Anchor Loss Calculation for Ring Shape Anchored Contour Mode Disk Resonators

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Miniaturization is the most sophisticated method for achieving UHF and SHF resonance frequencies in RF MEMS resonators. However, by reducing the dimensions of the resonators, the size of their supports become more comparable with the size of the resonator and anchor loss becomes the dominant loss mechanism, thereby suppressing the quality factor. This study considers, Ring Shape Anchored Contour Mode Disk Resonator and calculates anchor loss effects using both energy loss and acoustic impedance ratio methods. Results of analytical calculations are verified using finite element harmonic analysis. Simulation results show that RSACMDRs have an acceptable quality factor in comparison with the other state-of-the-art resonators.

1. INTRODUCTION

In the last two decades, RF MEMS technology has changed the structure of RF and microwave transceivers as well as other communication related systems like radars and antennae.1–3 Now, many scientists and research centres are involved with development, design and fabrication of such devices worldwide.

Of all RF MEMS devices, the RF MEMS resonator has the most impact in transceivers structures due to its low power consumption, high quality factor (Q), integrability alongside CMOS transistors, and ability to operate in GHz frequencies.4 The Ring Shape Anchored Contour Mode Disk Resonator (RSACMDR) is highly reliable5 and could serve as an alternative for the bulky off-chip quartz and crystal resonators which often occupy significant space on transceiver boards. Additionally, RSACMDRs work in higher even modes while suppressing both spurious modes6 and low velocity locations in order to couple with other resonators and construct higher order filters7 or mechanical pattern recognition applications.8,9

One of the most essential features for resonators is the ability to work in GHz frequencies. The most sophisticated method for achieving such frequencies is scaling down the resonators. Unfortunately, as the dimensions of resonators are miniaturized toward nm, energy loss in the resonator anchor becomes more significant and, consequently, drastically decreases the quality factor.

Calculation of anchor loss is essential when designing resonators in order to attain maximum achievable Q. In studies on anchor loss, several calculation methods have been used, such as employing acoustic impedance,10 considering a substrate as a perfectly matched layer,11 and and calculating dissipated energy through a substrate.12,13 This paper provides an analytical calculation of the anchor loss for RSACMDRs by first calculating lost energy from the anchor into the substrate, then calculating the ratio of the acoustic impedances of the anchor and the substrate. Calculations will consider a more precise 3D model of the resonator, describing its vibration in all directions.

The remainder of this paper will address, device structure and modelling and anchor loss calculation, followed by a comprehensive discussion of results.

2. DEVICE STRUCTURE AND MODELLING

As described by Baghelani and Ghavifekr,5,6 the RSACMDR (as shown in Fig. 1) is a hollow disk supported by eight segments of a thin rings that connect to the substrate as anchors and use an electrostatic mechanism for excitation and sensing parts. The eight segments provide sufficient connections to other resonators, suppress the close-to carrier spurious modes, and prevent complications in the interconnections.6 However, if the connection to other resonators is not important for a special application (e.g. as a frequency stabilizer in oscillators), the total number of segments could be reduced.

There are two sets of electrodes in most electrostatic resonators. As illustrated in Fig. 1, the RSACMDR possesses...
sensing electrodes and excitation electrodes, as well as excitation, sensing, and DC bias connections. Through the DC bias connection, DC biasing voltage is applied to the central anchor segment. Output AC signal could be directly extracted from the sensing electrodes. As the AC input signal is applied both outside and inside electrodes, it causes an electrically-induced alternative force, which (when paired with DC bias voltage) allows symmetric distribution of force on the outer side of the disk in a radial direction.

The desired mode of vibration is the radial contour mode which is the radial expansion and contraction of the disk. The generated output current is calculated using:

\[ i_o = V_p \frac{\partial C}{\partial r} \frac{\partial r}{\partial t} \]  

where \( \frac{\partial C}{\partial x} \), the variation rate of resonator, overlaps capacitance in respect to displacement. As a current passes through the output electrodes, the current can be converted to voltage using a trans-impedance amplifier. In this type of resonator, the anchor is placed in the nodal point, located at the centre of the disk.

The vibration equation at the resonance frequency of the resonator could be calculated as

\[ u(r, z, t) = (R(r)a_r + Z(r, z)a_z) e^{j2\pi f_n t}. \]  

From the basic elasticity theorem, the radial displacement function of the disk resonators in the radial contour mode could be calculated from the differential equation

\[ r^2 \frac{d^2 R(r)}{dr^2} + rv \frac{dR(r)}{dr} + (k^2 r^2 - v) R(r) = 0. \]  

The boundary conditions could be considered using

\[ R(r_{in}) = R_0 \quad \text{and} \quad \left. \frac{dR(r)}{dr} \right|_{r=r_{in}} = 0; \]  

where \( \xi_0 \) is normalization factor, and the maximum of \( R(r) \) is adjusted on a desired value.

The function \( f_r \) is the resonance frequency and \( R(r) \) is given by:

\[ R(r) = \frac{-1}{14.4} \left\{ \left[ \frac{a ((k - 1)Y_p(q))}{a \sqrt{5}} + \frac{3k Y_{p+1}(q)}{\sqrt{5}} \right] J_p(kr) + \ldots \left[ a (v-1) J_p(q) + 3k (J_{p+1}(q)) \right] \right. \]

\[ - \left. 2apY_p(q) J_p(kr) + \ldots \left[ a (v-1) J_p(q) + 3k (J_{p+1}(q)) \right] \right\} C_f^b \]  

where \( J_p(.) \) and \( Y_p(.) \) are Bessel functions of the types 1 and 2 in and the order of \( x \). The other constants in the above equation are explicated as

\[ \begin{align*}
  p &= \frac{1}{2} \sqrt{5v^2 - 2v + 1} \\
  q &= \frac{3k}{\sqrt{5}} \\
  a &= 125000 \\
  m &= 0.5(1 + v) \\
  b &= 0.5(1 - v) \\
  C_f &= \frac{3}{\pi} \left( \frac{a^{m+1} - a^m}{(p_{p+1}-p_p)^2} \right)
\end{align*} \]  

\[ \text{Table 1. RSACMD resonator design summary.} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_p )</td>
<td>12 V</td>
</tr>
<tr>
<td>( H )</td>
<td>2 ( \mu )m</td>
</tr>
<tr>
<td>( E )</td>
<td>160 GPa</td>
</tr>
<tr>
<td>( \rho )</td>
<td>2330 ( kg/m^3 )</td>
</tr>
<tr>
<td>( r_{in} )</td>
<td>11.5 ( \mu )m</td>
</tr>
<tr>
<td>( r_{out} )</td>
<td>16.1 ( \mu )m</td>
</tr>
<tr>
<td>Electrodes overlap arc</td>
<td>5( \pi/6 )</td>
</tr>
<tr>
<td>( A_f )</td>
<td>1 nm</td>
</tr>
<tr>
<td>( Q ) from energy approach with 8 anchor segments</td>
<td>4994</td>
</tr>
<tr>
<td>( Q ) from energy approach with 4 anchor segments</td>
<td>9988</td>
</tr>
<tr>
<td>( Q ) from acoustic impedance approach 8 anchor segments</td>
<td>5621</td>
</tr>
<tr>
<td>( Q ) from FE harmonic analysis with 8 anchor segments</td>
<td>5369</td>
</tr>
<tr>
<td>Resonance frequency (analysis)</td>
<td>935 MHz</td>
</tr>
<tr>
<td>Resonance frequency (simulation)</td>
<td>934.3 MHz</td>
</tr>
<tr>
<td>AirGap</td>
<td>100 nm</td>
</tr>
</tbody>
</table>

Figure 2 shows the modal analysis of the resonator at its contour mode. The above equations and constants are achieved for the resonator dimensions and structural material and summarized in Table 1.

3. ANCHOR LOSS CALCULATION

Energy loss through the anchor(s) is the main bottleneck of designing high \( Q \) resonators in SHF and/or higher frequency bands. It is also even the dominant loss mechanism in UHF and VHF when comparing squeeze film and thermo-elastic damping mechanisms. This section calculates anchor loss in resonators using two different methods.

3.1. Anchor Vibrations

Vibration of the resonator in radial contour mode causes contraction and dilation in the vertical direction (see Fig. 3). These vibrations result in anchor resonance with the same frequency, consequently coupling energy to the substrate. The transferred energy to the substrate is mostly lost due to the semi-infinite dimensions of the substrate. Figure 4 illustrates the radial resonance profile of RSACMDR. As shown, at the anchor location, radial displacement becomes zero. Therefore, ideally no flexural vibrations could be induced to the anchor.
To garner better insight into torsional vibrations of the anchor, considering the spurious modes may be useful. Figure 5 shows the spurious modes of the resonator. As illustrated, all modes have radial vibrations, but the main mode ($L_0$) could only vibrate in radial direction. The vibration profile of the resonator in the direction of $\theta$ in both fundamental and spurious modes is sketched in Fig. 6 at the radius of anchor.

As shown, there is no vibration in this direction at the anchor location for the fundamental mode. The anchor has no torsional vibrations. Hence, the only vibration direction of the anchor is longitudinal vibration in $z$ direction. Figure 7 illustrates the vertical resonance profile of RSACMDRs. As expected, the maximum vertical vibration amplitude occurs at the anchor location. For a comparison between the vertical vibration amplitude of the anchors of the RSACMDR and centrally supported disk resonators at the second radial mode, Fig. 8 illustrates the normalized vertical vibration amplitude of centrally supported contour mode disk resonator. By considering Fig. 7 and Fig. 8, one can see that the vertical vibration amplitude at the anchor location of the centrally supported contour mode disk resonator is much higher than that of the RSACMDR, which means a much higher density of dissipated energy in addition to aforementioned to reliability problems in centrally-supported contour mode disk resonators.

### 3.2. Lost Energy Calculation

This section calculates the coupled energy from the structure to the anchor (and consequently the substrate). The energy loss related to the vertical vibrations of the anchor into the substrate could be calculated as

$$\Delta W = \pi \int_{\text{anchor area}} \text{stress} \times \text{displacement.} \quad (7)$$

To promote accuracy, we consider a 3D model of the resonator to calculate $u_z$, using the 3-dimensional relationship
between stress and strain

\[
\begin{bmatrix}
\varepsilon_z \\
\varepsilon_\phi \\
\varepsilon_r
\end{bmatrix} = \frac{1}{E} \begin{bmatrix}
1 & -v & 1-v \\
-v & 1 & -v \\
1-v & 1 & 1
\end{bmatrix} \begin{bmatrix}
\sigma_r \\
\sigma_\phi \\
\sigma_z
\end{bmatrix}.
\]

As it neglects the radius or thickness of the anchors and their vertical effect on the system, no stress in \( z \)-direction exists. Thus, the principal strain in \( z \)-direction can be calculated using

\[
\varepsilon_z = \frac{\partial u_z}{\partial z} = -\frac{v}{1-v} \left( \sigma_r + \sigma_\phi \right) - \frac{v}{1-v} \partial_z u_z.
\]

From calculations of \( R(r) \) we have:

\[
\sigma_r = \frac{E}{1-v^2} \left( \frac{\partial u_r}{\partial r} + \frac{u_r}{r} \right),
\]

\[
\sigma_\phi = \frac{E}{1-v^2} \left( \frac{\partial u_\phi}{\partial r} + \frac{u_\phi}{r} \right).
\]

Applying Eqs. (10) and (11) to Eq. (9) establishes the relationship between \( u_z \) and \( u_r \) below:

\[
\varepsilon_z = \frac{\partial u_z}{\partial z} = -\frac{v}{1-v} \left( \frac{\partial u_r}{\partial r} + \frac{u_r}{r} \right).
\]

By integrating over the \( z \), the vertical component of displacement in any point of the disk can be achieved using

\[
u_z(r,z,t) = -z \frac{v}{1-v} \left( \frac{\partial u_r}{\partial r} + \frac{u_r}{r} \right).
\]

The anchor is located at \( z = -d/2 \), where \( d \) presents the thickness of the disk. As a result, important information about vertical displacement at the anchor location can be calculated. Calculations demonstrate that this alternative displacement pushes and pulls the anchor vertically, thereby causing anchor loss.

\[
u_{zA}(r,t) = \frac{d}{2} \left( \frac{v}{1-v} \left( \frac{\partial u_r}{\partial r} + \frac{u_r}{r} \right) \right).
\]

Based on Eq. (2), the vertical displacement \( u_z \) can be rewritten in the terms of amplitude and time functions, similarly to \( u_r \)

\[
u_{zA}(r,t) = Z(r)e^{j\omega t},
\]

\[
Z(r) = \frac{d}{2} \left( \frac{v}{1-v} \left( \frac{dR(r)}{dr} + \frac{R(r)}{r} \right) \right).
\]

From the above calculations, the applied stresses over the anchor could be analysed using

\[
\sigma_{zz} = E A_l \frac{2\pi}{\lambda};
\]

where \( \lambda \) is the mechanical wavelength and \( A_l \) is the vibration amplitude of the anchor. Calculations of \( A_l \) are made possible using

\[
A_l = \frac{Z(r,z)|_{r=r_a,z=-d}}{\sin \left( \frac{2\pi h}{\lambda} \right)}.
\]

Hence, calculations of the displacement of the anchor at the substrate attachment location use

\[
u_z = \frac{\sigma_0 A_c \omega_r \Pi}{2\pi \rho c^2_T};
\]

where the propagation velocities of the longitudinal and transverse waves are \( c_L \) in Eq. (20) and \( c_T \) in Eq. (21), respectively:

\[
c_L = \sqrt{\frac{E(1-v)}{\rho(1+v)(1-2v)}};
\]

\[
c_T = \sqrt{\frac{E}{2\rho(1+v)}}.
\]

Equation (22) calculates \( \Pi \) using

\[
\Pi = \text{Im} \int_0^\infty \sqrt{\frac{\gamma^2 - 1}{(2\zeta^2 - \gamma^2)^2 - 4\zeta^2 \sqrt{\gamma^2 - \zeta^2}} \zeta d\zeta};
\]

\[
\gamma = \sqrt{\frac{2}{1-v}}.
\]

The integral of Eq. (22) does not have an explicit solution and should be solved numerically. However, since the boundary is goes to infinity, its numerical solution may appear a complicated problem. The imaginary part of integrand in Eq. (22) (see Fig. 9) is limited to 1.8, with

\[
\sigma_0 = \sigma_{zz} \mid_{\text{anchor attachment point to the substrate}}.
\]

In sum, the calculation for total lost energy is explained by (25) (see the top of the next page).
\[ \Delta W = \left( \pi \int_{\varphi = \varphi_{a0}}^{\varphi = \varphi_{a1}} \int_{r = r_{ai}}^{r = \infty} u_z \sigma_{zz} \mid z = \text{anchor attachment to the substrate} \ r \ dr \ d\varphi \right) \times \text{number of anchor segments.} \]  

(25)

3.3. Acoustic Impedance Approach

Another approach to calculate anchor loss is the consideration of acoustic impedances using an electrical equivalent circuit analogy. In this approach (see Fig. 10), the coupled vibration energy from the resonator to the substrate is calculated by \( \Delta W' \). Calculations result in \( \Delta W' \), which refers to transferred energy into the substrate, or, in other words, the lost energy. Since all the transferred energy to the substrate is lost, the substrate could be modelled as a pure resistance. Transferred energy, or \( \Delta W' \), could be calculated using

\[ \Delta W' = R_{\text{substrate}} \Delta W'. \]  

(26)

The acoustic resistance of the substrate \( R \) could be calculated using

\[ R = \frac{2 \pi C_3 \rho}{\gamma F(\gamma) \omega^2}; \]  

(27)

where \( F(\gamma) \) is the denominator of the integrand from Eq. (22). \( Z_{\text{anchor}} \) is the longitudinal acoustic impedance of the anchor and could be calculated using

\[ Z_{\text{anchor}} = v_L \rho S; \]  

(28)

where \( S \) is the anchor area and \( v_L \) in Eq. (29) is acoustic velocity

\[ v_L = \sqrt{\frac{E(1 - v)}{\rho(1 + v)(1 - 2v)}}. \]  

(29)

\( \Delta W' \) could also be calculated as:

\[ \Delta W' = \pi \sigma_{zz} u_{zz} A. \]  

(30)

Equation (26) provides the simplest calculation of \( \Delta W' \), allowing an analysis of results from the analytical approaches and a finite element analysis.

4. RESULTS

The previous section shows calculations of lost or transferred energy from the resonator to the substrate. To calculate anchor loss-related \( Q \), a calculation of total energy of the structure is required. Since the disk is supported at a nodal region within its radius (see the lumped model in Fig. 11) the disk could be considered two separate resonators vibrating with the same frequencies. The total resonance energy of the structure is the sum of vibration energies of these separated disks is calculated using

\[ W = \frac{\pi E}{2} \frac{H \cdot A_p^2 \Sigma}{1 - v^2}; \]  

(31)

where \( H \) is the resonator thickness and \( A_p \) is the vibration amplitude of the resonator, and

\[ \Sigma = \lambda_p^2 \left[ J_1^2(\lambda_p) - J_0(\lambda_p) J_2(\lambda_p) \right]. \]  

(32)

To calculate \( \lambda_p \) one must use

\[ \lambda_p = \frac{\omega R}{\sqrt{\frac{E}{\rho(1 - v^2)}}}; \]  

(33)

where \( R \) is the radius of the object of study.

Figure 12 illustrates the result of harmonic analysis of the resonator. From this analysis, the resonator \( Q \) is

\[ Q = \frac{J_f}{BW}; \]  

(34)

where \( BW \) is the 3 dB bandwidth of the resonator. Also, for the energy-based calculations, \( Q \) could be calculated using

\[ Q = \frac{2 \pi}{\Delta W}. \]  

(35)

It is clear that, \( Q \) has an inverse relation with the anchor area. Therefore, increasing the anchor area in a RSACMDR will directly reduce its \( Q \). Thus, for applications where coupling with other resonators is unnecessary, the number of anchor segments could be decreased.

The design summary of the resonator is addressed in Table 1. As illustrated, these calculations of \( Q \) using either approach have reasonable agreements and are comparable with the state of the art resonators in literature.
5. CONCLUSION

Anchor loss of RSACMD resonator designed for low GSM applications has been calculated using different methods. The results in this paper show an acceptable agreement between different two approaches. Based on these calculations, the $Q$ of the resonator could increase by reducing the number of its supports (i.e., anchor area). For example, attaining a $Q$ as high as 20000 in GHz frequencies is possible by reducing the number of anchor segments from 8 to 2 segments, at the price of increasing the effect of 2nd, 3rd and 4th longitudinal spurious modes. Further research on design and modelling of RF MEMS resonators is ongoing.

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