The static and dynamic stability of a rotating tapered beam having an elliptical cross-section subjected to a pulsating axial load with a thermal gradient is investigated under three different boundary conditions, such as clamped-clamped (C-C), clamped-pinned (C-P), and pinned-pinned (P-P). The governing equations of motion have been derived by using Hamilton’s energy principle. A set of Hill’s equations have been obtained by the application of generalized Galerkin’s method. The effects of taper parameter, hub radius, rotational speed, thermal gradient, and geometric parameter on the static buckling loads and the regions of instability have been studied and the results are presented graphically.

**NOMENCLATURE**

- $A(x), A(\xi)$: Area of the generic section of the beam
- $A_1$: Cross sectional area
- $C_0$: Hub radius
- $l$: Length of the beam
- $c_0$: Dimensionless hub radius, ($C_0/l$)
- $b(x), b(\xi)$: Minor diameter of a generic section of the beam
- $B$: Minor axis diameter at $x = 0$
- $E$: Young’s modulus
- $I(x), I(\xi)$: Moment of inertia at a generic section of the beam
- $I_1$: Moment of inertia $x = 0$
- $m(\xi)$: Mass distribution function
- $P_0$: Static axial load
- $P_1$: Dynamic axial load
- $p(\tau)$: Dimensionless load
- $p_0$: Dimensionless static axial load
- $p_1$: Dimensionless dynamic axial load
- $S(\xi)$: Moment of inertia distribution function
- $t$: Time period
- $w(x,t)$: Transverse deflection of the beam
- $q(\xi)$: Geometric parameter
- $\alpha^*$: Minor axis taper parameter
- $\eta$: Dimensionless transverse deflection, ($= w/l$)
- $\xi$: Dimensionless length, ($= x/l$)
- $\tau$: Dimensionless time
- $\rho$: Density of the beam material
- $\Omega$: Uniform angular velocity of the beam about $z'$ axis
- $\Omega_0$: Rotational speed parameter
- $\omega$: Excitation frequency
- $\omega_0$: Dimensionless fundamental natural frequency
- $\overline{\omega}$: Non-dimensional excitation frequency
- $b$: Minor axis diameter at $a$
- $a$: Major axis diameter

**1. INTRODUCTION**

The vibration analysis of rotating beams is of great importance in the design of many engineering examples, such as helicopter rotor blades, turbine blades, turbo engine blades, etc. The effect of thermal gradient is also a vital aspect as the modulus of elasticity for most of the elastic materials are greatly affected by the temperature. Lo and Renbarger obtained the differential equation of motion of a cantilever blade mounted on a rotating disc at a stagger angle.\(^1\) The natural frequencies and mode shapes of a rotating uniform cantilever beam with tip mass were studied by Bhat.\(^2\) The theoretical expression for the work done due to centrifugal effects was derived by Carnegie.\(^3\) The effects of rotational speed, disc radius, and stagger angle of the blade on the frequencies of the lateral vibration were studied by Rao.\(^4\) Liu and Yeh obtained the natural frequencies of a non-uniform rotating beam.\(^5\) Bauer studied the effects of spin, hub radius, and aspect ratio on the vibrational behavior of a rotating beam.\(^6\) The effect of rotational speed and root flexibilities on static buckling loads and the first order simple resonance zones were studied by Abbas by using finite element technique.\(^7\) Ishida et al. investigated the influence of rotational speed on the unstable regions of a system consisting of a disc mounted on an elastic shaft subjected to a pulsating axial load.\(^8\) The explicit stability conditions for a rotating shaft under parametric excitation were derived by...