Characteristics of In-Plane Waves in Composite Plates

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The high frequency dynamic excitations generate both in-plane as well as bending waves in structures. In aerospace applications, many of these structures are made of composite materials. There are two types of in-plane motions, longitudinal and in-plane shear. Although these motions are uncoupled in isotropic materials, composite structures show coupled behaviour. The works reported on in-plane waves in composite structures assume that two in-plane motions are uncoupled as in isotropic plates. In this work, characteristics of the in-plane waves in composite laminated plates are investigated. Expressions for wavenumber, phase speed and group speed are derived. It is seen that in composite plates the two in-plane waves are coupled in longitudinal and shear propagations and are non-dispersive. The phase speeds of in-plane waves in composite plates can be much different from those determined using the expressions for isotropic plates where the waves are uncoupled. To validate the expressions derived the phase speeds of in-plane waves in a typical composite panel are determined experimentally. It is seen that the experimentally obtained phase speeds match well with the theoretical results.

1. INTRODUCTION

Broadband high frequency acoustic excitation is one of the loads for the design of spacecraft elements. These excitations generate both in-plane as well as bending waves in structures. Characteristics of these waves should be well understood in order to study the response behaviour of structures in the high frequency region. The wavenumber, phase speed and group speed are the most important parameters that govern the wave motion.

The characteristics of bending waves in various forms of structural elements are well discussed.1,2 Many of the spacecraft panels are made of honeycomb sandwich construction with composite face sheets. Closed form expressions for the bending wave characteristics are derived for such panels considering even the transverse shear deformation.3 Though the structure-borne sound caused by acoustic excitation is dominated by bending waves, the in-plane waves become important at locations far away from the excitation as well as in structural health monitoring.4 Most of the work on the wave characteristics carried out in structural health monitoring relate to bending waves.5–7 Very few works are reported on the characteristics of in-plane waves in composite plates.

Datta et al. studied the wave propagation in laminated composite plates, but were limited to bending waves.8 Due to the fact that they used a numerical method, no expressions were derived. Some studies used spectral elements for modelling the in-plane wave propagation and hence no expressions were derived for their characteristics.9 Though there are a few more studies reported on the characteristics of the in-plane waves, no expression is derived for these characteristics,10–12 These characteristics must instead be obtained through numerical methods. The works reported in13,14 also do not present any expression for the wave characteristics, but instead use numerical techniques to determine the characteristics at every frequency. In a few other references, although the transmission of in-plane waves are discussed, no expressions for computing the speeds of these waves are derived.15–17

There are two types of in-plane waves, in-plane longitudinal and in-plane shear. These planes are uncoupled in isotropic materials. In composite plates, which will be shown later, they are coupled. There are several works dealing with the speed of in-plane waves in composite panels. All of them assume that these two in-plane waves are uncoupled.18–20 Thus, the results do not include the coupled in-plane shear and longitudinal behaviour. Instead, the results give the speeds as though they are uncoupled, as in isotropic plates.

In the present work, expressions for the characteristics of in-plane waves in a composite laminate are derived. It takes in to account the coupled motion of both the in-plane waves. Sensitivities of these characteristics to various parameters of the composite plate are discussed. Experimental results are obtained for a typical composite plate. The experimentally determined phase speeds are compared with the phase speeds determined using the expression derived in this work.

2. DIFFERENTIAL EQUATIONS OF WAVE MOTION

Consider a plate as shown in Fig. 1. The coordinate axes are in-plane (x and y) and normal (z). The displacements along the in-plane directions are represented by u and v and the normal displacement is represented by w. Corresponding mid-plane displacements are \( u_0, v_0 \) and \( w_0 \). The equilibrium equations for the free vibration of thin plates, neglecting rotary inertia, are as follows:

\[
\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = \rho_m \frac{\partial^2 u_0}{\partial t^2};
\]

\[
\frac{\partial N_{yy}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = \rho_m \frac{\partial^2 v_0}{\partial t^2};
\]

\[
\frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} = \rho_m \frac{\partial^2 w_0}{\partial t^2}.
\]
where $\rho_m$ represented the mass per unit area, $N_{xx}, N_{yy}$ and $N_{xy}$ are the stress resultants (per unit width) and $M_{xx}, M_{yy}$ and $M_{xy}$ are the moment resultants (per unit width).

Assuming Classical Lamination theory, based on Kirchoff’s theory, in which the displacements in the in-plane directions vary linearly through the plate thickness while the normal displacement is independent of the thickness, the displacement field is determined by:

\begin{align}
 u &= u^0 - z \frac{\partial w}{\partial x}, \\
v &= v^0 - z \frac{\partial w}{\partial y}, \\
w &= w_0.
\end{align}

Here, the transverse shear effects are neglected and the corresponding shear strains are zero.

The relationships of in-plane strains, curvature and twist with the displacements are called the generalized strain-displacement or kinematic equations. The strains are expressed in terms of the mid-plane strains $\epsilon_{0xx}, \epsilon_{0yy}$ & $\gamma_{0xy}$ and the curvature and the twist $\kappa_{xx}, \kappa_{yy}$ & $\kappa_{xy}$.

\begin{align}
 \epsilon_{xx} &= \epsilon_{0xx} + z \kappa_{xx}; \\
 \epsilon_{yy} &= \epsilon_{0yy} + z \kappa_{yy}; \\
 \gamma_{xy} &= \gamma_{0xy} + z \kappa_{xy}.
\end{align}

As the stresses in a laminate vary from layer to layer, it was convenient to deal with a simpler but equivalent system of forces and moments on a laminate cross-section, commonly called as the stress resultants and moment resultants. These six quantities formed a system that is equivalent to the stress system on the laminate, but it is applied at the geometrical mid-plane.

As the laminate is in a plane-stress state, normal stress and the transverse shear stresses were neglected. Only $\sigma_{xx}, \sigma_{yy}$ and $\tau_{xy}$ were considered. In order to find a system of forces and moments acting at the geometric mid-plane that was equivalent to the effect of these stresses, three stress resultants were defined which are equal to the sum or integral of these stresses in the thickness direction. These forces and moments were defined as:

\begin{equation}
 \begin{bmatrix}
 N_{xx} \\
 N_{yy} \\
 N_{xy}
 \end{bmatrix} = \sum_{k=1}^{k=n} \int_{h_k}^{h_{k-1}} \begin{bmatrix}
 \sigma_{xx} \\
 \sigma_{yy} \\
 \tau_{xy}
 \end{bmatrix} dz; \quad (10)
\end{equation}

\begin{equation}
 \begin{bmatrix}
 M_{xx} \\
 M_{yy} \\
 M_{xy}
 \end{bmatrix} = \sum_{k=1}^{k=n} \int_{h_k}^{h_{k-1}} \begin{bmatrix}
 \sigma_{xx} \\
 \sigma_{yy} \\
 \tau_{xy}
 \end{bmatrix} z dz. \quad (11)
\end{equation}

For a thin plate, using the kinematic equations as described before and combining Eqs. (10) & (11) we determined that:

\begin{equation}
 \begin{bmatrix}
 N_{xx} \\
 N_{yy} \\
 N_{xy} \\
 M_{xx} \\
 M_{yy} \\
 M_{xy}
 \end{bmatrix} = \begin{bmatrix}
 A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\
 A_{12} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\
 A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\
 B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\
 B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\
 B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66}
 \end{bmatrix} \begin{bmatrix}
 \epsilon_{0xx} \\
 \epsilon_{0yy} \\
 \gamma_{0xy} \\
 \kappa_{xx} \\
 \kappa_{yy} \\
 \kappa_{xy}
 \end{bmatrix}. \quad (12)
\end{equation}

Here, $A_{ij} = \sum_{k=1}^{k=n} (Q_{ij})_k (h_k - h_{k-1}),$ called as extensional stiffness terms; $B_{ij} = \frac{1}{2} \sum_{k=1}^{k=n} (Q_{ij})_k (h_k^2 - h_{k-1}^2),$ called as coupling stiffness terms; $D_{ij} = \frac{1}{8} \sum_{k=1}^{k=n} (Q_{ij})_k (h_k^3 - h_{k-1}^3),$ called as bending stiffness terms.

In the present work, symmetric laminate was considered, therefore $B_{ij} = 0.$ It was assumed that the laminate had negligible values of $A_{16}, A_{26}, D_{16}$ and $D_{26}.$ Hence the above relation became,

\begin{equation}
 \begin{bmatrix}
 N_{xx} \\
 N_{yy} \\
 N_{xy} \\
 M_{xx} \\
 M_{yy} \\
 M_{xy}
 \end{bmatrix} = \begin{bmatrix}
 A_{11} & A_{12} & 0 & 0 & 0 & 0 \\
 A_{12} & A_{22} & 0 & 0 & 0 & 0 \\
 0 & 0 & A_{66} & 0 & 0 & 0 \\
 0 & 0 & 0 & D_{11} & D_{12} & 0 \\
 0 & 0 & 0 & D_{12} & D_{22} & 0 \\
 0 & 0 & 0 & 0 & D_{66}
 \end{bmatrix} \begin{bmatrix}
 \epsilon_{0xx} \\
 \epsilon_{0yy} \\
 \gamma_{0xy} \\
 \kappa_{xx} \\
 \kappa_{yy} \\
 \kappa_{xy}
 \end{bmatrix}. \quad (13)
\end{equation}

Using Eq. (13) in the first two equations of motion given by Eqs. (1) & (2) (corresponding to the inplane direction) the equilibrium equations were cast in terms of displacements.

\begin{equation}
 A_{11} \frac{\partial^2 \epsilon_{0xx}}{\partial x^2} + A_{66} \frac{\partial^2 \epsilon_{0xx}}{\partial y^2} + (A_{12} + A_{66}) \frac{\partial^2 \gamma_{0xy}}{\partial x \partial y} = \rho_m \frac{\partial^2 u^0}{\partial t^2}; \quad (14)
\end{equation}

\begin{equation}
 A_{66} \frac{\partial^2 \epsilon_{0xx}}{\partial x^2} + A_{22} \frac{\partial^2 \epsilon_{0xx}}{\partial y^2} + (A_{12} + A_{66}) \frac{\partial^2 \gamma_{0xy}}{\partial x \partial y} = \rho_m \frac{\partial^2 v^0}{\partial t^2}. \quad (15)
\end{equation}

3. EQUATIONS FOR WAVEMOTIONS

Using the above governing differential equations (Eqs. (14) and (15)), the characteristics of the waves were obtained.

The solution to the wave equation was as follows:

\begin{equation}
 u^0 = U_0 e^{j(\omega t - k_x x - k_y y)} \quad \text{and} \quad v^0 = V_0 e^{j(\omega t - k_x x - k_y y)}, \quad (16)
\end{equation}

where $k_x$ and $k_y$ represent the wavenumbers in the $x$ and $y$ directions respectively and $\omega$ represents the radian frequency.
Upon the substitution of Eq. (16) into Eqs. (14) and (15), we produced equations for wave motions.

\[
\begin{bmatrix}
A_{11}k_x^2 + A_{26}k_y^2 + \frac{k_x^2 k_y^2}{-\rho_m \omega^2} \\
(12 + A_{66})k_xk_y
\end{bmatrix}
\begin{bmatrix} U_0 \\ V_0 \end{bmatrix} = 0 \tag{17}
\]

4. WAVENUMBER AND WAVE SPEED

The above determinant being zero leads to the equation relating the wavenumbers. From the expressions for wavenumber, the phase speeds and group speeds of the waves are determined.

4.1. Wavenumber

Determinate of Eq. (17) is given below.

\[
A_{11}A_{66}k_x^4 + A_{22}A_{66}k_y^4 + k_x^2 k_y^2(11_{11}A_{22} + A_{66}^2 \\
-2A_{22}^2 - 2A_{11}A_{66} - \rho_m \omega^2(A_{11}k_x^2 + A_{66}k_y^2 \\
+ A_{22}k_x^2 + A_{26}k_y^2) + \rho_m \omega^4 = 0. \tag{18}
\]

By defining \( \alpha = \frac{A_{11}}{A_{11}}; \beta = \frac{A_{66}}{A_{11}}; \delta = \frac{A_{22}}{A_{11}} \), the determinant became:

\[
A_{11}A_{66}(k_x^2 + \delta k_y^2) + A_{11}(\delta - \alpha^2 - 2\alpha \beta)k_x^2k_y^2 - \rho_m \omega^2A_{11}(k_x^2 + \beta k_y^2 + \beta k_x^2 + \delta k_y^2) + \rho_m \omega^4 = 0. \tag{19}
\]

For a panel having \( A_{11} = A_{22} = 1 \),

\[
A_{11}A_{66}(k_x^2 + k_y^2) + A_{11}(1 - \alpha^2 - 2\alpha \beta)k_x^2k_y^2 - \rho_m \omega^2A_{11}(k_x^2 + \beta k_y^2 + \beta k_x^2 + k_y^2) + \rho_m \omega^4 = 0. \tag{20}
\]

Since \( k_x^2 + k_y^2 = k^2 \) and \( k_x^4 + k_y^4 = k^4 - 2k_x^2k_y^2 \), we determined that:

\[
A_{11}A_{66}k^4 + A_{11}^2\{((1 - \alpha(\alpha + 2\beta)) - 2\beta)k_x^2k_y^2 - \rho_m \omega^2A_{11}k^2(1 + \beta) + \rho_m \omega^4 = 0. \tag{21}
\]

Since \( k_x = k \cos \theta; k_y = k \sin \theta \) in the wavenumber domain,

\[
\rho_m^2 \omega^4 - \rho_m \omega^2A_{11}(1 + \beta)k^2 + A_{11}A_{66}k^4 + A_{11}^2k^2 = 0. \tag{22}
\]

By defining:

\[
\epsilon = \{1 - \alpha^2 - 2\alpha \beta - 2\beta \} \sin^2 \theta \cos^2 \theta;
\]

the above equation can be simplified as:

\[
\rho_m^2 \omega^4 - \rho_m \omega^2A_{11}(1 + \beta)k^2 + A_{11}A_{66}k^4 + A_{11}^2k^2 \epsilon = 0. \tag{24}
\]

Eq.(24) is the dispersion relation which is a function of \( \omega \) and wavenumber \( k \), from which the expression for the wavenumber can be derived as:

\[
k^2 = \frac{\rho_m \omega^2}{2A_{11}} \left\{ (1 + \beta) \pm \sqrt{(1 + \beta)^2 - 4\epsilon} \right\}. \tag{25}
\]

It is to be noted that there are two wavenumbers at a particular frequency. Also, it depends on the orientation of the wave which is represented by the parameter \( \epsilon \).

4.2. Phase Speed

Using the definition of the wavenumber, that is \( k = \frac{\omega}{c} \), in Eq. (24), we determined that:

\[
\rho_m^2 \omega^4 - \rho_m \omega^2A_{11}(1 + \beta)\left(\frac{\omega}{c}\right)^2 + A_{11}A_{66} \left(\frac{\omega}{c}\right)^4 + A_{11}^2 \left(\frac{\omega}{c}\right)^4 \epsilon = 0; \tag{26}
\]

which, on simplification, became:

\[
c^2 \rho_m^2 - \rho_m A_{11}(1 + \beta)c^2 + (A_{11}A_{66} + A_{11}^2c^2) = 0. \tag{27}
\]

This can be further simplified as:

\[
c^2 \rho_m - \rho_m A_{11} - c^2(1 + \beta) + A_{11} \rho_m (\beta + \epsilon) = 0. \tag{28}
\]

Eq. (28) is quadratic in \( c^2 \), the solution of which gives the velocity of the in-plane wave as:

\[
c^2 = \frac{1 + \beta \pm \sqrt{(1 + \beta)^2 - 4\left(\frac{\rho_m}{A_{11}}\right) \frac{A_{11}}{\rho_m} (\beta + \epsilon)}}{2\left(\frac{\rho_m}{A_{11}}\right)}; \tag{29}
\]

\[
c^2 = \frac{A_{11} \rho_m}{2\rho_m} [(1 + \beta) \pm \sqrt{(1 - \beta)^2 - 4\epsilon}]. \tag{30}
\]

Eq. (30) gives the phase speed of the in-plane wave in a composite plate. It is to be noted that the results showed the occurrence of two waves, as in isotropic plates and they depended on the orientation of the wave which is represented by the parameter \( \epsilon \).

4.3. Group Speed

The gradient \( \frac{\partial \omega}{\partial k} \) is the group velocity, denoted by \( c_g \). From Eq. (24) we determined that:

\[
\rho_m^2 \omega^4 - k^2(1 + \beta)\omega^2 + \frac{A_{11}}{\rho_m} k^4(\beta + \epsilon) = 0. \tag{31}
\]

The above expression is quadratic in \( \omega^2 \) and the solution was:

\[
\omega^2 = \frac{k^2(1 + \beta) \pm \sqrt{k^4 (1 + \beta)^2 - 4\left(\frac{A_{11}}{\rho_m}\right) \frac{A_{11}}{\rho_m} k^4(\beta + \epsilon)}}{2\left(\frac{1}{A_{11}}\right)}. \tag{32}
\]

Upon applying certain algebraic operations, we determined that:

\[
\omega = k \sqrt{\frac{A_{11}}{2\rho_m} \left( (1 + \beta) \pm \sqrt{(1 - \beta)^2 - 4\epsilon} \right)}; \tag{33}
\]

\[
c^2_g = \frac{A_{11}}{2\rho_m} \left( (1 + \beta) \pm \sqrt{(1 - \beta)^2 - 4\epsilon} \right). \tag{34}
\]

Eq. (34) represents the group speed of the in-plane wave in a composite plate. It can be seen that the group speed is the same as the phase speed, which was as expected since the group speed was independent of frequency.
4.4. Assumptions

The assumptions used in arriving at the above expressions are summarized below. Some of these assumptions are part of any two-dimensional structural analysis but given here for completeness.

1. The plate is thin, i.e., thickness of the plate was much less compared to the other dimensions.
3. \( \epsilon_z = 0 \), i.e., the displacement \( w \) was independent of \( z \).
4. The transverse planes that were normal to the undeformed layers remained plane and normal after deformation (CLPT is used). This means the shear deformations were negligible.
5. Material was linearly elastic.
6. The laminate was symmetric, therefore \( B_{ij} = 0 \).
7. The laminate had negligible values of \( A_{16}, \ A_{26} \). This was satisfied in most of the practical cases. If the laminate is balanced, these parameters vanished.
8. The parameters \( D_{16}, D_{26} \) of the laminate were negligible.
9. The laminate had \( A_{11} = A_{22} \), otherwise the expressions for the characteristics were more complex.
10. Mass distribution was uniform, i.e. mass per unit area was constant.
11. Rotary inertia was neglected.

5. SIMPLIFIED EXPRESSION

Eq. (30) gives the phase speed of the in-plane waves in a composite plate. It is to be noted that the speed depends on the orientation of the wave which is represented by the parameter \( \epsilon \). This expression can be simplified in certain cases.

5.1. Simplification of the Expression

Consider the equation for the phase speed, \( c^2 = \frac{A_{11}}{2\rho_m} [(1 + \beta) \pm \sqrt{1 + \beta^2 - \beta - (1 - \alpha^2 - 2\alpha\beta)\frac{1}{2}} \]. Here, \( \epsilon \) is given by the expression \( (1 - \alpha^2 - 2\alpha\beta - 2\beta)\sin^2 \theta \cos^2 \theta \) which serves as a function of \( \theta \). The parameter establishes the relation between the wavenumber components along the two directions. For a particular value of wavenumber, it is possible for one to have various combinations of wavenumber components such that the resultant wavenumber is the same. Since there was no specific directional preference, i.e. equal probability of occurrence for the wave components, an average value of \( \sin^2 \theta \cos^2 \theta \) was used. The average value of \( \sin^2 \theta \cos^2 \theta \) can be shown to be equal to 1/8 using \( \epsilon \approx \{1 - \alpha^2 - 2\alpha\beta\}^{1/2} \). Now the term inside the square root of the expression for the phase speed became \( 1 + \beta^2 - \beta - (1 - \alpha^2 - 2\alpha\beta)\frac{1}{2} \).

Using the above term, the phase speed of the in-plane wave can be simplified as:

\[
\begin{align*}
  c^2 &= \frac{A_{11}}{2\rho_m} [(1 + \beta) \pm \sqrt{1 + \beta^2 - \beta - (1 - \alpha^2 - 2\alpha\beta)\frac{1}{2}}]. \\
  &\approx \frac{A_{11}}{2\rho_m} \left[ (1 + \beta) \pm \sqrt{1 + \beta^2 - \beta - (1 - \alpha^2 - 2\alpha\beta)\frac{1}{2}} \right]. \\
  &= \frac{A_{11}}{2\rho_m} \left[ (1 + \beta) \pm \sqrt{1 - \beta + \frac{1 - \alpha^2 - 2\alpha\beta}{2}} \right]. \\
  &= \frac{A_{11}}{2\rho_m} \left[ (1 + \beta) \pm \sqrt{1 - \beta + \frac{1 - \alpha^2 - 2\alpha\beta}{2}} \right].
\end{align*}
\]

For small values of \( \beta, \beta^2 \) was neglected and hence:

\[
\begin{align*}
  c^2 &= \frac{A_{11}}{2\rho_m} \left[ (1 + \beta) \pm \sqrt{1 - \beta + \frac{1 - \alpha^2 - 2\alpha\beta}{2}} \right]. \\
  &= \frac{A_{11}}{2\rho_m} \left[ (1 + \beta) \pm \sqrt{1 - \beta + \frac{1 - \alpha^2 - 2\alpha\beta}{2}} \right].
\end{align*}
\]

The square-root appearing in Eq. (36) can be eliminated if \( |\beta + \frac{1 - \alpha^2 - 2\alpha\beta}{2}| \approx 1 \), and the expression for the phase speed results in:

\[
\begin{align*}
  c^2 &= \frac{A_{11}}{2\rho_m} \left[ (1 + \beta) \pm \sqrt{1 - \beta + \frac{1 - \alpha^2 - 2\alpha\beta}{2}} \right]. \\
  &= \frac{A_{11}}{2\rho_m} \left[ (1 + \beta) \pm \sqrt{1 - \beta + \frac{1 - \alpha^2 - 2\alpha\beta}{2}} \right].
\end{align*}
\]

Considering the positive root of Eq. (37):

\[
\begin{align*}
  c^2 &= \frac{A_{11}}{2\rho_m} \left[ 1 + 0.25\beta - \left\{ \frac{1 - \alpha^2 - 2\alpha\beta}{8} \right\} \right]. \\
  &= \frac{A_{11}}{2\rho_m} \left[ 1 + 0.25\beta - \left\{ \frac{1 - \alpha^2 - 2\alpha\beta}{8} \right\} \right].
\end{align*}
\]

Considering the negative root of Eq. (37):

\[
\begin{align*}
  c^2 &= \frac{A_{11}}{2\rho_m} \left[ 0.75\beta + \left\{ \frac{1 - \alpha^2 - 2\alpha\beta}{8} \right\} \right]. \\
  &= \frac{A_{11}}{2\rho_m} \left[ 0.75\beta + \left\{ \frac{1 - \alpha^2 - 2\alpha\beta}{8} \right\} \right].
\end{align*}
\]

Thus, we found two waves with phase speeds that are determined by Eqs. (38) and (39). The significance of these two wave speeds will be clear when we discuss the results for the isotropic plates. The additional assumptions used for the simplified expression were the equal probability of occurrence for the waves. The value of \( \beta \) was so small that \( \beta^2 \) could be neglected and the value \( |\beta + \frac{1 - \alpha^2 - 2\alpha\beta}{2}| \approx 1 \), which on simplification, became \( |2\beta - \alpha^2 - 2\alpha\beta| \ll 1 \).

5.2. Typical Results

Phase speed is obtained for a typical honeycomb sandwich plate with orthotropic face sheets. The material properties are given below for reference. The face sheet material is two layers of bi-directional CFRP (0°/90°) M18/43090. The elastic modulus of the material (each layer) is 1.47 E11 N/m² and the shear modulus is 4.09 E9 N/m² with a Poisson’s ratio of 0.03. The honeycomb core has a thickness of 12 mm. The mass per unit area of the panel is 0.92 kg/m². The panel has \( A_{11} = 4.71 \times 10^7 \) N/m; \( A_{12} = 1.41 \times 10^7 \) N/m; \( A_{66} = 1.28 \times 10^8 \) N/m; \( \alpha = 0.03, \beta = 0.027, \epsilon = 0.117 \) and \( 1 - \alpha^2 - 2\alpha\beta = 0.125 \). Using these parameters, the phase speeds are 6650 m/s and 2941 m/s. The phase speeds obtained using the simplified expression are 6737 m/s and 2731 m/s and they are very close to those obtained using the general expression.

Thus, if \( \beta^2 \) is negligible and \( |2\beta - \alpha^2 - 2\alpha\beta| \ll 1 \), Eqs. (38) and (39) can give the values of phase speeds. Otherwise, the general expression given by Eq. (35) can be used in determining the phase speeds.

5.3. Isotropic Plates

In the case of isotropic material,

\[
\begin{align*}
  A_{11} &= \frac{E_h}{1 - \mu^2}; \ A_{12} = \frac{\mu E_h}{1 - \mu^2} \ &\ A_{66} = \frac{E_h}{\pi(1 + \mu)}; \\
  \alpha &= \frac{A_{12}}{A_{11}} = \mu, \beta = \frac{A_{66}}{A_{11}} = \frac{1 - \mu}{2} \ &\ 1 - \alpha^2 - 2\alpha\beta = 1 - \mu.
\end{align*}
\]
Considering positive root given by Eq. (38) we get:

$$c^2 = \frac{A_{11}}{\rho_m}. \quad (40)$$

Considering negative root given by Eq. (39) we get:

$$c^2 = \frac{A_{11}}{\rho_m} \left[ \frac{1 - \mu}{2} \right]. \quad (41)$$

Thus, we get two phase speeds. One speed is related to the in-plane longitudinal and the other is related to in-plane shear. The two in-plane waves present in the composite plate correspond to the in-plane longitudinal and in-plane shear. In an isotropic plate they are decoupled and this decoupling results in two independent waves. However, the in-plane waves are coupled in a composite plate.

6. WAVE CHARACTERISTICS

Two in-plane waves exist in a composite plate, which are coupled in longitudinal and shear. These waves are non-dispersive. Both the phase and group speeds are the same. The phase speeds depend on two parameters, namely $\beta$ and $\alpha$. The parameter $\beta$ represents the shear properties and $\alpha$ represents Poisson’s effect. Relationships of phase speeds with the above parameters are discussed here.

In Fig. 2, phase speed is shown against various values of $\beta$ for a value of 0.3 for $\alpha$. As discussed earlier, phase speed will have two values. One of the waves has a higher phase speed, denoted in the figure as wave 1, compared to the other, denoted as wave 2. As in most of the composite structures the values of $\beta$ are much lower and the results are provided for various values of $\beta$ up to a value of 0.4. For an isotropic material $\beta = \frac{1-\mu}{2}$ and $\alpha = \mu$. For a Poisson’s ratio of 0.3, $\beta = 0.35$ and $\alpha = 0.3$.

For convenience, the phase speed is normalized with respect to $\frac{A_{11}}{\rho_m}$. This factor $c^2/(A_{11}/\rho_m)$, termed here as normalized phase speed, is taken as the ordinate in the plots.

Therefore the figures give $1 + 0.25\beta - \left(\frac{1-\alpha^2-2\alpha\beta}{8}\right)$ and $0.75\beta + \left(\frac{1-\alpha^2-2\alpha\beta}{8}\right)$ for various values of $\beta$. The term $\frac{A_{11}}{\rho_m}$ is the phase speed of the longitudinal wave if the material was isotropic.

6.1. Influence of Parameter $\beta$

Consider wave 1. For an isotropic plate, the phase speed converges to that of the longitudinal wave. In this case, the wave is termed here as the longitudinal dominant wave. It can be seen that the phase speeds of such waves increase with parameter $\beta$. Typical values of the parameters of a composite panel are ($\beta = 0.03$ and $\alpha = 0.03$). Fig. 3 gives the phase speed for $\alpha = 0.03$ for various values of $\beta$. It can be seen that $c^2/(A_{11}/\rho_m)$ can be about 0.5 times that of the equivalent isotropic plate.

Consider wave 2. For an isotropic plate the phase speed converges to that of shear wave therefore termed here as shear dominant wave. It can be seen that the phase speed increases with parameter $\beta$. For an isotropic plate ($\beta = 0.35$ and $\alpha = 0.3$), and hence $c^2/(A_{11}/\rho_m)$ is equal to 0.35. For a typical composite panel ($\beta = 0.03$ and $\alpha = 0.03$), it can be seen that $c^2/(A_{11}/\rho_m)$ can be about 0.5 times that of the equivalent isotropic plate.

6.2. Influence of Parameter $\alpha$

The influence of parameter $\alpha$ is now explored. Variations of phase speeds with $\beta$ are plotted for various values of $\alpha$. Fig. 4 gives these results for the longitudinal dominant wave and Fig. 5 gives these results for the shear dominant wave. As $\alpha$ increases, the phase speeds of longitudinal waves decrease while the phase speeds of shear dominant waves increase. Nevertheless, the influence of $\alpha$ is negligible, meaning that the phase speeds do not significantly change with $\alpha$.

6.3. Use of Simplified Expression

The above results are obtained using the simplified expressions for the phase speeds. The characteristics are now ob-
6.4. Summary of the Results

The phase speeds of composite panels can be much lower than those computed using the expression for the isotropic plate, thus justifying the need for using the expressions derived in this work for determining the phase speeds of in-plane waves in composite plates. The impact is very significant for the shear dominant waves than the longitudinal dominant waves. Influence of the parameter $\beta$ is quite significant though the influence of the parameter $\alpha$ is relatively negligible.

7. EXPERIMENTAL RESULTS

Experiments are conducted on a typical composite panel to validate the expressions derived here for the phase speeds of the in-plane waves. The panel is excited using a modal hammer. Accelerations are measured at salient points and at the point of impact. The time lag of the acceleration response from the acceleration at the excitation point is used to determine the phase speed. Experimentally obtained phase speeds are compared with the phase speeds estimated by the expressions derived in this work.

7.1. Details of the Panel

A honeycomb sandwich panel with composite face sheet was used for the experiment. The panel had dimensions 1440 mm $\times$ 480 mm $\times$ 15 mm. The face sheet had two layers of CFRP (+45/-45). The CFRP material was M18 / BD43090. The elastic modulus of the material (each layer) was 1.47 E11 N/m$^2$ and the shear modulus was 4.09 E9 N/m$^2$ with a Poisson’s ratio of 0.03. The honeycomb core had a thickness of 15 mm. The mass per unit area of the panel was 1.21 kg/m$^2$. The panel has $A_{11} = 4.71 \times 10^7$ N/m, $A_{22} = 4.71 \times 10^7$ N/m, $A_{12} = 1.41 \times 10^6$ N/m, $A_{66} = 1.28 \times 10^6$ N/m, $\alpha = 0.03$, $\beta = 0.027$. The above values were determined with respect to the principal material directions.

7.2. Test Setup

The panel was kept on a foam and isolated from the ground as shown in Fig. 8. The panel was impacted at the center of an edge with a modal hammer of Kistler make (Model no: - Kistler 9722A500). Using the Impact hammer, an impact was generated at location 1 along $Y$ direction. A Teflon tip (Serial no 9904A), with additional dead weight, was used for impact. Acceleration responses were measured at three locations as shown in Fig. 9. Response was measured at location 1. Portable data acquisition system of the LMS was used for the data acquisition. All the channels were connected in IEPE mode directly to the data acquisition systems.

7.3. Data Acquisition Parameters

To obtain the wave speed properly, the data acquisition parameters had to be selected properly. For a speed of 10000 m/s, the in-plane wave was expected to reach the other edge in 45 $\mu$s. A resolution of about 10 $\mu$s, wherein there will be 4 lines in the duration of the arrival time, was expected to give good results. Therefore, the data was acquired with a sampling rate of 102.4 kHz. This resulted in a bandwidth of 51.2 kHz. The number of spectral lines selected was 64. This resulted in a frequency resolution of 800 Hz. The above settings provided the acquisition for a duration of 0.00125 s = 1/(800). The data had a high resolution of 0.00001 s (10 $\mu$s). Several trials were carried out in arriving at these parameters. Use of 20.48 kHz as a bandwidth with a resolution of 24.4 $\mu$s was not sufficient to capture the arrival of the wave. Usage of higher sampling rate, e.g. a bandwidth of 102.4 kHz caused several issues with the measured acceleration response like resonance of the accelerometers etc.
7.4. Test Results

The acceleration responses measured at the impact location (1Y) and at location 2Y are shown in Fig. 10 and impact location (1Y) and at location 4Y are shown in Fig. 11.

The wave speeds are determined from the delay in the disturbance reaching the edge from the initiation of the disturbance. The time or arrival of the in-plane wave at location 2 gives a speed of 5861 m/s. Based on the position of location 2 this wave is expected to be a longitudinal dominant wave. The time or arrival of the in-plane wave at location 4 gives a speed of 3806 m/s. Based on the position of location 4 this wave is expected to be an in-plane shear dominant wave. Thus, the experimental results reveal existence of two in-plane waves and show that these in-plane waves are coupled.

7.5. Comparison with Theoretical Estimation

The phase speeds of the in-plane waves in the above panel are theoretically estimated using the expressions derived in this work (Eq. (30)).

The speeds thus determined are 5772 m/s (against the experimentally obtained speed of 5861 m/s) and 2561 m/s (against the experimentally obtained speed of 3806 m/s). The phase speed of the longitudinal dominant wave is obtained by using the expression derived here. In this case, the phase speed of the longitudinal dominant wave is in good agreement with the experimentally determined phase speed. However, the experimentally determined phase speed of the in-plane shear dominant wave is quite higher than the estimated phase speed. This is investigated further.
It is important to analyse the way in which the waves are generated in the experimental set up. It can be seen that the impact is given at location 1 along direction Y. This impact set a longitudinal dominant wave in both face sheets as there is a metallic embedment, called an insert, at the impact point connecting both face sheets. Since the longitudinal dominant wave is coupled to in-plane shear motion, the in-plane shear waves are now generated from the progressing longitudinal dominant waves. In this case, the longitudinal dominant waves are already set in the face sheets. Since the core is very flexible, the in-plane shear waves that are generated in the face sheets travel independently. Considering the above logic, the in-plane shear waves are expected to be set independently in both the face sheets. Therefore, considering the properties of one face sheet alone (the axial stiffness becomes half) and the core does not move along with the face sheets (mass per unit area is 0.3656 kg/m²) we determined that the phase speed of the in-plane shear is 3296 m/s (against the experimentally obtained speed of 3806 m/s). The phase speed of the in-plane shear is about 0.87 of the experimentally obtained phase speed.

Thus, it can be seen that the phase speeds estimated using the expression derived here are in good agreement with the experimentally obtained phase speeds.

It is interesting to compare the experimentally obtained results with those computed using the existing expressions. As discussed earlier (section 6.1), the speeds computed using the existing expression can cause large errors in the speed of the shear dominant in-plane wave. The speed of this wave, which is determined by the existing expression, is 1323 m/s (against the experimentally obtained speed of 3806 m/s). There is a significant difference between the experimentally determined speed and those determined using the existing expressions. The speed of this wave, which is computed using the expression derived here, is 3296 m/s. The experimental results signy the coupled motion of both the in-plane waves and also the need for the expression derived in this work.
8. SUMMARY AND CONCLUSIONS

Expressions for the phase speed and group speed of in-plane waves in a composite laminate plate are derived. Two in-plane waves exist in a plate and they are non-dispersive. In an isotropic plate they are uncoupled and form independent in-plane longitudinal and in-plane shear waves. In a composite plate these waves are coupled. The phase speeds in a typical composite panel are determined experimentally. The experimental results show the existence of the two in-plane waves with different phase speeds. The phase speeds determined using the expressions derived here match very well with the experimental results.

The phase speeds of in-plane waves in composite plates can be much lower than those computed using the expression for isotropic plates in which the two waves are uncoupled. The additional parameters that govern the phase speed are the in-plane shear stiffness and Poisson’s ratio related parameters. The influence of the in-plane shear stiffness parameter on the phase speed is quite significant, but the influence of Poisson’s ratio related parameters is relatively less. The impact is considerable in the case of the shear dominant waves.

REFERENCES


