Free Vibration of Axially Loaded Multi-Cracked Beams Using the Transfer Matrix Method

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In this paper, an analytical method is developed to study the free vibration of multi-cracked, axially loaded beams with differing boundary conditions, namely, hinged-hinged, clamped-clamped, clamped-hinged, and clamped-free. The cracked beam system is modelled as a number of beam segments connected by massless rotational springs with sectional flexibility. Each segment is assumed to obey the Euler-Bernoulli beam theory. The characteristic equation of the cracked beam with differing boundary conditions, which is a function of the natural frequency, sizes and location of the cracks, and the physical parameters of the beam, as well as the corresponding mode shapes, is derived using a simple transfer matrix method. In this paper, a detailed parametric study is conducted to show the effects of cracks and axial load on vibrational properties of the cracked beam. The results obtained in this study agree well with analytical results available in the literature.

1. INTRODUCTION AND LITERATURE REVIEW

Beams are one of the primary structural elements that are widely used in various aeronautical, civil, and mechanical engineering fields. The response of structures under service loads is one way of evaluating structural systems. Structures, such as bridges, may experience fatigue or overloading, which may cause cracking. These cracks change the dynamic properties of the structure, such as natural frequencies and corresponding mode shapes. Much research has been done to study the effect of cracks on the vibrational properties of structures.

In recent decades, experimental research on the effects of crack and damage on the safety of structures has had the attention of specialists in this area. Liebowitz and Claus, some of the pathfinders in this area, studied the load capacity of columns with cracks and suggested a failure gauge based on a stress concentration factor. A state-of-the-art review was presented by Dimarogonas for papers published between 1971 and 1995 on the dynamic response of beams and rotor structures with cracks. The reviewed papers were classified into three categories, according to the way the problem was addressed: the equivalent reduced section method, the continuous cracked beam method, and the local flexibility method.

Kirmshier and Thomson both used the equivalent reduced section method. A local bending moment and a reduced section are used to simulate cracks. This method is considered to be the first attempt to measure the stiffness reduction in a cracked region. The theory of cracks and exact numerical simulation methods were not available at that time (1943), so the stiffness reduction due to a crack was estimated by experiment.

Christides and Barr used the continuous cracked beam method and found that the existence of cracks would lead to material changes in strain distributions and stresses in the cracked region. Near the end of a crack, a large stress concentration appeared. The stresses over the cross-section were distributed nonlinearly. The changes in stress and strain distribution in the cracked region were approximated by an unknown function, which had a maximum value at the tip of the cracks. Although the boundary conditions and some elements of the stress distribution in the cracked region were simulated by the function, the discontinuity in the stress distribution was an obvious weakness of the function, as stress and strain should distribute continuously throughout a structure. Zheng and Ji showed that the stress distribution in the vicinity of an abrupt change of cross-section remains continuous by using a fine meshed finite element model. In 1998, Chondros et al. modelled the cracked area as a continuous flexible system. This model gave a better approximation of the stress and displacement distributions.

In this paper, the analytical approach was based on the concept of the local flexibility method. This method considered the cracked beam as a number of beam segments with the same properties, connected by massless rotational springs at the cracked sections. In 1976, Dimarogonas estimated the local flexibility of the cracked region and determined the stiffness of the massless springs, which represented the cracks. Local flexibility was used to estimate the crack Stress Intensity Factor (SIF). SIF is an important concept in fracture mechanics, representing the stress intensity near the tip of a crack due to remote loads or residual stresses. Castigliano’s theorem and fracture mechanics were used by Liebowitz and Claus Jr., Liebowitz et al., and Okamura et al. to compute the flexibility of the cracked region of a uniform beam with a transverse surface crack, which is a function of the ratio of crack depth to the height of the cross-section. The function was obtained experimentally, according to the measurements provided by Brown and Srawley. This factor can also be obtained in Tada et al.’s handbook. Yoon, et al. studied the dynamic behaviour of a simply-supported Euler-Bernoulli beam with double cracks, both analytically and experimentally. Hamilton’s principle was used to derive the equation of motion. The influence of the crack position and its depth on the natural frequencies and corresponding mode shapes was studied. A simplified method was used by Fernandez-Saez et al. to evaluate the natural frequencies of a simply-supported Euler-Bernoulli beam with one