2} - \lambda_2 \tan \beta - 2 \varepsilon_{z2} \right] + c_2 \left[ \dot{z}_{p2} - \dot{z}_{q2} - \lambda_2 \tan \beta - 2 \dot{\varepsilon}_{z2} \right] + k_2 \varepsilon_c + c_2 \varepsilon_z \right]. \]  \hspace{1cm} (A4)

The transformation equations of Eq. (10)–Eq. (25) is represented as

\[ m_y \ddot{z}_{q1} + c_{q1z} \dot{z}_{q1} + k_{q1z} f_{q1z}(z_{q1}) + c_{qz} (\dot{z}_{q1} - \dot{z}_{q2}) + k_{qz} (z_{q1} - z_{q2}) = -F_{z1}; \]  \hspace{1cm} (A10)

\[ m_y \ddot{z}_{q1} + c_{q1z} \dot{z}_{q1} + k_{q1z} f_{q1z}(z_{q1}) + c_{qz} (\dot{z}_{q1} - \dot{z}_{q2}) + k_{qz} (z_{q1} - z_{q2}) = -F_{z1}; \]  \hspace{1cm} (A11)

\[ m_y \ddot{x}_{p2} + c_{p2z} \dot{x}_{p2} + k_{p2z} f_{p2z}(x_{p2}) + c_{px} (\dot{x}_{p2} - \dot{x}_{p1}) + k_{px} (x_{p2} - x_{p1}) = \chi_2 \mu_2 F_{p2y}; \]  \hspace{1cm} (A12)

\[ m_y \ddot{y}_{p2} + c_{p2y} \dot{y}_{p2} + k_{p2y} f_{p2y}(y_{p2}) + c_{py} (\dot{y}_{p2} - \dot{y}_{p1}) + k_{py} (y_{p2} - y_{p1}) = -F_{p2y}; \]  \hspace{1cm} (A13)

\[ m_y \ddot{x}_{q2} + c_{q2z} \dot{x}_{q2} + k_{q2z} f_{q2z}(x_{q2}) + c_{qx} (\dot{x}_{q2} - \dot{x}_{q1}) + k_{qx} (x_{q2} - x_{q1}) = -\chi_2 \mu_2 F_{p2y}; \]  \hspace{1cm} (A14)

\[ m_y \ddot{y}_{q2} + c_{q2y} \dot{y}_{q2} + k_{q2y} f_{q2y}(y_{q2}) + c_{qy} (\dot{y}_{q2} - \dot{y}_{q1}) + k_{qy} (y_{q2} - y_{q1}) = F_{p2y}; \]  \hspace{1cm} (A15)

\[ m_y \ddot{z}_{q2} + c_{q2z} \dot{z}_{q2} + k_{q2z} f_{q2z}(z_{q2}) + c_{qz} (\dot{z}_{q2} - \dot{z}_{q1}) + k_{qz} (z_{q2} - z_{q1}) = -F_{z2}; \]  \hspace{1cm} (A16)

\[ m_y \ddot{z}_{q2} + c_{q2z} \dot{z}_{q2} + k_{q2z} f_{q2z}(z_{q2}) + c_{qz} (\dot{z}_{q2} - \dot{z}_{q1}) + k_{qz} (z_{q2} - z_{q1}) = -F_{z2}; \]  \hspace{1cm} (A17)

\[ m_y \ddot{p}_{1y} + c_{p1z} \dot{p}_{1y} + k_{p1z} f_{p1z}(p_{1y}) + c_{px} (\dot{p}_{1y} - \dot{p}_{1x}) + k_{px} (p_{1y} - p_{1x}) = \chi_1 \mu_1 F_{p1y}; \]  \hspace{1cm} (A18)

\[ m_y \ddot{p}_{1y} + c_{p1z} \dot{p}_{1y} + k_{p1z} f_{p1z}(p_{1y}) + c_{px} (\dot{p}_{1y} - \dot{p}_{1x}) + k_{px} (p_{1y} - p_{1x}) = \chi_1 \mu_1 F_{p1y}; \]  \hspace{1cm} (A19)

\[ m_y \ddot{y}_{2} + c_{p2y} \dot{y}_{2} + k_{p2y} f_{p2y}(y_{2}) + c_{py} (\dot{y}_{2} - \dot{y}_{1}) + k_{py} (y_{2} - y_{1}) = F_{p2y}; \]  \hspace{1cm} (A20)