Analytical and Numerical Solution of 2D Problem for Transversely Isotropic Generalized Thermoelastic Medium with Green-Naghdi Model II

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In this paper, a comparison was made between the analytical and numerical solution of a two-dimensional problem for a transversely isotropic generalized thermoelastic medium. The study is carried out in the context of generalized thermoelasticity proposed by Green and Naghdi’s theory of type II. The problem has been solved analytically using the normal mode method with the eigenvalue approach and numerically using a finite element method. The accuracy of the finite element formulation was validated by comparing the analytical and numerical solutions for the field quantities.

1. INTRODUCTION

Thermoelasticity theories, which admit a finite speed for thermal signals (second sound), have aroused much interest in the last three decades. In contrast to the conventional coupled thermoelasticity theory based on a parabolic heat equation,\(^1\)\(^-\)\(^^3\) which predicts an infinite speed for the propagation of heat, these theories involve a hyperbolic heat equation and are referred to as generalized thermoelasticity theories. Among these generalized theories, the theory proposed by Lord and Shulman\(^4\) involving one relaxation time and the theory developed by Green and Lindsay\(^5\) involving two relaxation times have been subjected to a large number of investigations. In the Lord–Shulman theory, a modified Fourier’s law of heat conduction including both the heat and its time derivative replaces the conventional Fourier’s law whereas in the Green-Lindsay model, Fourier’s law of heat conduction is left unchanged but the classical energy equation and stress-strain temperature relations are modified. Verma and Hasebe\(^6\) studied wave propagation in transversely isotropic plates in the context of generalized thermoelasticity proposed by the Lord-Shulman theory. Othman\(^7\) studied the dependence of the modulus of elasticity on the reference temperature in a two-dimensional problem of generalized thermoelasticity under Lord-Shulman theory. In view of some experimental evidence available in favour of finiteness of heat propagation speed, generalized thermoelasticity theories are considered to be more realistic than the conventional thermoelasticity theory in dealing with practical problems involving very large heat fluxes and short time intervals, like those occurring in laser units and energy channels. The relevant literature can be found in Chandrasekharaiah,\(^8\) Othman,\(^9\)\(^-\)\(^11\) and Ignaczak.\(^12\)

On the experimental side, available evidence in support of the existence of finite thermal wave speed in solids is rather sparse, although an experimental study for second sound propagation in dielectric solids and some related experimental observations were reported nearly four decades ago.\(^13\)\(^-\)\(^18\) Most engineering materials such as metals possess a relatively high rate of thermal damping and thus are not suitable for use in experiments concerning second sound propagation. But, given the state of recent advances in material science, it may be possible in the foreseeable future to identify (or even manufacture for laboratory purposes) an idealized material for the purpose of studying the propagation of thermal waves at finite speed.

Relevant theoretical developments on the subject were made by Green and Naghdi.\(^19\) They developed three models for generalized thermoelasticity of homogeneous isotropic materials which are labelled as model I, II, and III. The nature of these theories is such that when the respective theories are linearized, model I\(^20\) reduces to the classical heat conduction theory based on Fourier’s law. The linearized versions of model II and III permit propagation of thermal waves at finite speed. Model II, in particular, exhibits a feature that is not present in the other established thermoelastic models as it does not sustain dissipation of thermal energy.\(^20\) In this model, the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among other constitutive variables. Green-Naghdi’s third model\(^21\) ad-