Thermo-Mechanical Vibration Analysis of Imperfect Inhomogeneous Beams Based on a Four-Variable Refined Shear Deformation Beam Theory Considering Neutral Surface Position

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In this disquisition, an exact solution method is developed for analyzing the vibration characteristics of porous functionally graded (FG) beams by considering neutral surface position and different thermal loadings via a four-variable shear deformation refined beam theory. Four types of environmental conditions through the z-axis direction are supposed as: uniform (UTR), linear (LTR), nonlinear (NLTR) and sinusoidal (STR) temperature rises. Mechanical properties of porous FG beams are supposed to vary through the thickness direction and are modeled via the modified power-law. The modified power-law is formulated using the concept of even and uneven porosity distributions. Since the variation of pores along the thickness direction influences the mechanical properties, porosity plays a key role in the mechanical response of FG structures. The governing differential equations and related boundary conditions of porous FG beams are subjected to temperature field that is derived by Hamilton’s principle based on a four-variable refined theory which verifies shear deformation regardless of any shear correction factor. The Navier-type solution procedure is used to achieve the natural frequencies of porous-FG beams supposed to various thermal loadings which satisfies the simply-simply boundary condition. A parametric study is led to carry out the effects of material graduation exponent, porosity volume fraction, different porosity distribution, and thermal effect on dimensionless frequencies of porous FG beams. It is concluded that these parameters play noticeable roles in the vibration behavior of imperfect FG beams. Presented numerical results can be applied as benchmarks for future designs of imperfect FG structures with porosity phases.

1. INTRODUCTION

Technology development in the field of making materials with functional properties introduces Functionally Graded Materials (FGMs) as a new class of smart composite structures. FGMs have led many researchers to analyze the mechanical specifications of these materials with engineering structures like beams, plates and shells. Due to the high strength and temperature resistance of FGMs, they are increasingly utilized in mechanical, civil, and aerospace engineering as structural components.1–3 FGMs are advanced types of composite materials with inhomogeneous micromechanical structure where the concentration, shape and orientation of constituent phases vary in one or more directions for optimizing the performance. Typically, FGMs are composed of two different parts such as ceramics with excellent characteristics in heat and corrosive resistances and metal with good toughness. Functional grading of the material properties is often in one direction. However, grading can be implemented in several directions. These materials have been developed for general purpose structural components such as rocket engine components or turbine blades where the components are exposed to extreme temperatures. The FGMs were introduced by Japanese scientists in mid-1980s as aerospace application for the first time. FGMs possess various advantages in comparison with traditional composites. Some of these advantages are: multi functionality, the ability to control deformation, corrosion resistance, dynamic response, the minimization or removal of stress concentrations, smoothing the transition of thermal stress and resistance to oxidation. Recently, due to the need for new materials for engineering applications in modern industries including aerospace, nuclear energy, turbine components, rocket nozzles, critical furnace parts, FGMs have received wide attention.4–7 The advantages of using FGM structures in general engineering structures have been increasingly recognized in recent decades. Therefore it is important to understand the behaviors of engineering structures made of FGMs. For example, the vibration, static and dynamic behavior of FG beams and plates are often found in general engineering structures.8,9 A large number of investigations are documented in literature dealing with the static, buckling and dynamic characteristics of FGM structures.10,11 Pradhan and Chakraverty have presented free vibration FG beam characteristics using Euler and Timoshenko beam theories.12 The Rayleigh-Ritz method was used to obtain frequencies in their analysis. Thai and Vo investigated the bending and free vibrations of FG beams within the framework of different higher-order shear deformation beam theories.6 Due to the huge application of FGM structures in different fields such as civil, marine and aerospace engineering, and the difference between the making and working temperatures of FG structures, it is important to take into account the thermal effect when designing FG structures. Mahi et al. studied the temperature-dependent vibration behavior of FG beams.13 The important influence of temperature change on the vibration re-
response of FG beams is also taken into account. Recently, thermal post-buckling and the vibration of thermally post-buckled FGM beams were analyzed by Esfahani et al. The generalized differential quadrature method was employed to solve the equations of motion.

With the rapid development in technology of structural elements, structures with graded porosity can be introduced as one of the latest developments in FGMs. The structures treat pores as microstructures by taking the local density into account. Researches focus on the development of preparation methods for FGMs such as powder metallurgy, vapor deposition, self-propagation, centrifugal casting, and magnetic separation. These methods have their own ineffectiveness such as the complexity of the technique and its high costs. An efficient way to manufacture FGMs is sintering process in which, due to difference in solidification of the material constituents, porosities or micro-voids through material can be created. An investigation has been carried out on porosities that exist in FGMs that have been fabricated by a multi-step sequential infiltration technique. According to this information, for building more secure and accurate structures, it’s important to consider the porosity impact on designing FGM structures. Porous FG structures have many interesting combinations of mechanical properties, such as high stiffness in conjunction with very low specific weight. Since porous FG structures have reached remarkable attention by many engineers, recent papers in the field of FG structures discuss the mechanical responses of structural ingredients made of porous FGMs. Wattanasakulpong and Unghbakorn examined the linear and non-linear vibration of porous FGM beams with elastically restrained ends. Ebrahimi and Mokhtari provided differential transformation methods to examine the vibration behavior of rotating Timoshenko FG beams with even porosities. They reported that porosity volume fraction plays a key role in the vibrational response of the FG beams. In order to predict flexural vibration of porous FG Timoshenko beams, Wattanasakulpong and Chaikittiratana used the Chebyshev collocation method. Ebrahimi and Zia applied the Galerkin and multiple scales methods to solve nonlinear vibration of porous FGM beams. Ebrahimi and Zia presented that the thermo-mechanical vibration response of temperature-dependent porous FG beams are subject to various temperature risings based on the classic beam theory (CBT). CBT disregards the influence of shear deformation. In other words, CBT is unable to model thick beams and higher modes of vibration. Hereupon, the first order shear deformation theory (FSDT) is suggested to overcome the defects of CPT with supposition a shear correction factor in the thickness direction of beam. As respects FSDT isn’t able to evaluate the zero-shear stress on the top and bottom surfaces of the beam, there appeared a need to develop new theory. In order to bypass these defects, higher order shear deformation theory (HSDT) was introduced. This theory predicts transverse shear stresses without need of any shear correction factors. Many papers are published which framework HSDT to investigate mechanical response of FG structures. Moreover, Ait Yahia et al. studied the porosity effect on the wave propagation of FG plates by using various higher-order shear deformation theories. Recently, Mechab et al. developed the nonlocal two-variable refined plate theory for the free vibration of FG porous nanoplates that are resting on elastic foundations. Most recently Ebrahimi et al. studied the vibration of porous FG Euler beams subjected to thermal loadings.

It is worth mentioning that, although various inclusion-related studies about the vibration of FG beams have been conducted in recent years, no published works consider the porosity and thermal effect on the vibration response of imperfect FG beams with the concept of exact neutral axis position and different porosity distributions based on the four-variable refined shear deformation theory. This complicated problem is not well-investigated and there is a need for further studies. The present research developed a four-variable refined shear deformation theory for the thermo-mechanical vibration of FG beams with porosities. Reined beam theory considers a constant transverse displacement and higher-order variation of axial displacement through the depth of the beam so that there is no need for any shear correction factors. Two kinds of porosity distribution, namely even and uneven through the thickness directions, are considered. Four types of environmental conditions through the z-axis direction are supposed as: uniform (UTR), linear (LTR), nonlinear (NLTR) and sinusoidal (STR) temperature rises. The modified power-law model is used to describe the gradual variation of the material properties of porous beams. By applying Hamilton’s principle, governing equations of higher order MEE-FG beams are obtained together based on four-variable refined shear deformation theory and are solved by applying an analytical solution method. Several numerical exercises indicate that various parameters such as thermal environment, power-law exponent, porosity parameters and types of porosity distribution have remarkable influence on fundamental frequencies of porous FG beam.

2. THEORETICAL FORMULATIONS

2.1. Power-low Functionally Graded Beams with Porosities

Figure 1 shows a uniform functionally graded beam with a rectangular cross-section of length \( L \), width \( b \) and thickness \( h \). The beam was made up of homogeneous and isotropic functionally graded materials in which the volume fraction and micro-structural morphology of the material compositions varied continuously in thickness direction. Functionally graded materials are the new generation of composite materials. In this case, composite materials were usually produced from two or more different materials. For this study, FG materials were made from a mixture of ceramic and metal. The effective material properties of FG beams including Young’s modulus (\( E \)), shear modulus (\( G \)), mass density (\( \rho \)) and thermal expansions (\( \alpha_{\text{exp}} \)) change continuously in the thickness direction (z-axis) based on power-law distribution. Poisson’s ratio was assumed to be constant in the z-axis direction. In this paper, FG beams were assumed to have porosities spreading within the thickness due to defects during production.

The effective material properties (\( P_f \)) of FG beams with two kind of porosities that distributed identical in two phases of ceramic and metal were expressed by using the modified rule of mixture: \( P_f = P_c(V_c - \frac{\alpha}{2}) + P_m(V_m - \frac{\alpha}{2}) \).

Where \( \alpha \) denotes the volume fraction of porosities (\( \alpha \ll 1 \)), for perfect FGM \( \alpha \) was set to zero, \( P_c \) and \( P_m \) were the ma-
Material properties of ceramic and metal. \( V_c \) and \( V_m \) were the volume fractions of ceramic and metal. The compositions represented in relation to:  

\[
V_c + V_m = 1. \tag{2}
\]

Then the volume fraction of ceramic phase \((V_c)\) was defined as follows:

\[
V_c = \left(\frac{z_{ns} + c}{h} + \frac{1}{2}\right)^n; \tag{3}
\]

where \((0 \leq n)\) was the volume fraction index that determined the material distribution through the thickness of the beam. According to this distribution, we had a metal beam for the large value of \(n\) and when \(n\) equals zero, A ceramic beam remained and \(Z\) was the distance from the mid-plane of the graded beam.

According to Eqs. (1), (2), the effective material properties of FG beams with even porosities (FGM-I) were expressed in the following forms:  

\[
P(z_{ns}) = (P_c - P_m) \left(\frac{z_{ns} + c}{h} + \frac{1}{2}\right)^n + P_m - \frac{\alpha}{2} \left(P_c + P_m\right) \frac{1}{2} \left(2\left|z_{ns} + c\right|\right); \tag{5a}
\]

\[
G(z_{ns}) = (G_c - G_m) \left(\frac{z_{ns} + c}{h} + \frac{1}{2}\right)^n + G_m - \frac{\alpha}{2} \left(G_c + G_m\right) \frac{1}{2} \left(2\left|z_{ns} + c\right|\right); \tag{5b}
\]

\[
E(z_{ns}) = (E_c - E_m) \left(\frac{z_{ns} + c}{h} + \frac{1}{2}\right)^n + E_m - \frac{\alpha}{2} \left(E_c + E_m\right) \frac{1}{2} \left(2\left|z_{ns} + c\right|\right); \tag{5c}
\]

\[
\rho(z_{ns}) = (\rho_c - \rho_m) \left(\frac{z_{ns} + c}{h} + \frac{1}{2}\right)^n + \rho_m - \frac{\alpha}{2} \left(\rho_c + \rho_m\right) \frac{1}{2} \left(2\left|z_{ns} + c\right|\right); \tag{5d}
\]

\[
\alpha(z_{ns}) = (\alpha_c - \alpha_m) \left(\frac{z_{ns} + c}{h} + \frac{1}{2}\right)^n + \alpha_m - \frac{\alpha}{2} \left(\alpha_c + \alpha_m\right) \frac{1}{2} \left(2\left|z_{ns} + c\right|\right). \tag{5e}
\]

Here, it should be noted that the FGM-I had porosity phases with even distributions of volume fraction over the cross section. The FGM-II had porosity phases that spread frequently near the middle zone of the cross-section and the amount of porosity seemed to decrease linearly to zero at both the top and bottom of the cross-section. Figur 2 shows examples of cross-section areas of FGM-I and-II with porosity phases. For the second type, uneven distribution of porosities (defined as FGM-II), the effective material properties were replaced by the form below:  

\[
P(z_{ns}) = (P_c - P_m) \left(\frac{z_{ns} + c}{h} + \frac{1}{2}\right)^n + P_m - \frac{\alpha}{2} \left(P_c + P_m\right) \frac{1}{2} \left(2\left|z_{ns} + c\right|\right); \tag{6a}
\]

\[
G(z_{ns}) = (G_c - G_m) \left(\frac{z_{ns} + c}{h} + \frac{1}{2}\right)^n + G_m - \frac{\alpha}{2} \left(G_c + G_m\right) \frac{1}{2} \left(2\left|z_{ns} + c\right|\right); \tag{6b}
\]

\[
E(z_{ns}) = (E_c - E_m) \left(\frac{z_{ns} + c}{h} + \frac{1}{2}\right)^n + E_m - \frac{\alpha}{2} \left(E_c + E_m\right) \frac{1}{2} \left(2\left|z_{ns} + c\right|\right); \tag{6c}
\]

\[
\rho(z_{ns}) = (\rho_c - \rho_m) \left(\frac{z_{ns} + c}{h} + \frac{1}{2}\right)^n + \rho_m - \frac{\alpha}{2} \left(\rho_c + \rho_m\right) \frac{1}{2} \left(2\left|z_{ns} + c\right|\right); \tag{6d}
\]

\[
\alpha(z_{ns}) = (\alpha_c - \alpha_m) \left(\frac{z_{ns} + c}{h} + \frac{1}{2}\right)^n + \alpha_m - \frac{\alpha}{2} \left(\alpha_c + \alpha_m\right). \tag{6e}
\]

According to Fig. 2, the position of neutral axis of the imperfect FG beams were determined to satisfy the first moment with respect to elastic stiffness being zero as follows:

\[
\int_{-\frac{b}{2}}^{\frac{b}{2}} E(z_{ms}) (z_{ms} - C) dz_{ms} = 0. \tag{7}
\]

Consequently, the position of neutral surface can be obtained as:

\[
C = \int_{-\frac{b}{2}}^{\frac{b}{2}} E(z_{ms}) z_{ms} dz_{ms} = \frac{1}{\int_{-\frac{b}{2}}^{\frac{b}{2}} E(z_{ms}) dz_{ms}}. \tag{8}
\]

For more precise anticipation of FGMs behavior under extreme temperature fields, material properties must be dependent on temperature. Therefore, temperature-dependent coefficients of material phases were expressed according to the non-linear equation below:  

\[
P = P_0 (P_1 T^{-1} + P_1 T + P_1 T^2 + P_1 T^3). \tag{9}
\]

In this case, \(P_0, P_1, P_2, P_3\) and \(P_4\) were the temperature dependent coefficients that were tabulated in the table materials properties (Table 3) for \(Si_3N_4\) and SUS304. The bottom and top surface of FG beams were supposed to be fully metal (SUS304) and fully ceramics \(Si_3N_4\), respectively.
2.2. Kinematic Relations

Based on the four-variable refined shear deformation beam theories, the displacement field at any point of the beam were supposed to be in the form below:

\[ u_1(x, z_{ns}, t) = u(x, t) + z_{ns} \frac{dw_b}{dx} - f(z_{ns}) \frac{dw_s}{dx}; \]  
\[ u_2(x, z_{ns}, t) = 0; \]  
\[ u_3(x, z_{ns}, t) = w_b(x, t) + w_s(x, t). \]  

In this form, \( u \) was the displacement of the mid-plane along \( x \), \( w_b, w_s \) represented the bending and shear components of the transverse displacement of a point on the mid-plane of the beam and \( t \) represented time. \( f(z) \) denoted a shape function that estimated the distribution of shear stress across the beam thickness. \( f(z) \) was used to satisfy the stress-free boundary conditions on the top and bottom sides of the beam. So, it is not required to use any shear correction factor. The displacement relation of the new hyperbolic shear deformation theory based on Mahi and Tounsi was obtained by using new hyperbolic shape function. The new hyperbolic shape function was expressed as follows:

\[ f(z_{ns}) = \frac{h}{2} \tanh \left( 2 \frac{z_{ns}}{h} \right) - \frac{4}{3 \cosh^2(1)} \left( \frac{z_{ns}^3}{h^2} \right). \]  

Then the nonzero strains displacement relation of new shear deformation beam theory was expressed as follows:

\[ \epsilon_{xx} = \frac{\partial u}{\partial x} - z \frac{\partial^2 w_b}{\partial x^2} - f \left( \frac{\partial^2 w_s}{\partial x^2} \right); \]  

\[ \gamma_{xz} = \left( 1 - \frac{\partial f}{\partial x} \right) \frac{\partial w_s}{\partial x} = g \frac{\partial w_s}{\partial x}; \]  

where \( \epsilon_{xx}, \gamma_{xz} \) was the normal and shear strains and \( g(z) = (1 - \frac{\partial f}{\partial x}) \) was the shape function of the transverse shear strains as following:

\[ g(z_{ns}) = 1 - \left[ 1 - \tanh^2 \left( \frac{z_{ns} + c}{h} \right) \right] \frac{4}{\cosh^2(1)} \left( \frac{z_{ns} + c}{h} \right)^2. \]  

The Euler Lagrange equations have been used to derive the equation of motion by using Hamilton’s principle, in which the motion of an elastic structure in the time interval \( t_1 < t < t_2 \) is so that the integral with respect to time of the total potential energy is extremum:

\[ \int_{t_1}^{t_2} \delta(U - T + V) dt = 0; \]  

where \( U \) was strain energy, \( V \) was work done by external forces and \( T \) was kinetic energy. The virtual variation of strain energy \( \delta U \) was calculated as:

\[ \delta U = \int_v \sigma_{ij} \delta \epsilon_{ij} dV = \int_A \int_0^L \left( \sigma_{xx} \delta \epsilon_{xx} + \sigma_{xz} \delta \gamma_{xz} \right) dx da; \]  

where \( \delta \) was the variation symbol, \( A \) was the cross-section area of the uniform beam, \( \sigma_{xx} \) and \( \sigma_{xz} \) the axial and shear stress was the shear stress, by substituting the expressions for \( \epsilon_{xx} \) and \( \gamma_{xz} \) into Eq. (15) as:

\[ \delta u = \int_0^L \int_A \sigma_{xx} \delta \left[ \frac{\partial u}{\partial x} - z \frac{\partial w_b}{\partial x^2} - f \frac{\partial^2 w_s}{\partial x^2} \right] dA dx + \int_0^L \int_A \sigma_{xz} \delta \left[ g \frac{\partial w_s}{\partial x} \right] dA dx; \]  

\[ \delta u = \int_0^L \left[ \frac{\partial N}{\partial x} \delta u - \frac{\partial^2 M_b}{\partial x^2} \delta w_b - \frac{\partial^2 M_s}{\partial x^2} \delta w_s + \frac{\partial Q_{xz}}{\partial x} \delta w_s \right] dx. \]  

In which the variables introduced in arriving at the last expression were defined as follows:

\[ (N, M_b, M_s) = \int_A \sigma_{xx}(1, z_{ns}, f) dA, \quad (Q) = \int_A \sigma_{xz}(g) dA. \]  

The first variation of the virtual kinetic energy was written in the form:

\[ \delta T = \frac{1}{2} \int_0^L \left[ I_0 \frac{\partial^2 u}{\partial x^2} \delta u + I_2 \frac{\partial^2 w_b}{\partial x^2} \delta w_b + I_3 \frac{\partial^2 w_s}{\partial x^2} \delta w_s + I_1 \left[ \frac{\partial^2 u}{\partial x^2} \delta w_b + \frac{\partial^2 w_b}{\partial x^2} \delta w_b - J_1 \frac{\partial^2 w_b}{\partial x^2} \delta u \right] dx \right. \]

\[ \left. + \frac{\partial^2 u}{\partial x^2} \delta w_s + \frac{\partial^2 w_s}{\partial x^2} \delta w_s + I_0 \left[ \frac{\partial^2 w_b}{\partial x^2} \delta w_b + \frac{\partial^2 w_s}{\partial x^2} \delta w_s + \frac{\partial^2 w_s}{\partial x^2} \delta w_b \right] \right]. \]
\[ (I_0, I_1, I_2) = \int_A \rho(z_{ns}, T)(1, z_{ns}, z_{ns}^2) dA; \]
\[ (J_1, J_2, K_2) = \int_A \rho(z_{ns}, T)(f, f z_{ns}, f^2) dA. \]  

Also, the first variation of potential energy was written in the form:

\[ \delta V = \int_0^L [f(x)\delta u + q(x)\delta (w_s + w_b) + N^T \delta \sigma (w_s + w_b)] dx. \]

In this study, for analyzing vibration of porous FG beam in thermal environment, the first variation of external loadings due to thermal loadings was obtained as:

\[ \delta V = \epsilon_b N^T \delta \sigma (w_s + w_b) dx; \]

where \( N^T \) was defined as following:

\[ N^T = \int_{-h/2-c}^{h/2+c} E(z_{ns}, T) \alpha_{\text{exp}}(z_{ns}, T) \Delta T dz. \]

In which \( \alpha_{\text{exp}} \) is the coefficient of thermal dilatation that is typically positive and very small (\( 0 < \alpha \ll 1 \)). By inserting Eqs. (18), (21) and (24) into Eq. (14) and setting the coefficients of \( \delta u, \delta w_b \) and \( \delta w_s \) to zero, the following Euler-Lagrange equations were obtained:

\[ (\delta u : 0), \frac{\partial N}{\partial x} = I_0 \frac{\partial^2 u}{\partial t^2} - I_1 \frac{\partial^2 w_b}{\partial t^2 \partial x} - J_1 \frac{\partial^2 w_s}{\partial t^2 \partial x}; \]
\[ (\delta w_b : 0), \frac{\partial^2 M_b}{\partial x^2} - K_0 \frac{\partial^2 (w_s + w_b)}{\partial x^2} = I_0(\frac{\partial^2 w_s}{\partial t^2} + \frac{\partial^2 w_b}{\partial t^2}) + I_1 \frac{\partial^3 u}{\partial t^2 \partial x^2} - I_2 \frac{\partial^4 w_b}{\partial t^2 \partial x^2} - J_1 \frac{\partial^4 w_s}{\partial t^2 \partial x^2}; \]
\[ (\delta w_s : 0), \frac{\partial^2 M_s}{\partial x^2} + \frac{\partial Q_{xz}}{\partial x} - K_0 \frac{\partial^2 (w_s + w_b)}{\partial x^2} = I_0(\frac{\partial^2 w_s}{\partial t^2} + \frac{\partial^2 w_b}{\partial t^2}) - k_2 \frac{\partial^4 w_b}{\partial t^2 \partial x^2} + J_1 \frac{\partial^3 u}{\partial t^2 \partial x} - J_2 \frac{\partial^4 w_s}{\partial t^2 \partial x^2}. \]

For a material that is linearly elastic and obeys the 1D Hooke’s law, the relation between stress-strain was described as:

\[ \sigma_{xx} = E(z_{ns})\varepsilon_{xx}; \]
\[ \sigma_{zz} = G(z_{ns})\gamma_{zz}; \]

where \( G \) was the shear modulus and \( E \) was the Young’s modulus, by substituting the Eqs. (11), (12) into Eqs. (31) and (32) and subsequent results into Eq. (17) and integrating over the beam’s cross-section, stress resultant were derived as:

\[ N = A_x \frac{\partial u}{\partial x} - B_{xx} \frac{\partial^2 w_b}{\partial x^2} - C_{xx} \frac{\partial^2 w_s}{\partial x^2}; \]
\[ M_b = B_{xx} \frac{\partial u}{\partial x} - D_{xx} \frac{\partial^2 w_b}{\partial x^2} - E_{xx} \frac{\partial^2 w_s}{\partial x^2}; \]
\[ M_s = C_{xx} \frac{\partial u}{\partial x} - E_{xx} \frac{\partial^2 w_b}{\partial x^2} - F_{xx} \frac{\partial^2 w_s}{\partial x^2}; \]
\[ Q = A_x \frac{\partial w_s}{\partial x}. \]

In which the cross-section rigidities were calculated as follows:

\[ (A_{xx}, B_{xx}, C_{xx}, D_{xx}, E_{xx}, F_{xx}) = \int_A (1, z, f, z_{ns}, f^2) E(z_{ns}, T) dA; \]
\[ A_{zz} = \int_A g^2 G(z_{ns}, T) dA. \]

The last form of Euler-Lagrange equations for porous FG beam subjected to thermal loading based on new hyperbolic shear deformation beam theory in terms of displacement \( u, w_b \) and \( w_s \) were derived as:

\[ A_{xx} \frac{\partial^3 u}{\partial t^2} - B_{xx} \frac{\partial^3 w_b}{\partial t^2 \partial x} - C_{xx} \frac{\partial^3 w_s}{\partial t^2 \partial x} = I_0 \frac{\partial^2 u}{\partial x^2} - I_1 \frac{\partial^2 w_b}{\partial x^2 \partial x} - I_2 \frac{\partial^2 w_s}{\partial x^2 \partial x} - J_1 \frac{\partial^3 w_s}{\partial t^2 \partial x}; \]
\[ B_{xx} \frac{\partial^3 u}{\partial x^2 \partial x} - D_{xx} \frac{\partial^4 w_b}{\partial x^2} - E_{xx} \frac{\partial^4 w_s}{\partial x^2} - N^T \frac{\partial^2 (w_b + w_s)}{\partial x^2} = I_0 \frac{\partial^2 (w_s + w_b)}{\partial x^2} - I_1 \frac{\partial^3 u}{\partial x^2 \partial x} - I_2 \frac{\partial^4 w_b}{\partial x^2 \partial x^2} - J_2 \frac{\partial^3 w_s}{\partial x^2 \partial x}; \]
\[ C_{xx} \frac{\partial^3 u}{\partial x^2 \partial x} - F_{xx} \frac{\partial^4 w_b}{\partial t^2 \partial x} - E_{xx} \frac{\partial^4 w_s}{\partial t^2 \partial x} + A_x \frac{\partial^2 w_s}{\partial x^2} = I_0 \frac{\partial^2 (w_s + w_b)}{\partial t^2} - k_2 \frac{\partial^4 w_b}{\partial t^2 \partial x^2} + J_1 \frac{\partial^3 u}{\partial t^2 \partial x} - J_2 \frac{\partial^4 w_s}{\partial t^2 \partial x^2}. \]

### 3. SOLUTION METHOD

#### 3.1. Analytical Solution

In this section, an analytical solution of the Euler-Lagrange equations for vibration of S-S porous functional grading beam based on Navier type method were provided. The displacement variables were adopted as combinations of non-significant coefficients and known trigonometric functions to satisfy Lagrange equation and boundary conditions. The following displacements variables were assumed to be formed as:

\[ u(x, t) = \sum_{m=1}^{\infty} u_m \cos \left( \frac{m\pi}{L} x \right) e^{i\omega_m t}; \]
in which \((w_m, w_{b,m}, w_{s,m})\) are the unknown Fourier coefficient that will be calculated for each value of \(m\).

By substituting Eqs. (38), (39), (40) into Eqs. (35), (36), (37) respectively, leads to Eqs. (41), (42), (43):

The analytical solutions were obtained from the following equation:

\[
\left[ -A_{xx} \left( \frac{m\pi}{L} \right)^2 + I_0 \omega^2 \right] w_m + \left[ B_{xx} \left( \frac{m\pi}{L} \right)^3 - I_1 \omega^2 \left( \frac{m\pi}{L} \right) \right] w_{b,m} + \left[ C_{xx} \left( \frac{m\pi}{L} \right)^3 - J_1 \omega^2 \left( \frac{m\pi}{L} \right) \right] w_{s,m} = 0;
\]

\[
[41]
\]

\[
\left[ B_{xx} \left( \frac{m\pi}{L} \right)^3 - I_1 \omega^2 \left( \frac{m\pi}{L} \right) \right] w_m + \left[ -D_{xx} \left( \frac{m\pi}{L} \right)^4 \right] w_{b,m} + \left[ -E_{xx} \left( \frac{m\pi}{L} \right)^4 + \tilde{N}^T \left( \frac{m\pi}{L} \right)^2 \right] w_{s,m} = 0;
\]

\[
[42]
\]

\[
\left[ C_{xx} \left( \frac{m\pi}{L} \right)^3 - J_1 \omega^2 \left( \frac{m\pi}{L} \right) \right] w_m + \left[ -E_{xx} \left( \frac{m\pi}{L} \right)^4 \right] w_{b,m} + \left[ -F_{xx} \left( \frac{m\pi}{L} \right)^4 + A_{xx} \left( \frac{m\pi}{L} \right)^2 + \tilde{N}^T \left( \frac{m\pi}{L} \right)^2 \right] w_{s,m} = 0;
\]

\[
[43]
\]

\[
\left( a_{11} a_{12} a_{13} a_{21} a_{22} a_{23} a_{31} a_{32} a_{33} \right) \begin{pmatrix} u_m \cr w_{b,m} \cr w_{s,m} \end{pmatrix} = 0.
\]

By finding determinant of the coefficient matrix and setting this multinomial to zero, natural frequencies \(\omega_n\) was obtained by:

\[
\text{det} \left( a_{11} a_{12} a_{13} a_{21} a_{22} a_{23} a_{31} a_{32} a_{33} \right) = 0. \quad (44)
\]

4. THERMAL ENVIRONMENT AND TEMPERATURE DISTRIBUTIONS

For a porous FG beam in a thermal environment, it was assumed that temperature varied along the thickness directions at two ways as:

\[
w_b(x,t) = \sum_{m=1}^{\infty} w_{b,m} \sin \left( \frac{m\pi}{L} x \right) e^{i\omega_n t}; \quad (38)
\]

\[
w_s(x,t) = \sum_{m=1}^{\infty} w_{s,m} \sin \left( \frac{m\pi}{L} x \right) e^{i\omega_n t}; \quad (39)
\]

4.1. Uniform Temperature Rise (UTR)

A porous FG beam that had a reference temperature equal to \(T_0 = 300\) and is free of stresses at \(T_0\) was used. The temperature of the FG beam was uniformly raised to a final temperature with the difference of \(\Delta T\) as:

\[
\Delta T = T - T_0. \quad (45)
\]

4.2. Linear Temperature Rise (LTR)

By assuming that the temperature of the top surface of the porous FG beam was \(T_1\) and varied linearly from \(T_1\) to \(T_t\), the bottom surface temperature and the temperature rise was finally able to be determined as:

\[
T = T_m + \Delta T \left( \frac{1}{2} + \frac{z_{ns} + c}{h} \right). \quad (46)
\]

And \(\Delta T\) was defined as:

\[
\Delta T = T_1 - T_b. \quad (47)
\]

4.3. Nonlinear Temperature Rise (NLTR)

By solving the following equation, the steady-state one-dimensional heat conduction equation with the known temperature boundary conditions on bottom and top surfaces of the FG beam was achieved:

\[
[41] - \frac{d}{dz} \left( \kappa(z, T) \frac{dT}{dz} \right) = 0;
\]

\[
T \left( \frac{h}{2} \right) = T_c, \quad T \left( \frac{h}{2} \right) = T_m. \quad (48)
\]

The following equation solved the solution of Eq. (51) subjected to the boundary conditions:

\[
T = T_m + \Delta T \int_{z_{ns}}^{z_{ns} + c} \frac{1}{\kappa(z, T)} \frac{dz}{\frac{h}{2} - c - \frac{z_{ns}}{2}}; \quad (49)
\]

where \(\Delta T = T_c - T_m\).

4.4. Sinusoidal Temperature Rise (STR)

The temperature field when FG beam was exposed to sinusoidal temperature rise across the thickness can be defined as:

\[
T = T_m + \Delta T \left( 1 - \cos \frac{\pi}{2} \left( \frac{1}{2} + \frac{z_{ns}}{h} \right) \right); \quad (50)
\]

where \(\Delta T = T_c - T_m\) is temperature change.

5. NUMERICAL EXAMPLE

In the following section, the validation of porous FG beams with S-S boundary conditions was confirmed in Table 3.

Then, the influence of porosity distributions, porosity volume fraction, power-law exponent and temperature rises on the non-dimensional frequencies of the porous FG beam were explored. The functionally graded porous beam was comprised of Steel (SUS304) and Silicon nitride (\(\text{Si}_3\text{N}_4\)) where its properties are given in Table 1. It was supposed that the temperature...
rise in fully metal surface to reference temperature $T_0$ of the FG beam is $T_m - T_0 = 5K$.\(^{30}\)

The non-dimensional natural frequencies ($\lambda$) were calculated by relations in Eq. (51).

$$\lambda = \frac{\omega^2 L^2}{h^3} \sqrt{\frac{\rho_m}{\epsilon_m}} \left(\frac{\rho_m}{\epsilon_m}\right) \left(\frac{\epsilon_m}{\rho_m}\right)$$

To verify the accuracy of the present method, the numerical results obtained were compared with those available in the literature to demonstrate the performance of the present study. For this purpose, non-dimensional frequencies of FG beams with S-S boundary condition were compared with those of Şimşek and Thai for different volume fraction indexes and slenderness.\(^{6,11}\) Hereupon, $f(z)$ was considered based on the Reddy beam theory as \(\frac{z^3}{3 L^3}\) and the FG beams made of alumina and aluminum were compared with the results from Şimşek and Thai which had been obtained by using Lagrange’s equations and the Navier type solution.\(^5\) Computations had been carried out for the following material and beam properties: \((E_{AI} = 70GPa, \rho_{AI} = 2702kg/m^3, \nu_{AI} = 0.3, E_{Al_{2}O_{3}} = 380GPa, \rho_{Al_{2}O_{3}} = 3960kg/m^3, \nu_{Al_{2}O_{3}} = 0.3)\) for $L/h = (5, 20)$. Table 2 found that the non-dimensional frequency parameters obtained in the present investigation were in agreement with the results provided in these literatures and thus validated the proposed method of solution.

In the present section, results were extracted for various temperature changes, power-law indexes, porosity parameters with four temperature rises (UTR, LTR, NLTR, STR) and two porosity distributions (even, uneven) to present an adequate sensitivity analysis.

As a first verification and investigation example, effects of porosity volume fraction, thermal loading, porosity distribution and power-law exponents on the first non-dimensional frequency of the porous FG simply supported beams were assessed. The results were extracted for different porosity parameters \((n = 0, 0.1, 0.2, 0.5, 1)\), given for three temperature changes \((\Delta T = 20, 40, 80K)\) and constant value of \((L/h = 20)\) at Table 3. Two types of porosity distributions were considered (even and uneven), as well as temperature rises included uniform, linear, nonlinear and sinusoidal distributions. Present results are derived using the Navier type solution method.

Results given in Table 3 show that the growing of the power-law exponents provide lower non-dimensional frequencies of porous FG beams. In fact, the $n = 0$ beam is made from ceramics and has the greatest frequency. Increasing the power law exponents from 0 to 5 changes the composition of the FG beams from a fully ceramic beam to a beam with a combination of ceramic and metal. By increasing the metal percentage and having the smaller value of Young’s modulus in metal with respect to ceramic, the stiffness of the system decreases. Thus, as also known from mechanical vibrations, natural frequencies decrease as the stiffness of a structure decreases. In addition, it is obvious from this Table that increasing temperature change (UTR, LTR, NLTR, STR) yields decreasing of natural frequencies. This indicates that increasing changes in temperature yield a decrease in Young’s modulus $E$. This effect would increase if the temperature was raised. It is concluded that four temperature rises have a considerable effect on the vibration behavior of FG porous beams. It is found that the non-dimensional frequency of porous FG beams under sinusoidal temperature rise is higher than FG beam subjected to NLTR, and the frequency of FG beam subjected to NLTR is higher than that subjected to LTR, which is higher than that under UTR. The difference between non-dimensional frequencies of different temperature rises (UTR, LTR, NLTR, STR) becomes greater by increasing changes in temperature. The reason is that the rigidity of the FG beams for sinusoidal temperature rise is greater than the other cases of temperature rises. According to results of this table, it is beheld, when the power-law indexes are in the range of [0-1], the natural frequencies grow with the increase in the porosity parameters for every temperature increase and porosity distributions. Because of the internal pores in FG growth rigidity beams, this situation is more eminent for lower values of material graduation. Increasing porosity decreases fundamental frequencies when the percentage of the metal is higher than ceramic \((1 < n)\) and temperature change is constant. However, this trend is opposite with increasing changes in temperature. Comparing the frequency of porous FGM beams with even and uneven porosity distribution revealed that when the power index is in the range of [0–0.5], natural frequencies of the even porosity are higher than uneven however, this treatment is vice versa in the range of power law index over 0.5. In addition, for a certain values of temperature change and gradient index, changes in the porosity parameter \((a)\) leads to more variations in frequencies of even distribution in comparison of uneven. In other words, in FGM I, the porosity has more significant impact on natural frequency of beam than that of FGM II.

Table 4 presents the effect of various temperature changes, porosity parameters, material graduations on the non-dimensional frequency of the S-S imperfect FG beams subjected uniform temperature rise with both porosity distributions. Here again, it is seen that by increasing the material power law index, the non-dimensional frequencies decrease. This is due to the increment in flexibility of the FG beams, since the percentage of metal phase increases when power in-
Increasing changes in temperature decreases the frequency parameters so that the effect of this parameter on the fundamental frequency cannot be concealed. Table 4 shows that the variations of frequencies depend on both changes in temperature and the volume fraction index. For example, when $0 < n < 1$ (beams with more percentage of ceramic), increasing of volume fraction of porosity leads to increment of fundamental frequency for all temperature changes and porosity distributions. While, at even distribution the trend of fundamental frequency changes is different for $1 < n$ (beams with more percentage of metal). For example, increasing volume fraction of porosity yields decrease in the non-dimensional frequencies. However, this trend is opposite with increasing the temperature changes. When the temperature changes increase, the fundamental frequency changes due to the increase of the volume fraction index and porosity are also presented in this table. It is evident that at uneven distributions of porosity, increasing porosity parameters yields an increase in non-dimensional frequencies for all temperature changes and power-law indexes.

It can be stated from Fig. 2 that the first dimensionless natural frequency variations of the FG porous beams with simply supported boundary condition subjected to uniform temperature rising for different values of porosity and gradient index parameters is plotted. Non-dimensional natural frequencies of FG beams that are near zero, where the critical point is, decrease with the increase of temperature change. This is because of the reduction in the total stiffness of the beam. Geometrical stiffness shows a decrease when temperature rises. We can get higher frequency results before the critical temperature if the porosity volume fraction is higher in value for a porous FG beam. On the other hand, after the critical temperature this behavior is vice versa. Furthermore, it can be stated that the temperature change can soften FG beam at pre-buckling region in a way that when the temperature rise this effect will be increased. Lower porosity indexes will cause to a decrease of stiffness of the structure. By consideration of the lower porosity parameter, this is the main reason for postponing of branching point of the FG beam. Also, it can be seen that increasing the material graduation exponents leads to reduction in the non-dimensional frequency for every type of porosity distribution. In fact, when power-law is equal to zero beam is made from fully $\text{Si}_3\text{N}_4$ and has the greatest frequency. Increasing the material graduation exponent from 0 to 10 changes the composition of the FG beams from a full $\text{Si}_3\text{N}_4$ beam to a beam with a combination of $\text{Si}_3\text{N}_4$ and SUS304. By increasing the metal percentage and having the smaller value of Young’s modulus of SUS304 with respect to $\text{Si}_3\text{N}_4$, the stiffness of the system decreases. Thus, natural frequencies decrease as the stiffness of a structure decreases. Moreover, it can be seen that, depending on an increase in the material gradient index and porosity parameter, the buckling temperatures can decrease.

In order to clearly understand the difference between different temperature risings, Fig. 3 displays the variations of the first dimensionless frequencies of simply supported FG porous beams under four cases of thermal loadings (UTR, LTR, NLTR and STR) for different fraction of porosity volume and constant of $(L/h = 50, n = 1)$. A comparison between Figs. 3(a–d) revealed that the difference of variant porosity volume fractions is more considerable under sinusoidal temperature rise. It can be found that critical temperature point of porous FG beams subjected to sinusoidal temperature rises is higher than the other temperature risings. Comparison of the first non-dimensional natural frequencies of the FG (I) beam respected to NLTR with the changing of porosity volume fraction and material graduation are presented in Fig. 4 at $(L/h = 20)$. Four types of temperature changes are considered as 0, 20, 40 and 80. It is observed from the results of Fig. 4 that if the power indexes increase, the non-dimensional natural frequencies of porous FG beam will decrease. When the $n$ (power-law exponent) is in the range of 0 to 2, reducing is higher than where power exponent is in range between 2 to 10. The effect of temperature change is obvious, the non-dimensional natural frequencies will be decreased by increasing temperature changes for all gradient indexes, thus various thermal environments have an important effect on the non-dimensional frequency of the porous FG beam. The porosity effect in even distributions depends on power indexes and temperature changes. For example, at a constant value of temperature changes, increasing the porosity parameters causes the natural frequen-

<table>
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<tr>
<th>Power-law Exponent</th>
<th>$L/h$</th>
<th>present</th>
<th>Şimşek\textsuperscript{11} (2010)</th>
<th>Lagrange\textsuperscript{11} equations</th>
<th>Analytical</th>
<th>That\textsuperscript{6} (2012)</th>
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<td>5.46030</td>
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</table>
| $n = 0.2$          | 5    | 4.8080746 | 4.80924          | ...           | ...         | ...
|                    | 20   | 5.0815249 | 5.08286          | ...           | ...         | ...
| $n = 0.5$          | 5    | 4.4106620 | 4.41108          | 4.4111        | 4.4111      |
|                    | 20   | 4.6511086 | 4.65139          | 4.6516        | 4.6516      |
| $n = 1$            | 5    | 3.9904189 | 3.99042          | 3.9904        | 3.9904      |
|                    | 20   | 4.2050549 | 4.20503          | 4.2050        | 4.2050      |
| $n = 2$            | 5    | 3.6264396 | 3.62643          | 3.6264        | 3.6264      |
|                    | 20   | 3.8361340 | 3.83611          | 3.8361        | 3.8361      |
| $n = 5$            | 5    | 3.4012044 | 3.40120          | 3.4012        | 3.4012      |
|                    | 20   | 3.6484863 | 3.64850          | 3.6485        | 3.6485      |
| $n = 10$           | 5    | 3.2816647 | 3.28160          | 3.2816        | 3.2816      |
|                    | 20   | 3.5389891 | 3.53896          | 3.5390        | 3.5390      |
| Full metal         | 5    | 2.66086   | 2.67732          | ...           | ...         | ...
|                    | 20   | 2.83602   | 2.83716          | ...           | ...         | ...
The effect of porosity volume fraction, porosity distribution, temperature rise and power law exponent on the non-dimensional frequency of a S-S FG porous beam subjected to different temperature rises. ($L/h = 20$).

$$\Delta T = 20\,[K]$$

<table>
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<th>FGM type</th>
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<th>Load type</th>
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<th>$n = 0.1$</th>
<th>$n = 0.2$</th>
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</table>

| FGM II   |    | UTR       | 6.30395 | 5.55819   | 5.07277   | 4.27880   | 3.72769  |
|          |    | LTR       | 6.35810 | 5.61597   | 5.13246   | 4.3404    | 3.78915  |
|          |    | NLTR      | 6.35810 | 5.61702   | 5.13423   | 4.34342   | 3.79273  |
|          |    | STR       | 6.39225 | 5.64926   | 5.16531   | 4.37273   | 3.82096  |
|          | 0.1| UTR       | 6.64116 | 5.78827   | 5.24679   | 4.38053   | 3.79119  |
|          |    | LTR       | 6.69143 | 5.84224   | 5.30269   | 4.43833   | 3.84869  |
|          |    | NLTR      | 6.69143 | 5.86219   | 5.31963   | 4.44858   | 3.85849  |
|          |    | STR       | 6.72282 | 5.97471   | 5.33269   | 4.46776   | 3.87764  |
|          | 0.2| UTR       | 7.04850 | 6.05528   | 5.44402   | 4.49159   | 3.85849  |
|          |    | LTR       | 7.09510 | 6.10568   | 5.49636   | 4.54575   | 3.91228  |
|          |    | NLTR      | 7.09849 | 6.12149   | 5.50462   | 4.56483   | 3.92360  |
|          |    | STR       | 7.12384 | 6.34584   | 5.52359   | 4.57244   | 3.93845  |

$\Delta T = 40\,[K]$
Table 3 (continued). The effect of porosity volume fraction, porosity distribution, temperature rise and power law exponent on the non-dimensional frequency of a S-S FG porous beam subjected to different temperature rises. \((L/h = 20)\).

\[
\Delta T = 80[K]
\]

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<td>5.13579</td>
<td>4.21250</td>
<td>3.59499</td>
</tr>
<tr>
<td></td>
<td>NLTR</td>
<td>6.68032</td>
<td>5.84659</td>
<td>4.16353</td>
<td>4.25462</td>
<td>3.64128</td>
</tr>
</tbody>
</table>
|          | STR       | 6.80574     | 5.64576     | 5.25394     | 4.32791     | 3.70836     

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ity distribution increases as the increment in porosity parameter for every power-law exponents and temperature changes. To indicate the influences of temperature change on the non-dimensional frequency of the FG (I) beam subjected to STR for various temperature changes and power-law indexes, Fig. 6 presents the frequency results versus material graduation for perfect \((\alpha = 0)\) and porous \((\alpha = 0.2, \alpha = 0.1)\) FG beam at \((L/h = 20)\). It can be seen that an increase in temperature change gives rise to decrease in the non-dimensional frequency for every value of power-law exponent. Comparison of the non-dimensional frequencies of simply-simply FG(I) beam subjected to UTR and STR with the changing of porosity volume fractions and power exponents at \(\Delta T = 40\), \(L/h = 20\) are presented in Fig. 7. It is concluded that the frequency of the beam subjected to uniform is less than sinusoidal temperature rises and the difference will be increased by raising the power exponents. By comparing diagrams with different porosity parameters, it can be found that increasing of porosity parameter yields to increasing of frequencies.

6. CONCLUSIONS

In this research thermo-mechanical vibrational characteristic of FG porous beams are subjected to various thermal loadings with two porosity distributions. The equations of motion are derived by using four-variable refined shear deformation beam theory and simply-simply boundary condition is considered. The material’s properties are temperature-dependent and vary in the thickness direction established upon the modified rule of mixture. The governing equations are derived by using the new hyperbolic shear deformation theory and by using Hamilton’s principle. The Navier-based analytical model is used to solve governing partial differential equations. According to the numerical results, it is found that the proposed modeling can provide accurate frequency results of the FG beams as compared to the other solution results. As a result, the characteristics of vibration for FGM porous beams are significantly influenced by temperature field, volume fraction of porosity, power-law indexes and porosity distributions. The effects of the induced thermal environment, volume fraction of porosity, power-law index and porosity distribution on non-dimensional frequencies of porous FG beams are investigated. Numerical results show that:

Figure 3. Effect of porosity and temperature change on the non-dimensional frequency of the S-S porous FG beam respect to various temperature rises \((L/h = 50, n = 1)\).
Table 4. The effect of porosity volume fraction, temperature and power law exponent on the non-dimensional frequency of a S-S FG porous beam subjected to uniform temperature rise. \((L/h = 20)\).

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{FGM type} & \alpha & \Delta T = 0 & \Delta T = 20 & \Delta T = 40 & \Delta T = 80 & \Delta T = 100 & \Delta T = 120 & \Delta T = 140 \\
\hline
\text{FGM-I} & 0.1 & 6.55963 & 6.30395 & 6.03316 & 5.43503 & 5.09989 & 4.73312 & 4.32591 \\
& 0.2 & 7.15262 & 6.88946 & 6.64120 & 6.10115 & 5.80425 & 5.48483 & 5.13800 \\
\hline
\text{FGM-II} & 0.1 & 6.55963 & 6.30395 & 6.03316 & 5.43503 & 5.09989 & 4.73312 & 4.32591 \\
& 0.2 & 6.88460 & 6.64115 & 6.38460 & 5.82320 & 5.51234 & 5.17580 & 4.80749 \\
\hline
\end{array}
\]

1. By increasing the gradient index value, the non-dimensional frequencies are found to decrease.

2. Fundamental frequencies decrease by increasing the four-temperature rising and all two porosity distributions.

3. The responses of the non-dimensional frequencies in the FG porous beams according to geometric parameters, under sinusoidal temperature rise are very similar to that under nonlinear, linear and uniform temperature rise. However, the critical temperature gradient under sinusoidal temperature rise is higher than those under the other temperature rises.

4. The non-dimensional frequency predicted by STR is always greatest than those UTR, LTR, NLTR and the uniform temperature rise has more significant effect on the non-dimensional frequencies than the other temperature rise.

5. For FGM-I, at a constant value for changes in temperature, increasing the porosity first causes the increase in fundamental frequencies, however this trend is vice versa for upper values of gradient indexes. This behavior is dependent on power law indexes and temperature changes. For FGM-II, increasing the porosity causes the increase in fundamental frequencies for all values of gradient indexes.
and temperature changes.

6. In FGM I, the porosity has more significant impact on natural frequencies of the beam in comparison of FGM II.

It is concluded that various factors such as porosity parameter, porosity distribution, temperature rising and power-law index have a notable effect on the non-dimensional frequencies of FG beams with porosities. This emphasizes the importance of the inspected porosity volume fraction effect in thermal environments. Therefore, the porosity and thermal effects should be considered in the analysis of vibration behavior of FG structures.

REFERENCES


Figure 6. The variation of the first dimensionless frequency of S-S FGM(I) beam subjected to STR for different porosities and temperature changings ($L/h = 20$).


Figure 7. Comparison of the first non-dimensional frequency of S-S FG(I) beam subjected to UTR and STR for different porosity volume fractions and material graduations. ($\Delta T = 40, L/h = 20$).


