Volume 23, Number 2, June 2018

EDITORIAL

The ICSV25: Sound of Peace Bell
Akira Ikuta .................................................. 123

ARTICLES

Design and Testing of a Mechatronic Device to Actively Self Suppress Vibration in Structures
Simone Cinquemani, Marco Bassetti and Ferruccio Resta .................. 124

Condition Monitoring of Single Point Cutting Tools Based on Machine Learning Approach
N. Gangadhar, Hemantha Kumar, S. Narendranath and V. Sugumaran .......... 131

Dynamic Analysis of Half Car Model with MR Damper as Semi-Active Suspension Element
K. Hemanth, Hemantha Kumar and K. V. Gangadharan ......................... 138

Application of Spectral Element Method Combining Dilatation Theory to Sound Generated by a Co-rotating Vortex Pair
Zhenzhong Bao and Guoliang Qin ........................................... 147

Rolling Bearing Fault Detection in the Initial Stage of Degradation Based upon Optimized NLM and TKEO
Hongru Li, Baigian Chen, He Yu, Xiaolong Li and Hongqiang Gu ............. 157

Dynamic Characteristics of the Flange Joint with a Snap in Aero-Engine
Yu Liu, Jianjun Wang and Liqiang Chen .......................................... 168

Vibration Analysis of a Spring Supported FG Beam Under Harmonic Force
Cihan Demir and Merter Alkınsoy .................................................. 175

Roughness Evaluation Approach for Nonstationary Vehicle Noise Based on Wavelet Packet and Neural Network Techniques
Hui Guo, Jinsong Wang, Xiaolan Wang, Xingning Liu and Jiarong Li ............ 185

Numerical Simulations of Tap Test on Composite Structures
Sung Joon Kim ................................................................. 195

FFT-ApEn Analysis for the Vibration Signal of a Rotating Motor
Yiping Zhou, Shixun-Ho Chang, Shuangwu Wu, Xiong You Cai, Lin Yang and Z. Y. Xu ...... 203

Attributes of a Vibration Isolator Design with Stiffness Nonlinearities
Sudhir Kaul .......................................................... 208

Rolling Bearing Fault Trend Prediction Based on Composite Weighted KELM
He Yu, Hong-rui Li, Zai-ke Tian and Yu-kai Wang .......................... 217

Vibration Analysis of an Axially Moving Plate Based on Sound Time-Frequency Analysis
Qianqian Lu, Wei Shao, Jingfeng Wu and Chunlin Xia ................................ 226

Acoustic Emission Signal Analysis and Event Extraction through Tuned Wavelet Packet Transform and Continuous Wavelet Transform While Tensile Testing the AA 2219 Coupon
P. Arun Bose, T. Sasikumar, P. Arul Jose and Jey Philip ........................ 234

Analysis on a Kinetic Theoretical Model of the Straight-Curved Pipe Conveying Fluid
Hua-bin Wen, Yi-ren Jiang, Yun-dong Li and Yun Huang .......................... 240

Location Optimization of Monopole Equivalent Sources in Wave Superposition Method
Shaowei Wu and Yang Xiang ..................................................... 254

Free Vibration of Angle-ply Laminated Conical Shell Frusta with Linear and Exponential Thickness Variations
K. K. Viswanathan, A. K. Njar Hafizah and Z. A. Aziz ............................ 264

About the Authors .................................................................. 277
The ICSV25: Sound of Peace Bell

It is our great pleasure to invite you and your accompanying persons to participate in the 25th International Congress on Sound and Vibration (ICSV25), to be held from 8 to 12 July 2018 at the Grand Prince Hotel Hiroshima, Hiroshima, Japan.

The ICSV25 is jointly organized by the International Institute of Acoustics and Vibration (IIAV), the Acoustical Society of Japan (ASJ) and the Institute of Noise Control Engineering of Japan (INCE-J) in cooperation with the Prefectural University of Hiroshima, the Hiroshima City University and the Kindai University. This is the first major international congress to be held in Hiroshima covering the whole spectrum of acoustics and vibration. This congress is a leading event in the area of acoustics and vibration and provides an important opportunity for scientists and engineers to share their latest research results and exchange ideas on theories, technologies and applications in these fields. The congress will feature a broad range of high-level technical papers from around the world. In total, 1,040 abstracts from 55 countries, have been accepted, including abstracts of 180 papers that were submitted for peer review.

In the city of Hiroshima, delegates can enjoy Japanese traditional cultures, the beauty of nature, night-life, gastronomy, and shopping. Hiroshima City, world-renowned as “City of Peace,” is the capital of Hiroshima Prefecture. It is a beautiful city endowed with natural treasures including the Seto Inland Sea and six rivers, which flow through mountains and urban areas. Hiroshima Peace Memorial Park including A-bomb Dome (World Heritage Site) & Hiroshima Peace Memorial Museum, and the Miyajima Island & Itsukushima Shrine (World Heritage Site) are among the most interesting tourist attractions in Japan. The ICSV25 venue this year is the Grand Prince Hotel Hiroshima. This hotel is located at a 15 minutes driving distance from the city center of Hiroshima. The 23-floor, triangular building allows for views of Hiroshima Bay from all 510 rooms with a seasonal outdoor pool and hot spring baths. The hotel is also conveniently located for tourism: you can reach Miyajima Island in only 26 minutes by high speed boat from Moto-Ujina Pier in front of the hotel. The scenic view of the Seto Inland Sea will give participants of the congress a new resort experience.

The ICSV25 Scientific Programme is structured in 14 Subject Areas, containing Structured Sessions and Regular Sessions. The themes include: acoustical measurement and instrumentation, active noise and vibration control, aeroacoustics and aviation noise, environmental noise mapping and urban soundscape, physical acoustics, ultrasound and wave propagation, industrial and occupational noise and vibration, machinery noise and vibration, materials for noise and vibration control, psycho-physio-bio acoustics, road and rail traffic noise and vibration, room and building acoustics, signal processing and simulation, underwater and maritime noise, and acoustics education.

The Scientific Programme will also be enriched by six distinguished plenary lectures: “Community noise — an overview of past, present and future” by Marion Burgess, Australia; “Acoustic metamaterial — from controlling the sound field to manipulating acoustic wave” by Jun Yang, China; “Unique vibration phenomena in high-speed, lightweight, compliant gears” by Robert Parker, USA; “Step forward in higher order signal processing for vibro-acoustical structural health monitoring and NDT — novel nonlinear higher order frequency response functions” by Len Gelman, UK; “35 years history of modal analysis” by Svend Gade, Denmark; and “Elucidation of mechanisms of bone-conduction perception and its applications to welfare devices” by Seiji Nakagawa, Japan.

The ICSV25 Technical Exhibition is an interesting and important part of the Congress and will provide delegates with a wide range of products and expert services on instrumentation, software, and consultancy. All refreshment breaks, including morning and afternoon tea, coffee and lunch, will be held within the exhibition area. The exhibition is centrally located near the main lecture hall in the congress hotel, and delegates can become aware of the latest advances in acoustics, noise & vibration technology.

A range of social and cultural activities are planned for ICSV25 delegates as well as accompanying persons. These activities include the Opening Ceremony and the Welcome Reception on Monday, 9th July, at the Grand Prince Hotel Hiroshima; IIAV & ASJ, INCE-J Members Boat Tour to Miyajima Island & Itsukushima Shrine on Tuesday, 10th July; Gala Dinner with Kagura performance (a music and dance dedicated to Shinto Gods) on the evening of Wednesday, 11th July at the Grand Prince Hotel Hiroshima; and lastly the Closing Ceremony will take place on Thursday, 12th July. Furthermore, the following exciting programmes have been prepared: Accompanying persons tour A on Monday, 9th July, visit to an elementary school in Hiroshima and Hiroshima Peace Memorial Park including A-bomb Dome & Hiroshima Peace Memorial Museum; Optional tour A on Tuesday, 10th July: half day visit to Iwakuni city & Kintai Bridge with some ancient technology; Accompanying persons tour B & Optional tour B on Wednesday, 11th July, visit to Onomichi city with its medieval setting.

Students specializing in sound and vibration are especially welcome at ICSV25 and can enjoy reduced fees as well as an opportunity to win the Best Paper Award. Along with the IIAV officers and the Local Organizing Committee, we look forward to welcoming you in Hiroshima this July.

Akira Ikuta
ICSV25 General Chair
Design and Testing of a Mechatronic Device to Actively Self Suppress Vibration in Structures

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(Received 22 April 2015; accepted 10 June 2016)

Application of active vibration control often require a complex setup. When large structures are considered, it is often necessary to have a high number of sensors and actuators, suitably cabled, in addition to all the devices necessary to condition and amplify the signals of measurement and control and to execute in real time the control algorithms synthesized. This work arises from the need to simplify this situation, developing a stand-alone device that is able of carrying out operations of vibration control in an autonomous way, thus containing in itself an actuator, the sensors needed to evaluate the vibratory state of the structure, and a controller. The design of the smart damper covers many aspects and requires a strong integration of different disciplines. A prototype has been realized and tested on a vibrating structure. The experimental results show good performance in suppress vibration.

1. INTRODUCTION

The aim of active vibration control is to enhance the performance of a system (e.g. comfort, fatigue life, etc.) by limiting vibrations. One of the most effective technique to reach this goal is to increase the equivalent damping of the system and then the dissipation of the kinetic energy (the so called sky-hook damping technique).

This practice can be advantageously realized by using inertial actuators to exert the desired control forces. An inertial actuator is a mass supported on a spring and driven by an external force. The force is normally generated electromagnetically or exploiting piezoelectric and magnetostrictive effects. Unlike reactive actuators, inertial actuators do not need to react off the base structure, so they can be used as modules that can be directly installed on a vibrating structure. Among the numerous applications in this field some recent studies are reported.

Although the feedback loop for the ideal sky-hook damper is unconditionally stable, it becomes conditionally stable once inertial actuator are used. Stability limits due to the use of proof-mass devices have been deeply studied and different control strategies have been implemented to increase the performance of these devices. Despite the use of these actuators is characterized by limitations on stability, their use is still intensive.

In practice, however, applications of active vibration suppression require a complex experimental setup. As a matter of fact, on large structures a large number of sensors and actuators have to be installed (often even in places difficult to be reached) and all the devices must be wired to a real time board to manage measurements and control signals. This paper presents a solution developed to achieve an active damper based on inertial actuator embedding both the sensors needed to obtain feedback signals, both the control logic that determines the control force to be exerted to perform the sky-hook damping technique. This solution therefore allows to have, in a single stand-alone device, all that is required to effectively perform the task of suppressing vibration. It is worth to note that, as to control a large structure a huge number of actuators and sensors is required, the new device allows wiring minimization as the feedback control loops only act locally, providing a decentralized control strategy.

The paper is structured as follows. Section 2 introduces the layout of the smart damper, while Sections 3 and 4 describe the model and the design of the device. In Section 5 the damper is realized and tested to assess its performance in suppress vibration autonomously. Finally conclusions are drawn in Section 6.

2. LAYOUT OF THE SMART DAMPER

The layout of the smart damper is shown in Fig. 1. The vibrating structure is modeled with an equivalent mass $m_s$, stiffness $k_s$ and damping $c_s$. The smart damper is made of an inertial actuator, whose main mechanical features are $m$, $k$ and $c$, and an electronic board embedding the sensors and the controller. The acceleration of the structure $(\ddot{y})$ is measured by an accelerometer placed on the fixed frame of the actuator. This signal must be integrated and suitably conditioned to estimate the velocity of the structure $(\dot{y})$. According to this feedback, the controller gives as output a control signal that has to be amplified and then sent to the actuator to exert the corresponding force $F_a$.

As all of these features are embedded on a PCB board, the damper does not require any other device and can work autonomously, thus reaching the goal of this work. Obviously the smart device does require an integrated design of all the constituent elements and their synergistic operation. In particular, the design of the electronics embedded on the device can not be separated from the knowledge of the dynamics of the actuator and its interaction with the structure.

https://doi.org/10.20855/ijav.2018.23.21072  
(Received 22 April 2015; accepted 10 June 2016)
3. DESIGN OF THE ACTUATOR

Magnetodynamics actuators consist on an inertial mass \( m \), mounted on a spring-damper element (where \( k \) and \( c \) are the stiffness and the damping respectively), excited by means of an electromagnetic force \( F_a \). According to variables depicted in Fig. 1 the equation of motion of the inertial actuators is:

\[
m \ddot{x} + c \dot{x} + k (x - y) = F_a;
\]

then:

\[
m \ddot{x} + c \dot{x} + kx = F_a + c \dot{y} + ky; \tag{1}
\]

while the force \( F_T \) transmitted to the structure is:

\[
F_T = m \ddot{x}. \tag{2}
\]

Equations (1) and (2) can be rewritten as:

\[
1 = \frac{F_T}{m \ddot{x}} = \frac{F_a + c \dot{y} + ky}{m \ddot{x} + c \dot{x} + kx}. \tag{3}
\]

Splitting the terms on the right side of the equation and performing the Laplace transform

\[
F_T(s) = \frac{F_a(s)}{ms^2 + cs + k} + \frac{(cs + k) Y(s)}{(ms^2 + cs + k)}, \tag{4}
\]

the transmitted force \( F_T \) can be obtained as:

\[
F_T(s) = \frac{ms^2}{(ms^2 + cs + k)} F_a(s) + \frac{(cs + k)}{(ms^2 + cs + k)} \cdot ms \cdot \dot{Y}(s) \tag{5}
\]

or, in a simplest form:

\[
F_T(s) = T_a(s) \cdot F_a(s) + Z_{aa}(s) \dot{Y}(s); \tag{6}
\]

where:

\[ T_a(s) \text{ is the so called blocked response transfer function between the force transmitted to the structure (when the structure is blocked, } \dot{y} = 0) \text{ and the electromagnetic force } F_a \text{ and it is:} \]

\[
T_a(s) = \frac{F_T(s)}{F_a(s)} = \frac{s^2}{s^2 + 2h_a \omega_a s + \omega_a^2}; \tag{7}
\]

where \( h_a = c/m \) and \( \omega_a = \sqrt{k/m} \) are the damping ratio and the natural frequency of the actuator itself respectively.

\[ Z_{aa}(s) \text{ is the actuator mechanical impedance that is the transfer function between the velocity of the structure } \ddot{x} \text{ and the transmitted force (when the actuator is not powered).} \]

\[
Z_{aa}(s) = \frac{F_T(s)}{s \cdot \dot{z}(s)} = \frac{m_a s \cdot 2h_a \omega_a s + \omega_a^2}{s^2 + 2h_a \omega_a \cdot s + \omega_a^2}. \tag{8}
\]

Figure 2 shows the blocked response transfer function of the inertial actuator and its mechanical impedance. For frequencies higher than \( \omega_a \), the blocked response \( T_a \) is constant and has no phase shift with respect to the electromagnetic force \( F_a \). In this condition the actuator can be considered as an ideal force generator. Vice versa, for frequencies lower than \( \omega_a \), the transmitted force is very small, with a phase shift shown in Fig. 2. For this reason it is better to use this kind of actuator to suppress vibration in a range of frequencies higher than the natural one for the actuator. The contribution of the mechanical impedance of the structure is quite negligible and does not lead to instability. \(^3\)

The active damper has been designed to have a natural frequency \( f_a \) equal to 50 Hz. A preliminary magnetic design of the system suggests to have an inertial mass equal to \( m_a = 450 \text{ g} \). Then the stiffness of the supporting element can be easily obtained as \( k_a = m_a / f_a^2 = 4.44 \cdot 10^4 \text{ N/mm} \).

The suspension shape must be designed so that it is easy to move the inertial mass along its main axis, while rotations and displacements along the others are constrained. To reduce the radial dimension, a S-like shape has been taken into account. The linear length of each arm and their thickness have been optimized by means of a finite element software.

The force generated by the actuator is the result of the interaction between a variable magnetic field (generated by a variable current \( I \) flowing into the coil) and a constant magnetic field generated by permanent magnets. The force \( F_a \) is directly proportional to the current \( I \) flowing into the windings:

\[ F_a = BI \cdot I; \tag{9} \]

where \( BI \) is the power coefficient describing the state of magnetization of the system. \( B \) being the flux density and \( l \) the total length of the coil. The topology of the actuator is similar to that of conventional devices in which, the magnetic circuit is obtained using a ferrite magnet ring designed to generate a constant magnetic flux intensity in a 1 to 2 mm air-gap. The driving coil inside the air gap moves in the uniform magnetic field. To estimate the force \( F_a \), the actuator can exert, it is necessary to develop a numerical model to evaluate the flux density at the air-gap. The results of the model give information on
the optimal geometry and size of the actuator. Figure 3 shows the trend of the magnetic field lines in a radial section of the actuator (see Fig. 4).

As known, the phase shift between the supply voltage $V$ and the current $I$ significantly varies with the frequency. This involves a phase shift, variable with frequency, between the control signal ($V$) and the force exerted by the actuator ($F_a$) that can not be accepted in applications of vibration control. For this reason, rather than compensating for it, it was considered preferable to use a current amplifier thus ensuring the absence of delay between the control signal and the force generated.

The flux density at the air-gap is estimated to be equal to 0.7 T and it should allow the device to exert a maximum force $F_T$ equal to 45 N with a current $I=1A$.

4. DESIGN OF THE CONTROLLER

The control logic to be implemented would be very simple from the theoretical point of view. The aim of the skyhook damping technique, in fact, is to generate a control force ($F_T$) that is:

$$F_T = -\bar{c}\dot{y};$$

(10)

where $\dot{y}$ is the velocity of the structure and $\bar{c}$ is the damping added by the control. The input signal is the acceleration of the vibrating structure ($\ddot{y}$) measured through an accelerometer mounted on the base of the actuator. The signal has to be integrated to obtain the speed $\dot{y}$. As known, the relationship described by Eq. (10) is hard to be obtained when using inertial actuators because of the dynamics of the device. This difficulty can be summarized looking to the control scheme depicted in Fig. 6.

The controller, in fact, has to take into account the need to integrate a signal and to filter out the low frequencies amplified by this operation. The gain of the controller may be adjustable and it physically represents the damping introduced into the structure by the active control. Subsequently, the control signal ($V$) must be amplified to generate the current ($I$) circulating in the windings from which depends the force generated by the actuator ($F_a(I)$). Finally it must be considered that the control force $F_T$ is a function of both the force $F_a$, through the blocked response function ($T_a$), and of the speed ($\dot{y}$) through the mechanical impedance function ($Z_{aa}$). The main difficulty in carrying out such operations lies in the need to introducing the lowest possible phase shift between the measured signal ($\ddot{y}$) and the control force ($F_T$).

The core of the electronic board is the controller. The analog integrator circuit is shown in Fig. 7. There are two MEMS accelerometers installed on-board, with resolution of ±2 g and ±16 g. Both of them are used only along z-axis. The signal is low-pass filtered with a single-pole RC filter, tuned at 550 Hz. This helps to remove high frequency noise. The signal coming from the MEMS accelerometer is then routed through the $W_{01}$ and $W_{02}$ jumpers. These jumpers can direct the signal to the integrator. The $C_6-R_7$ filter acts as high-pass filter, in order to remove the static acceleration of the system: a constant input signal, as known, saturates the analog integrator very quickly.

The fundamental integrator circuit is constructed by placing a capacitor $C_5$ in the feedback loop of an inverting amplifier. Assuming an ideal op-amp, current conservation at the indicated node gives:

$$IR_T = IC_5;$$

and

$$\frac{V_{in}}{R_T} = -C_5 \frac{d}{dt}V_{out}. \quad (11)$$

Rearranging equation (11) and integrating from 0 to $t$, we ob-
Expression (13) indicates that there is a 90° phase difference between the input and the output signals. This phase shift occurs at all frequencies. The gain of the amplifier given by the ratio of the output voltage to the input voltage due to the presence of small dc offset voltages at the input. This problem may be overcome by connecting a resistor, R3, in parallel with the feedback capacitor C5 as shown in Figure 7. The feedback path consists of the capacitor C5 in parallel with the resistor R3. The equivalent impedance of the feedback path is:

\[ Z_3 = \frac{R3 \cdot ZC5}{R3 + ZC5} = \frac{R3}{1 + j\omega R3 \cdot C5}. \]  

(14)

The transfer function becomes:

\[ \frac{V_{out}}{V_{in}} = -\frac{Z3(\omega)}{ZR7(\omega)} = -\frac{R3}{R7} \cdot \frac{1}{1 + j\omega H}; \]  

(15)

where

\[ \omega H = \frac{1}{R7C5}. \]  

(16)

At frequencies lower than \( \omega H \) the voltage gain becomes equal to \( R3/R7 \), while at frequencies higher than \( \omega H \) the gain decreases at a rate of 20 dB per decade. So we have seen that the integration is achieved by charging the feedback capacitor.

The power amplifier is shown in Fig. 8. It has been configured as voltage-to-current amplifier: this configuration is necessary because of the magnetic-dynamic actuator. The applied force in fact is proportional to the applied current, while the measured acceleration is proportional to the measured voltage. The transfer function of the power amplifier is equal to:

\[ \frac{I_{out}}{V_{in}} = \frac{1}{R14}, \quad \frac{R13}{R12} = \frac{1}{5\Omega}. \]  

(17)

The voltage compliance of the amplifier is equal to ±12 VDC. This active amplifier is not affected by the impedance variation of the load until it works within the voltage compliance. Otherwise it is quite simple to extend this range up to ±35 VDC.

Power supply has been designed to have as higher efficiency and simplicity as possible. To be able to achieve them, switching power converters have been used. As shown in Fig. 10, there are one dual-output dc/dc converter (±12 VDC) and one single-output dc/dc converter (+3.3 VDC). This kind of dc/dc power converter works at medium frequency. There are many advantages in using this system: in order to reduce the size and to increment the power density of the power supply stage. Otherwise it generates more noise than linear power supplies. In order to reject as much noise as possible, a passive L-C filter has been implemented. The filter has been placed in series on both arms of the power supply. The transfer function of this filter is calculated as:

\[ \frac{-R3 + sL + sC}{R2} + \frac{R3}{s} + R1 + sL \]  

where \( R1 \) is the eddy resistor associated to L and \( R2 \) is the load resistor. From equation (18) is it possible to calculate the resonance frequency of the filter and the ripple attenuation at the frequency of interest. For the dc/dc converter used in this project, the first harmonic of the switching frequency is equal to 100 kHz.

5. REALIZATION AND TEST OF THE SMART DAMPER

Figure 9 shows the PCB board of the first prototype. It contains a DC/DC Power Supply, placed near the mounting holes. This has been necessary, due to the mass of this component, to maximize the stiffness of the PCB Board. Below the DC/DC Supply there are the Output Signal Socket, the Feedback Power Resistor and The Power Amplifier. As shown in Fig. 9, all of them are surface mounted devices. This technique allows to use the PCB board as an heat-sink. The vias all around these components helps to improve the thermal transfer from the top layer to the bottom layer of the PCB.

Two multi-turn trimmers are placed on the PCB too (Fig. 7). They acts as gain control for the analog integrator. Also two MEMS accelerometers have been placed on-board. Both of them are made by Analog Devices, and can measure up to +/-2 g and +/-16 g. We have chosen to mount two accelerometer to be able to measure and control different kind of vibrations. Each accelerometer can be connected (or not) using the Accelerometer Selection Jumper. The smart damper is depicted...
Experimental blocked response transfer function has been obtained to evaluate the real performance of the inertial actuator (Fig. 12). It can be noted that the device natural frequency is slightly different from the desired one (50 Hz vs. 55 Hz), but it is reasonably close. For higher frequencies, the device is able to exert a force between 40 N and 50 N as planned.

To assess the performance of the smart damper, some preliminary tests have been carried out. The test rig (Fig. 13) consists of a 2 meters long cantilever beam made of steel that is forced by a piezoelectric patch placed in $\xi_D$. The smart damper is placed in point $\xi_C$ and an external accelerometer is placed at the end of the bar ($\xi_M$). Figure (14) shows the frequency response function of the system between the acceleration measured in $\xi_M$ and the driving force. The natural frequency of the smart damper ($\omega_0$) is depicted on the same graph.

As discussed in previous paragraphs, and more deeply by Elliott et al., inertial actuators can be profitably used to reduce vibration related to modes whose eigenfrequencies are higher than the natural frequency of the actuator itself ($\omega_0$). For this reason, experimental tests are carried out on modes 3, 4. Each test is performed by driving the system at one of its resonance frequencies. Once the system reaches its steady state condition, the smart damper is powered and the gain of the controller is increased till the gain margin of the controlled system is close to zero. Figures 15 and 16 show the effect of the smart damper on modes 3 and 4 respectively. In the first case the smart damper can reduce the vibration of 60% (from a root mean square value of $3.8 \cdot 10^{-2} \text{m/s}^2$ to $1.5 \cdot 10^{-2} \text{m/s}^2$), while in the second case vibration is reduced of 67% (from a root mean square value of $5.2 \cdot 10^{-2} \text{m/s}^2$ to $1.7 \cdot 10^{-2} \text{m/s}^2$)

Despite the system being a prototype, preliminary results confirm the correctness of the project and suggest interesting applications for this innovative device.

6. CONCLUSIONS AND FUTURE WORKS

The paper presented the design and testing of an active smart damper for vibration suppression. This device allows to have, in a single stand-alone device, all that is required to effectively perform the task of reducing vibration, without the need of complex set-up. As the system is complex, the design process is based on a multi-physic model considering the mechanical,
the electromagnetic and the electronic aspects. The device has been produced and tested. Preliminary results confirm the ability of the system to reduce vibration. The results encourage us to follow this direction. Future developments of the device plan to replace the analog control logic with a programmable microprocessor. This solution will allow to use different control logic and to improve the performance of the smart damper.

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Figure 15. Vibration suppression on mode 3.

Figure 16. Vibration suppression on mode 4.
**Condition Monitoring of Single Point Cutting Tools Based on Machine Learning Approach**

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(Received 15 August 2015; accepted 5 October 2016)

This paper presents the use of multilayer perceptron (MLP) for fault diagnosis through a histogram feature extracted from vibration signals of healthy and faulty conditions of single point cutting tools. The features were extracted from the vibration signals, which were acquired while machining with healthy and different worn-out tool conditions. Principle component analysis (PCA) used to select important extracted features. The artificial neural network (ANN) algorithm was applied as a fault classifier in order to know the status of cutting tool conditions. The accuracy of classification with MLP was found to be 82.5 %, which validates that the proposed approach is an effective method for fault diagnosis of single point cutting tools.

**NOMENCLATURE**

\[ \lambda \] Eigen values  
\[ \xi_i \] Input vector  
\[ \theta_j \] Threshold of the \( j^{th} \) neuron of the hidden layer  
\[ u \] Eigen vector  
ANN Artificial neural network  
BP Back propagation  
DAQ Data acquisition  
FP False positive  
HSV Hue saturation value  
\( i \) Input layer  
\( j \) Hidden layer  
\( k \) Output layer  
MLP Multilayer perceptron  
MSE Mean square error  
NI National instruments  
PCA Principle component analysis  
TP True positive  
\( W_{ij} \) Weight vector connecting the \( j^{th} \) neuron of the input layer to the \( i^{th} \) neuron of the hidden layer  
\( W_{jk} \) Weight vector connecting the \( j^{th} \) neuron of the hidden layer to the \( k^{th} \) neuron of the output layer

**1. INTRODUCTION**

Automated tool condition monitoring systems improves product quality and reduce defects and result in increased productivity. Automated tool monitoring systems prevents abrupt failure of the cutting tools and are extremely useful for modern automated machine tools.\(^1\) There are various machine learning techniques which have been applied for fault diagnosis in machinery systems. Ravikumar et al. used machine learning approaches for automated visual inspections of machine components.\(^2\) Sugumaran and Ramachandran studied the effects of a number of features on the classification of roller bearing faults, using both support vector machines and proximal support vector machines in their study.\(^3\) Indira et al. found a method for the calculation of optimum data and bin size of histogram features by employing fault diagnosis in a monoblock centrifugal pump.\(^4\) Taheri et al. used intelligent approach for a cooling radiator fault diagnosis, which was based on an infrared thermal image processing technique.\(^5\) Liu et al. presented a fog level detection method-based on an image hue saturation value (HSV) histogram — through an analysis of different HSV information contained in different weather images.\(^6\) Sakthivel et al. reported the use of histogram features for decision tree-based fault diagnosis in a monoblock centrifugal pump.\(^7\) Sugumaran and Ramachandran carried out fault diagnosis in roller bearing by using a fuzzy classifier and histogram features, with a focus on automatic rule learning.\(^8\)

1.1. Vibration Mode of a Turning Tool

In a turning process, three different types of mechanical vibrations are present due to a lack of dynamic stiffness/rigidity of the machine tool system, which comprises the tool, tool holder, workpiece, and machine tool itself, as explained by Tobias.\(^9\) These are free, forced, and self-excited chatter vibrations. Free vibrations are induced by shock and forced vibrations are due to an unbalanced effect in machine tool assemblies like gears, bearings, and spindles. Free and forced vibrations can be easily identified and eliminated. However, self-excited chatter vibrations are still not fully understood due to their complex nature. They are harmful for any machining process, including turning. Self-excited chatter vibrations are generally classified into either primary chatter or secondary chatter.\(^10\) Primary chatter is caused by friction between tool-workpiece, thermo-mechanical effects, or by mode coupling. Secondary chatter is caused by the regeneration of wavy surfaces on the workpiece. Regenerative vibration is the most destructive among the different vibrations. Bhuiyan et al. investigated tool wear, chip formation and the surface roughness of workpieces, each under different conditions while machining-, and used acoustic emission (AE) and vibration signature analysis in turning.\(^11\) They found that the AE and vibration com-
ponents can effectively respond to different occurrences in tool wear and surface roughness. Sevilla et al. presented a reconfigurable system using the vibration signals generated from machining tests, which were performed under different tool conditions, and cut parameters for tool condition monitoring in the high-speed machining (HSM) process. Patra et al. developed a tool condition (flank wear) monitoring system using the vibration signals of the machining process. They showed that the fuzzy radial basis function based neural network can recognize the features extracted from the time domain by applying the wavelet packet approach, which underlies the vibration signals more effectively than other methods (e.g., back propagation neural network, radial basis function network, and normalized radial basis function network). Kilundu et al. integrated signal processing methodology and different machine learning methodologies (e.g., decision trees, Bayesian networks, k-Nearest Neighbour and neural network) to handle the computational complexities in monitoring the tool-wear by using cutting vibration signals. Jemielniak et al. studied the force, vibration, and acoustic emission signals while turning a specific material and extracting some features from the time, frequency, and time-frequency domains of signals detecting the tool wear. Scheffer and Heyns, reported the simultaneous use of vibration and strain measurement for wear monitoring in turning operation. Rao et al. analysed the workpiece roughness, vibration of the workpiece, and volume of the metal removal rate through a laser Doppler vibrometer and a high speed FFT analyser in a boring process. Rao and colleagues observed that the amplitude of vibration increased as the tool wear increased and that the feed rate was the significant parameter for affecting surface roughness. Scandiffio et al. investigated the influence of toolpath direction and tool-workpiece surface contact on the machining force, surface roughness, tool wear, and tool life in freeform milling by a ball end cutting tool when milling hard-quenched and tempered AISI D6 steel. They identified that the most influential factor for tool life was tool vibration.

2. TOOL FAILURE MODES

It is important to identify the different tool failure modes in order to select appropriate operating conditions for machining. The most widely studied tool failure modes are flank wear, breakage (fracture), crater wear, and plastic deformation. Only a few researchers reported tool failure because of notching (groove wear), cracking, and chipping. Notching and chipping changes the tool nose curvature. Figure 1 shows the tool failure modes, as depicted by Rao et al. Flank and crater wear are generally accepted as the normal tool failure modes, because the other failure modes can be avoided by selecting the proper machining parameters. The growth of flank and crater wear is directly related to the cutting time (or length of cut), unlike some of the other failure modes such as notching (groove wear), cracking, and chipping, which can occur unexpectedly, even with a new tool.

2.1. Flank Wear

Flank wear is mainly caused by the friction between the newly machined work piece surface and the tool flank face. Flank wear is marked on the cutting tool and is shown in Fig. 1a. It is responsible for a poor surface finish, a decrease in the dimension accuracy of the tool, and an increase in cutting force, temperature, and vibration. The width of the flank wear-land is usually taken as a measure of the amount of wear, and the threshold value of the width is defined as a tool reshape criterion.

2.2. Crater Wear

Crater wear normally forms on rake face. It conforms to the shape of the chip’s underside and reaches the maximum depth at a distance away from the cutting edge – where the highest temperature occurs. At high cutting speeds, crater wear is the main factor that determines the life of the cutting tool, due to a weakened tool edge which results in severe cratering and eventually fractures. Crater wear is improved by selecting suitable cutting parameters and using coated tools or ultra-hard material tools. Crater and flank wear are shown in Fig. 1a and are the most common wear types.

2.3. Notch Wear

Notch wear is a single groove formation that occurs simultaneously on the face and flank of the tool at the depth of the cut. Machining parts with severe (hard or oxidized) surfaces will cause notch wear. Figure 1a shows the depth of the cut due to notch wear.

2.4. Chipping

Figure 1b shows the chipping that occurred on the cutting edge. Chipping is the result of an overloads of mechanical tensile stresses. These stresses can be due to a number of reasons, such as chip hammering, depth of cut or high feed rate, sand inclusions in the workpiece material, built-up edge, vibrations, or excessive wear on the insert.

2.5. Thermal Cracks

Thermal cracks appear on the rake face perpendicular to the cutting edge, as shown in Fig. 1b. Thermal cracking occurs when inserts go through rapid heating and cooling cycles. This failure mode is caused by interrupted cutting and by poor application of cutting fluids.

2.6. Breakage

Breakage is a mode of failure characterized by a breakaway of material on the tool edge. Breakage occurs when the feed rate is too high or when a tool is used with too low fracture.
The usual pattern of wear of turning inserts is shown in Fig. 1c.

The different wornout conditions of the tool inserts considered in this present study are as follows: breakage, thermal cracks, notch wear, and rake face chipping, which are all shown in Fig. 2.

Vibration signals are widely used in the condition monitoring of rotating elements in machines. However, the classification of tool conditions for single point cutting tools using histogram features has not yet been attempted. In this present study, vibration signals were acquired with healthy and industrial wornout inserts. Histogram features were extracted from vibration signals to identify the status of the tool’s condition. Fault detection is possible by comparing the signals of a machine running in both normal and faulty conditions. The methodology involved in the fault diagnosis of single point cutting tools, using a machine learning approach, is illustrated in Fig. 3. The vibration signals from the cutting tools, which were mounted on an engine lathe, was acquired using an accelerometer. Forty vibration signal samples were collected for each class of tool conditions. Histogram features were extracted from each of the collected samples using Excel. The most important features out of those extracted were selected using principle component analysis (PCA) as well as fault classification by use of multilayer perceptron (MLP).

The experiments were conducted on an engine lathe. Figure 4 shows a schematic diagram of the experimental setup. An accelerometer was used along with a data acquisition system for acquiring. A National Instruments (NI) piezoelectric accelerometer and its accessories formed the core equipment for vibration measurement and recording. The output from the accelerometer was connected to the NI-9234 data acquisition (DAQ) system and analysed by using LabVIEW software from NI. The vibration signals were acquired from single point cutting under healthy conditions (new) and by considering four different wornout inserts’ conditions at a constant cutting speed of 236 m/min. The sampling frequency used in the study was 25.6 kHz and each signal (sample) had a length of 25,600 data points. For each condition of the cutting tool, 40 samples were considered and recorded carefully.

One randomly selected vibration signal in time domain for each tool condition is shown in Fig. 5. From time domain signals, it was observed that the acceleration level increased with different fault conditions of cutting tools. A time domain technique for vibration signal analysis gave an overall vibration level, but it did not provide any diagnostic information.

A histogram is a graphical representation of the distribution of numerical data. To construct a histogram, the first step is to select data bin values, that is, divide the entire range of values into a series of small intervals and then count how many values fall into each interval. The histogram features were extracted from the time domain vibration signals. From the magnitude of the signal, it was found that the range varied from class to class. Corresponding histogram plots for different tool conditions are shown in Fig. 6.

The bin range should be from the lowest value of minimum amplitude (-20) to the highest value of maximum amplitude (+20) of all five classes. The number of bins for the fault diagnosis of single point cutting tools was obtained by carrying out a series of experiments using MLP with a different number of bins. At first, range of bins was divided into two equal parts. That is to say, number of bins used was two. The two histogram features, namely $F_1$ and $F_2$, were extracted and the corresponding classification accuracy was also obtained by using MLP. A set of similar experiments were carried out with a different number of bins — from two, three, four ... 97 — and
the corresponding results are shown in Fig. 7. Upon careful observation of the results, the best classification accuracy of 94% is obtained when the number of bins is 15, with a bin width of seven. Hence, these $F_1$ to $F_{15}$ histogram features were chosen as the bin parameters.

Table 1 shows the histogram features from $F_1$ to $F_{15}$. Out of 40 samples, only two samples pertaining to each class of tool condition are shown in Table 1. All extracted histogram features, $F_1$ to $F_{15}$ extracted from the vibration signals may not contain the information required for classification. The relevant features were selected using PCA.

5. DIMENSIONALITY REDUCTION USING PRINCIPLE COMPONENT ANALYSIS

Principal component analysis (PCA) is one of the most widely used multidimensional features reduction tools. PCA is the preferred choice because it is a simple and nonparametric method of extracting relevant information from complex data sets. The goal of PCA is to reduce the dimensionality of the data while retaining as much as possible of the variation in the original data sets. Elangovan et al. discuss the use of PCA with various classifiers — mainly to reduce the data dimensionality and report improvement in classifier efficiency. A similar approach for dimensionality reduction is attempted in this work. The basic workings of a PCA are presented below:

Let $F_1$, $F_2$, ..., $F_n$ be $N \times 1$ vectors.

Step 1: Mean value is calculated using the equation:

$$F = \frac{1}{N} \sum_{i=1}^{N} F_i.$$  (1)

Step 2: Each feature is used to subtract the mean value:

$$\phi_i = F_i - \bar{F}.$$  (2)

Step 3: Matrix $A = [\phi_1, \phi_2, ..., \phi_N]$ is generated by $N \times N$ matrix and covariance matrix $C$ with the same dimension size, which is computed as follows:

$$C = \frac{1}{M} \sum_{i=1}^{N} \phi_i \phi_i^T = AA^T.$$  (3)

The covariance matrix characterizes the distribution of the data.

Step 4: Eigenvalue is computed as:

$$C = \lambda_1 > \lambda_2 > \cdots \lambda_N.$$  (4)
Step 5: Eigenvector is computed as:

\[ C = \mu_1, \mu_2, \ldots, \mu_N. \]  (5)

Since C is symmetric \( \mu_1, \mu_2, \ldots, \mu_N \) form a basis, and \((F_1 - F)\) can be written as a linear combination of the Eigenvectors:

\[ F_1 - F = b_1u_1 + b_2u_2 + \cdots + b_Nu_N = \sum_{i=1}^{N} l_i. \]  (6)

Step 6: For dimensionality reduction, it keeps only the terms corresponding to the K largest Eigen values:

\[ F_1 - F = \sum_{i=1}^{K} b_iu_i, \text{where } K << N. \]  (7)

The representation of \( E.E \) into the basis \( u_1, u_2, \ldots, u_K \) is thus,

\[
\begin{bmatrix}
    b_1 \\
    b_2 \\
    \vdots \\
    b_K
\end{bmatrix}.
\]  (8)

6. ARTIFICIAL NEURAL NETWORK

An artificial neural network (ANN) is modelled on biological neurons and nervous systems. ANN’s have the ability to learn and have the processing elements known as neurons, which perform their operations in parallel. ANN’s are characterized by their topology, weight vector, and activation functions. They have three layers: an input layer (which receives signals from the external world), a hidden layer (which does the processing of the signals), and an output layer (which gives the result back to the external world).

6.1. Multilayer Perceptron (MLP)

The MLP is a unidirectional network, which has an input layer, hidden layer, and output layer, as shown in Fig. 8. The number of neurons in the input and output layers is equal to the number of input and output variables. Generally, one hidden layer is sufficient to map the problem in a correct way. The more complex the problem is: the more neurons are desirable.

As shown in Fig. 8, all input signals are received first by the input layer and then transmitted to other neurons in the hidden layer, where the processing task occurs. The information is then received by the output layer.

Each neuron in the hidden and output layer consists of an activation function, which is generally a nonlinear function like the logistic function, given by:

\[ f(x) = \frac{1}{1 + e^{-x}}. \]  (9)

Figure 8. MLP network structure.

where \( f(x) \) is differentiable and

\[ x = \sum_{i=1}^{n} W_{ij} \xi_i + \theta_j; \]  (10)

where \( W_{ij} \) is the weight vector connecting the \( i^{th} \) neuron of the input layer to the \( j^{th} \) neuron of the hidden layer, and \( \xi_i \) is the input vector and \( \theta_j \) is the threshold of the \( j^{th} \) neuron of the hidden layer. Similarly, \( W_{jk} \) is the weight vector connecting \( j^{th} \) neuron of the hidden layer with the \( k^{th} \) neuron of the output layer: \( i \)-represents the input layer, \( j \)-represents the hidden layer, and \( k \)-represents the output layer. The weights that are important in predicting the process are unknown. The weights of the network to be trained are initialized to small random values. The choice of value selected obviously affects the rate of convergence. The weights are updated through an iterative learning process known as the Error Back Propagation (BP) algorithm. The Error Back Propagation process consists of two passes through the different layers of the network — a forward pass in which input patterns are presented into the input layer of the network and its effect propagates through the network layer by layer. Finally, a set of outputs is produced as the actual response of the network. During the forward pass, the synaptic weights in the networks are all fixed. The error value is then calculated, which is the mean square error (MSE), given by

\[ E_{tot} = \frac{1}{n} \sum_{n=1}^{m} E_n; \]  (11)

where \( E_n = \frac{1}{2} \sum_{k=1}^{n} (\xi_k^n - O_k^n)^2 \) and \( n \) is the number of neurons in the output layer.

\( \xi_k^n \) is the \( k^{th} \) component of the desired or target output vector and

\( O_k^n \) is the \( k^{th} \) component of the output vector.

The weights in the links connecting the output and the hidden
layer $W_{jk}$ are modified as follows:

$$\Delta W_{jk} = \eta((-\partial E/\partial W_{jk}) = \eta \delta_j y_j, \text{where } \eta \text{ is the learning rate.}$$

Considering the momentum term (\(\alpha\)) $\Delta W_{jk} = \alpha \delta_j y_j$ and $W_{jk} + \Delta W_{jk}$. Similarly, the weights in the links connecting the hidden and input layer $W_{jk}$ are modified as follows:

$$\Delta W_{jk} = \alpha \delta_j \xi_j;$$

where,

$$\delta_j = y_j(1 - y_j) \sum_{k=1}^{m} \delta_k W_{jk};$$

$$W_{ij}^{\text{new}} = W_{ij}^{\text{old}} + \Delta W_{ij};$$

$$\delta_k = (\xi_k - O_k)O_k(1 - O_k);$$

for output neurons and for hidden neurons,

$$\delta_j = y_j(1 - y_j) \sum_{k=1}^{m} \delta_k W_{jk}.$$ 

The training process is carried out until the total error reaches an acceptable level (threshold). If $E_{\text{tot}} < E_{\text{min}}$, the training process is stopped and the final weights are stored, which is used in the testing phase for determining the performance of the developed network. The sigmoid transfer function was used in the hidden and output layers.

### 7. RESULTS AND DISCUSSIONS

The histogram features discussed in section three are considered as features to serve as the input for the algorithm. The corresponding condition, or status, of the categorized data will be the essential output of the MLP algorithm.

#### 7.1. Feature Extraction and Selection

From the obtained vibration signals, 15 histogram features (f1, f2 ... f14 and f15) were extracted. The process of selecting relevant features is known as feature selection and it was carried out by using PCA.

These selected features serve the purpose of classification using the MLP classifier algorithm. Classification accuracy of the MLP classifier algorithm is presented in the form of a confusion matrix, as shown in Table 2. The meaning of confusion matrix is explained as follows:

In the first row, the first element shows the number of data points belonging to the ‘healthy’ class and classified by the classifier as ‘healthy’. The total number of data points in the first row is 40; 31 of them are correctly classified. In the first row, the other elements are zero except for in the fifth column, which means that nine of the healthy conditions are misclassified as rake face chipping.

The second row represents the total number of data points corresponding to the breakage condition; the first column represents misclassification of those data points as the healthy condition, which in this case is one. The second row, second column entry represents how many of the breakage samples are correctly classified as breakage by the classifier. Out of 40 samples, 37 are correctly classified. In this case, two samples are misclassified as the thermal crack condition. In the second row, other elements came out to zero. This means none of the breakage conditions are misclassified as notch wear, rake face chipping, and so on.

Misclassifications are among the healthy as well as faulty conditions, and they are about 17.5 %. However, the misclassification of healthy conditions as rake face chipping is about 4.5 %. The misclassification percentages of breakage as healthy and thermal crack conditions are 0.5 % and 1 %, respectively. The misclassification percentages of notch wear as breakage and rake face chipping conditions are 1.5 % and 2 %, respectively. The misclassification percentages of rake face chipping as healthy, thermal cracks, and notch wear conditions are 3.5 %, 0.5 %, and 4 %, respectively. The MLP classifier performs absolutely well in classifying thermal crack samples. The performances differ slightly when classifying the healthy and faulty conditions. The misclassification in the MLP classifier is 17.5 %. Out of 200 samples, 35 samples were incorrectly classified by the MLP, with a classification accuracy of 82.5 % for vibration signals.

Table 3 shows the detailed class-wise accuracy of the multi-layer perceptron. In Table 3, ‘TP rate’ and ‘FP rate’ are very important. The ‘TP rate’ stands for true positive; its value should be close to ‘1’ for better classification accuracy. The ‘FP rate’ stands for false positive, and its value should be close to ‘0’ for better classification accuracy. From the study, one can recognize the closeness of ‘TP rate’ to ‘1’ and ‘FP rate’ to ‘0’. Both values confirm that the model built is acceptable. From the obtained results, the classification accuracy can be appreciated.

### 8. CONCLUSION

This paper discusses the fault diagnosis of single point cutting tools using the machine learning approach based on vibration signals. This methodology involved collecting 40 acceleration vibration signal samples for five different classes of industrial wornout conditions. Histogram features were extracted from acquired vibration signals pertaining to all classes of fault categories. PCA was used for important feature selection. The ANN algorithm was used for fault classification. Classification accuracy was found to be 82.5 %. Thus, MLP classifier can be practically utilized to monitor the condition of tungsten carbide inserts while machining die steel.

### 9. ACKNOWLEDGEMENTS

The authors acknowledge the Centre for System Design (CSD), a centre of excellence at NITK-Surathkal, for providing experimental facility.

### REFERENCES


Table 2. Classification accuracy of the multilayer perceptron classifier.

<table>
<thead>
<tr>
<th>Healthy</th>
<th>Breakage</th>
<th>Thermal cracks</th>
<th>Notch wear</th>
<th>Rake face chipping</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>37</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>40</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>0</td>
<td>33</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>24</td>
</tr>
</tbody>
</table>

Table 3. Detailed accuracies by class.

<table>
<thead>
<tr>
<th>Class</th>
<th>TP Rate</th>
<th>FP Rate</th>
<th>Precision</th>
<th>Recall</th>
<th>F-Measure</th>
<th>ROC Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Healthy</td>
<td>0.775</td>
<td>0.05</td>
<td>0.795</td>
<td>0.775</td>
<td>0.785</td>
<td>0.946</td>
</tr>
<tr>
<td>Breakage</td>
<td>0.925</td>
<td>0.019</td>
<td>0.925</td>
<td>0.925</td>
<td>0.925</td>
<td>0.951</td>
</tr>
<tr>
<td>Thermal cracks</td>
<td>0.825</td>
<td>0.019</td>
<td>0.93</td>
<td>1</td>
<td>0.964</td>
<td>0.992</td>
</tr>
<tr>
<td>Notch wear</td>
<td>0.6</td>
<td>0.081</td>
<td>0.649</td>
<td>0.6</td>
<td>0.623</td>
<td>0.861</td>
</tr>
<tr>
<td>Rake face chipping</td>
<td>0.6</td>
<td>0.081</td>
<td>0.649</td>
<td>0.6</td>
<td>0.623</td>
<td>0.861</td>
</tr>
</tbody>
</table>


Dynamic Analysis of Half Car Model with MR Damper as Semi-Active Suspension Element

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(Received 16 August 2015; accepted 17 November 2016)

This paper presents the dynamic analysis of a half-car model with a magnetorheological (MR) damper subjected to random excitation. Experimental studies have been conducted to predict the behavior of the prototype twin-tube MR damper. The mathematical model of the prototype MR damper has been proposed by using the Bouc-Wen model. The half-car model with the MR damper has been used to predict the ride comfort and road holding performance. Comparative studies between the half-car model with the passive and semi-active suspension system with a proportional-integral-derivative (PID) control shows that the MR damper suspension system offers a good performance.

1. INTRODUCTION

The vehicle suspension system is used to mitigate vibrations which are transmitted from the road surface to the vehicle body. A good ride comfort requires a soft suspension, whereas a hard suspension is required for carrying heavy loads. A good handling of vehicles requires a suspension system which makes for better trade-offs between the above stated two criteria. In order to fulfill these conflicting requirements, a fully active or semi-active suspension system is preferred over a conventional passive suspension system. Active and semi-active suspensions are attracting more attention in improvement of both ride comfort and road holding. In particular, semi-active suspension offers a desirable performance enhanced by an active suspension without requiring high power consumption and expensive hardware.

Magneto-rheological (MR) dampers are semi-active devices that use MR fluids to provide controllable damping force. A typical MR fluid consists of 20–40 percent by volume of relatively pure (3–10 micron diameter size) magnetizable particles suspended in a carrier liquid such as mineral oil, synthetic oil, water or glycol. When the MR fluid is exposed to a magnetic field, particles in the fluid form chain-like structures in the direction of the applied magnetic field, and the fluid becomes like a semisolid material in a few milliseconds, by which it creates a resistance against the fluid flow. The yield stress of the fluid can be controlled very accurately by varying the magnetic field intensity. Ashfak et al.,1 studied the design, fabrication and performance evaluation of the MR damper and MR damper’s applications in the field of vibration. Wen,2 proposed a mathematical model called the Bouc-Wen, which characterizes the MR damper behavior. Spencer et al.,3 reviewed several idealized mechanical models for controllable fluid dampers. Kwok et al.,4 and Giuclea et al.,5 conducted experiment of the MR damper and the Bouc-Wen model has been considered to characterize the MR damper. The parameters of the Bouc-Wen model were evaluated by using a genetic algorithm and the proposed model was validated with experimental results. Cseseci and Engin,6 have carried out experimental and theoretical studies to predict dynamic behavior of the MR damper, and the Bhingam plastic model is considered for theoretical study. Shivaram and Gangadharan,7 designed a statistical model of the MR damper using the design of the experimental approach. Various factors such as magnetic field strength, volume fraction of the magnetic particle, shearing gap between piston and cylinder, and amplitude and frequency of vibrations were considered in their experimentation. Avinash et al.,8 developed the twin-tube MR damper and conducted an experiment to analyze the damping characteristic of the MR damper under different conditions such as air damping, viscous damping and MR damping.

Dutta et al.,9 evaluated the performance of the sky-hook control strategy in the modified Bouc-Wen model of a quarter car model equipped with the MR damper under random and sinusoidal excitation. Song et al.,10 studied the non-parametric modelling approach to predict the behavior of the MR damper. They used a series of continuous and differentiable mathematical functions to represent the characteristics of the physical damper. Metered et al.,11 and Wang et al.,12 carried out experiments to study the dynamic characteristics of an MR damper and used the feed-forward recurrent neural network method to predict the behavior of the MR damper. Prabakar et al.,13 evaluated the control of stationary response of the quarter car model for random excitation with the MR damper, and the optimal parameters of the MR damper were found using multi-objective optimization technique. Butz et al.,14 provided an overview of properties of electro and MR fluids and their applications. They also discussed various phenomenological models of electrorheological (ER) and MR devices. Gopard and Narayanam,15 reported dynamic responses using the two-degree freedom quarter car model with nonlinear passive elements traversing a rough road with the sky-hook control strategy. Abdolvahab et al.,16 and Agharkakli et al.,17 studied the passive and active suspension system by using the quarter car model. Chen,18 proposed a sky-hook surface sliding mode control method to semi-active vehicle suspension system for its ride comfort enhancement. Rao et al.,19 evaluated the ride comfort of off-road vehicles by replacing the normal passive damper with a controllable, two-state, semi-active damper and compared it to the passive suspension system. Sireteanu and Stoia20 optimized the system nonlinear damping characteristics of passive and semi-active suspension with respect to ride comfort criterion. The proposed model shows that semi-active suspension with dry friction can provide good comfort improvement in comparison to passive suspensions. Turnip et al.,21 investigated the numerical aspect of sensitivity control of a MR damper. They modelled a MR damper by using a non-
parametric approach with a sixth order polynomial.

Prabakar et al., 22 studied the optimal preview control of the half-car model with a MR damper under random road excitation, and the MR damper has been modelled using a modified Bouc-Wen model. Rossi and Lucente, 23 used H-infinity control strategy to quarter car and half-car the semi-active suspension system in order to improve the ride comfort and road handling performance. Eltantawie, 24 developed a decentralized neuro-fuzzy controller to improve the ride and stability of the half-car model with the MR damper. They have evaluated the performance of the half-car model under two different road conditions (road bump and random road undulations) as inputs. They have also compared the obtained results with the passive suspension system. Karkoub et al., 25 conducted an analytical study to evaluate the effectiveness of the MR damper in reduction of vibration by using the half-car model with the optimal control strategy. Kasprzyk and Krauze, 26 evaluated the performance of the semi-active half-car model with sky-hook and LMS control algorithms and compared the results between them. Hrovat, 27 compared the performances of active and passive suspension systems on quarter, half and full car models using linear quadratic optimal control. Talib et al., 28 evaluated the performance of the half-car active suspension system with self-tuning PID controller under different road conditions.

Significant research work has been carried out on ride comfort and road holding performance of the vehicle by using the quarter car vehicle model (Vertical dynamics), but only few researchers have been reported on the dynamic analysis of the half-car vehicle model (vertical and lateral/longitudinal dynamics). In the present work, the Bouc-Wen model was considered for verifying the dynamic behavior of the MR damper. NSGA-II technique is used to identify the parameter of the Bouc-Wen model. By using the estimated parameters of the Bouc-Wen model, the ride comfort and road holding capabilities of a semi-active suspension system using the half-car model with PID control is analyzed.

2. METHODOLOGY

The methodology (Fig. 1) demonstrates the experimentation of the designed twin-tube MR damper and analytical studies to predict the behavior of the MR damper. Dynamic analysis of the half-car model with passive suspension and semi-active suspension with the MR damper under random excitation has been carried out.

3. MR FLUID PREPARATION

MR fluid consists of 20–40 percent by volume of relatively pure (3–10 micron diameter size) magnetizable particles, suspended in a carrier liquid such as mineral oil, synthetic oil, water or glycol. The MR fluids made from carbonyl iron particles exhibit maximum yield strengths of 50–100 kPa for applied magnetic fields of 150–250 kA/m. In the present study, 30 percent volume fraction of carbonyl iron particles with diameter of 6.23 µm and density of 2.33 g/cm³ is dispersed in silicone oil. The volume fraction of the suspended particle and carrier fluid were varied as per the requirements. The total volume of the MR fluid was 144 cm³, of which 30 percent volume fraction of carbonyl iron powder (around 100.6 g) and 70 percent volume fraction of silicone oil (around 100.8 cm³) were taken in a container, and the mixture was stirred continuously for one day (24 hours) with the help of a mechanical stirrer in order to get a uniform distribution of carbonyl iron particles in the silicone oil. To reduce the sedimentation of particles, small amounts of grease were added as a stabilizer (Kolekar et al. 29).

4. DAMPER DESIGN SCHEME

The design of the MR damper has been carried out similarly to the conventional hydraulic damper design with a necessary modification in the piston. In the MR damper, pistons having the electromagnetic coil capable of delivering the magnetic field in the fluid flow gap are provided around the piston. Pistons made up of magnetic steel material and piston rods with non-magnetic stainless steel having lower permeability is used to avoid flux leakage through the coil. The flow of the magnetic field depends upon the magnetic permeability of the material. According to the magnetic Ohm’s law, the magnetic potential is given as follows:

\[ NI = \Phi R_m; \]  
\[ \phi = BA; \]

where \( \phi \) is the magnetic flux, \( R_m \) is total reluctance of the magnetic circuit, \( I \) is the applied current, \( N \) is the number of turns in a coil, \( B \) is the magnetic flux density and \( A \) is the cross-sectional area magnetic circuit.

The magnetic circuit of the MR damper is as shown in Fig. 2. The magnetic circuits mainly consist of magnetic core, yoke, cylinder and annular flow path. The total reluctance of the electromagnetic circuit is composed of magnetic core reluctance (\( R_{core} \)), yoke reluctance (\( R_y \)) cylinder reluctance (\( R_c \)) and annular flow path reluctance (\( R_g \)).

\[ R_m = R_{core} + R_y + R_c + R_g; \]
\[ N = \frac{\phi(R_{core} + R_y + R_c + R_g)}{I}. \]

In the present study based on past literature, 1.5 mm fluid flow gap has been considered between the piston and cylinder of the MR damper (Sternberg et al. 30) for analysis. The designed dimension of the twin-tube MR damper is given in the Table 1.
5. CHARACTERIZATION OF MR DAMPER

The MR damper consists of a piston having an electromagnetic coil, inner cylinder and an outer cylinder. The MR damper is filled with MR fluid, which is controlled by a magnetic field, usually using an electromagnet. This allows the damping characteristics of the shock absorber to be continuously controlled by varying the current applied to the electromagnet. The components of the twin-tube MR damper are as shown in Fig. 3. The designed dimension of the twin-tube MR damper is given in Table 2. The electromagnetic circuit in the piston consist of a 1000 number of turns, and is subjected up to a maximum current of 1 A and 9 V electric potential.

To study the dynamic behavior of the designed MR damper, experimentation was carried out using a custom built damper testing machine as shown in Fig. 4. The main components of the damper testing machine are the shaker, linear variable differential transformer (LVDT), force transducer and a data acquisition system. The APS 420 ELECTRO-SEIS electrodynamics shaker was used for exciting the MR damper. The shaker has a rated sine peak force 900 N with a frequency range of 1 to 200 Hz. The rated peak to peak amplitude/displacement is 150 mm. It can be operated manually or by PC based control mode and is compatible to PC based data actuation. The APS 145 power amplifier was used in the voltage mode in order to produce constant velocity.

The damper test was performed at 2 Hz frequency for different current values. The current varied from 0.1 A to 0.4 A (increment of 0.1 A) with a sinusoidal signal of +/-0.005 m amplitude. The current was monitored and supplied through a DC power supply (0–64 V/5 A Max.). The damping force experienced by the piston rod was sensed by a force transducer fitted at the top of the piston rod, and the displacement was measured through LVDT. The dynamic characteristic of the MR damper is analyzed in the result and discussion section.

6. MR DAMPER MODELLING

The characteristics of the MR damper was analytically verified by using the Bouc-Wen model is shown in Fig. 5. The Bouc-Wen model consists of a set of differential equations describing the hysteresis behavior of the MR damper. The damping force developed by the Bouc-Wen model (Sapinski and Filus\(^31\)) is given in the Eq. (5).

\[
F_d = c_0\dot{x} + k_0x + \alpha z; \quad (5)
\]

\[
\dot{z} = \delta \dot{x} - \beta |x|^n - \gamma |x||\dot{x}|^{n-1}; \quad (6)
\]

where \(F_d\) is the damper force, \(k_0\) is the stiffness and \(c_0\) is the viscous coefficient respectively. \(\alpha, \beta, \delta, \gamma\) and \(n\) are the parameters that need to be adjusted in order to control the shape of the hysteretic curve. \(Z\) is the hysteretic variable, and the parameters of the Bouc-Wen model are identified by using the non-dominated sorting genetic algorithm II (NSGA II) technique (Deb\(^32\)). In the MR damper, the MR fluid properties vary with the current \(I\). Parameters \(K_0\) and \(C_0\) of the Bouc-Wen model is depending on the applied current to the damper, as current value changes the value of these two parameters also.
changes. Hence, field dependent parameters and the applied current can be related by the first order polynomial function, and the coefficient of the functions (constant) are obtained by the curve fitting method is given in Eq. (7) and (8).

\[ K_0 = 747l + 676.3; \]  
\[ C_0 = 526.4l + 79.73. \]  

Table 3. The Optimized Bouc-Wen model parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Optimized value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.0106696</td>
</tr>
<tr>
<td>( \beta )</td>
<td>1577.0759</td>
</tr>
<tr>
<td>( \delta )</td>
<td>113.5895</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>1926.2603</td>
</tr>
<tr>
<td>( n )</td>
<td>3</td>
</tr>
</tbody>
</table>

The parameters optimized for the Bouc-Wen model is obtained by minimizing the error between the Bouc-Wen model force and the experimental force as given in Eq. (9).

\[ J = \sum_{i=1}^{N} (F_{ei} - F_{pi})^2; \]

where \( F_{ei} \) and \( F_{pi} \) are the experimental force and model force respectively. The optimized value of the Bouc-Wen model is given in Table 3.

### 7. RANDOM ROAD PROFILE

The performance of the quarter car suspension model is evaluated under random road disturbance. The power spectral density (PSD) function of random road distribution can be expressed as (Prabakar et al., 13 Shinozuka and Jan 33).

\[ S_h(\omega) = \frac{\sigma^2 \alpha_r V}{\pi (\omega^2 + (\alpha_r V)^2)^2}; \]

where \( \sigma^2 \) is the variance of the road profile, \( \omega \) is the circular frequency, \( \alpha_r \) is the road roughness coefficient and \( V \) is the vehicle forward velocity. The PSD corresponding to the road excitation being the response of the first order linear filter to white noise excitation is given by:

\[ \dot{h}(t) + \alpha_r V h(t) = w(t); \]

where \( w(t) \) is the white noise process given by:

\[ w(t) = \sqrt{2} \sum_{k=1}^{N} S_0(\omega_k) \Delta \omega \frac{1}{2} \cos(\omega_k + \phi_k); \]

where \( S_0(\omega_k) \) is the single sided PSD of the road roughness at frequency \( \omega_k \) with \( \omega_k = \omega_l + (k - \frac{1}{2}) \) and \( k = 1, 2, 3...N \), \( \Delta \omega = (\omega_u - \omega_l)/N \), \( \omega_u \) and \( \omega_l \) are the upper and lower cutoff frequency, \( N \) is the number of the interval, \( \omega_k = \omega_k + \delta \omega \) in which \( \delta \omega \) is small random frequency and \( \phi_k \) is the independent random phase uniformly distributed between 0 to 2\( \pi \). The random road profile is as shown in Fig. 6.

### 8. HALF-CAR MODEL

A half-car suspension model with four degree of freedom system (Fig. 7) is analyzed by considering the vertical displacement and pitch movement \( \phi \) of the vehicle body. The nomenclature and detail of the half-car model (Patel et al. 34) are given in Table 4.

The equation of motion of a half-car suspension system can be derived by applying Newton’s second law of motion — see Eqs. (13) and (14) on the top of next page.

The above mentioned equation can be written in the form of state space variables as given by Eqs. (15)–(18) on the next page.
Values

\[ \text{K} \]

700 N

\[ \text{K} \]

62.2 kg

0.847 m

2500 N

2000 N

0.5

1.513 m

21000 N/m

28000 N/m

60 kg

2100 kg

Sheilza measured process variable and a desired set point (Gaur and Sheliza). The controller algorithm involves proportional (P), integral (I) and derivative (D) constants. The PID control strategy is given in Eq. (19).

\[ u(t) = K_p e(t) + K_i \int_0^t e(t) \, dt + K_d \frac{de(t)}{dt}; \quad (19) \]

where, \( u_t \) is the control signal, \( K_p \) is the proportional co-efficient, \( K_i \) is the integral co-efficient, \( K_d \) is the derivative co-efficient, \( e(t) \) is the error signal (\( e = u - u_{ref} \)), \( u \) is the

8.1. Control strategy

A proportional-integral-derivative (PID) controller is a closed loop feedback control system. The PID controller evaluates and minimizes the error value as a difference between a measured process variable and a desired set point (Gaur and Sheliza). The controller algorithm involves proportional (P), integral (I) and derivative (D) constants. The PID control strategy is given in Eq. (19).

\[ u(t) = K_p e(t) + K_i \int_0^t e(t) \, dt + K_d \frac{de(t)}{dt}; \quad (19) \]

where, \( u_t \) is the control signal, \( K_p \) is the proportional co-efficient, \( K_i \) is the integral co-efficient, \( K_d \) is the derivative co-efficient, \( e(t) \) is the error signal (\( e = u - u_{ref} \)), \( u \) is the

\[ \begin{align*}
M_s \ddot{x}_s + K_{sf}(x_{sf} - x_{uf}) + K_{sr}(x_{sr} - x_{ur}) + F_{df} + F_{dr} &= 0 \\
I \dot{\theta} + L_1 K_{sf}(x_{sf} - x_{uf}) - L_2 K_{sr}(x_{sr} - x_{ur}) + L_1 F_{df} - L_2 F_{dr} &= 0 \\
M_{ur} \ddot{x}_{ur} + C_{tf} (\dot{x}_{ur} - \dot{h}_f) + K_{sf}(x_{sf} - h_f) - K_{sr}(x_{sr} - h_f) - F_{df} &= 0 \\
M_{ur} \ddot{x}_{ur} + C_{tr} (\dot{x}_{ur} - \dot{h}_r) + K_{tr}(x_{tr} - h_r) - K_{sr}(x_{sr} - h_r) - F_{dr} &= 0 \\
\end{align*} \]

\[ \text{\{13\}} \]

\[ \begin{align*}
x_{sf} &= x_s + L_1 \dot{\theta} \\
x_{sr} &= x_s - L_2 \dot{\theta} \\
\end{align*} \]

\[ \text{\{14\}} \]

\[ \begin{align*}
\dot{x} &= A x + B u \\
y &= C x + D u \\
\end{align*} \]

\[ \text{\{15\}} \]

\[ \begin{align*}
A &= \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \\
B &= \begin{bmatrix}
-\frac{K_{sf} + K_{sr}}{M_s} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
L_2 K_{sr} - L_1 K_{sf} & \frac{L_1 K_{sf}}{M_s} & 0 & 0 & 0 & 0 & 0 & 0 \\
L_2 K_{sr} + L_1 K_{sf} & \frac{L_1 K_{sf}}{M_s} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \\
C &= \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
D &= \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \}
\end{align*} \]

\[ \text{\{17\}} \]

\[ \text{\{18\}} \]

Table 4. Nomenclature and details of the half-car model.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sprung mass (( M_s ))</td>
<td>1200 kg</td>
</tr>
<tr>
<td>Body mass moment of inertia (( J ))</td>
<td>2100 kg-m²</td>
</tr>
<tr>
<td>Front unsprung mass (( M_{uf} ))</td>
<td>62.2 kg</td>
</tr>
<tr>
<td>Rear unsprung mass (( M_{ur} ))</td>
<td>60 kg</td>
</tr>
<tr>
<td>Front suspension stiffness (( K_{sf} ))</td>
<td>22000 N/m</td>
</tr>
<tr>
<td>Rear suspension stiffness (( K_{sr} ))</td>
<td>21000 N/m</td>
</tr>
<tr>
<td>Front suspension damping (( C_{sf} )) for passive system</td>
<td>2500 N/s/m</td>
</tr>
<tr>
<td>Rear suspension damping (( C_{sr} )) for passive system</td>
<td>2000 N/s/m</td>
</tr>
<tr>
<td>Front and rear tyre stiffness (( K_{tr} ) and ( K_{fr} ))</td>
<td>13400 N/m</td>
</tr>
<tr>
<td>Front and rear tyre damping (( C_{tr} ) and ( C_{fr} ))</td>
<td>700 N/s/m</td>
</tr>
<tr>
<td>Location of center of gravity from front axle (( L_{f1} ))</td>
<td>0.847 m</td>
</tr>
<tr>
<td>Location of center of gravity from rear axle (( L_{r2} ))</td>
<td>1.513 m</td>
</tr>
</tbody>
</table>

Figure 7. Half-car suspension with the MR damper.

Table 5. Optimal parameter of the PID controller.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_p )</td>
<td>2</td>
</tr>
<tr>
<td>( K_i )</td>
<td>10</td>
</tr>
<tr>
<td>( K_d )</td>
<td>1</td>
</tr>
</tbody>
</table>

measured process variable and \( u_{ref} \) is the reference value. The relative displacement between sprung and unsprung is given as an input to the PID controller. The optimal parameter of the PID controller is given in Table 5.

9. RESULT AND DISCUSSION

The fabricated MR damper was tested at a 2 Hz frequency for different current values under the sinusoidal excitation. The
The dynamic behaviour of an MR damper analysed by using force v/s displacement and force v/s velocity curve are shown in Fig. 8. From an observation at the lower input current, the damping force is less and it increased gradually with an increase in current. The slope of the curve increased with an increase in current, which indicates an increase in stiffness.

It is observed that as the current increased, the area enclosed under the force v/s displacement curve also increased, which represents an increase of energy dissipation. Fig. 9 illustrates a comparison of the experimental and the proposed model damping force, a good agreement is observed between them.

The ride comfort and road holding performance of the passive and semi-active half-car suspension system has been evaluated under random excitation for different vehicle velocities. In the semi-active suspension system, the MR damper was modelled as the Bouc-Wen model, and the estimated parameter values of the Bouc-Wen model was used to carry out the simulation. The acceleration and its PSD response of the passive and semi-active half-car suspension system under random road distribution at constant speed of 35 m/s are illustrated in Figs. 10 and 11. It is observed that, the vibration level of vehicle body is significantly reduced in the case of a semi-active suspension system than a passive suspension system.

Figure 12(a) and (b) depicts the pitch rate of a passive and semi-active sprung mass. In this case, the semi-active suspension system also shows better performance.

Figures 13 and 14 depicts the root mean square (RMS) of the sprung mass acceleration, and the pitch angle acceleration response of the passive and semi-active suspension system for different vehicle velocities. In the semi-active suspension system, it can be observed that the increase in velocity results in significant reduction in the vertical acceleration of the sprung mass and pitch acceleration. This indicates that the semi-active suspension system provides better ride comfort for a vehicle.

Figure 15 illustrates the RMS road holding response of the front and rear wheel, which measures the relative displacement between the unsprung mass and the road displacement with respect to the different velocities. In this case also, road holding will be better with the semi-active suspension rather than the passive suspension for all velocities.

10. CONCLUSION

The MR damper has been designed for maximum load of 20 kN force and the electromagnetic circuit in the piston con-
sists of 1000 number of turns, and is subjected up to a maximum current of 1 A and 9 V electric potential. The experimental and analytical studies were conducted to predict the dynamic behavior of the MR damper. The designed twin-tube MR damper was tested under harmonic excitation. The result demonstrates that damping force increased with an increase in current, which leads to an increase in energy dissipation and stiffness. The dynamic force of an MR damper is analytically predicted by using the Bouc-Wen model. A good agreement has been observed between the proposed model and the experimental results. The passive and the MR damper based half-car suspension system is modeled and simulated under the random road profile with PID control. The MR damper based suspension system with the PID controller reduces around 33 percent of vibration amplitude at resonance frequency of the vehicle, and shows 40 percent improvement in road holding performance of the vehicle than with the passive suspension system.

ACKNOWLEDGEMENTS

The authors acknowledge the funding support from Department of Science and Technology (DST): No.SB/FTP/ETA-0071/2013 and also acknowledge SOLVE Lab: The Virtual Lab @ NITK (www.solve.nitk.ac.in) and Centre for System Design (CSD): A Centre of excellence at NITK-Surathkal, for providing experimental facility.

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Figure 12. Pitch rate of the passive and semi-active sprung mass.


Figure 13. Vertical acceleration of sprung mass.


18 Chen, Y., Skyhook surface sliding mode control on semi-active vehicle suspension system for ride com-
Figure 15. Road holding of the front and rear wheel.

Figure 15. Road holding of the front and rear wheel.
Application of Spectral Element Method Combining Dilatation Theory to Sound Generated by a Co-rotating Vortex Pair

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(Received 17 October 2016; accepted 2 February 2018)

A spectral element method for the simulation of an acoustic field is applied to the case of a co-rotating vortex pair in both stationary medium and mean flow. Based on the dilatation theory, the second time derivative of the pseudo-pressure is used as the source of the inhomogeneous convected wave equation which is discretized by a spectral element method in space and the Newmark-$\beta$ method in time marching. In addition, the nonreflecting boundary condition is adopted too. Then we compared the numerical results with the analytical solution. Numerical results are in good agreement with the analytical solution. Moreover, different grid spacings and time steps are investigated for evaluating the numerical accuracy. To study the frequency content of the sound, spectral analysis is also carried out. Finally, sound propagation in uniform flows and sheared mean flows are simulated and analyzed. This study shows the capabilities of the spectral element method combined with dilatation theory for the aeroacoustic problems.

1. INTRODUCTION

The acoustic analogy theory introduced by Lighthill to study jet noise is now over six decades, and it has been by far the most successful and versatile theory in dealing with aeroacoustic problems. Considering the pseudo-sound pressure, Ribner and Meecham offered a different approach to Lighthill’s acoustic analogy which posited fluctuating fluid dilatations as the acoustic source. One motivation for developing a new expression is that the pseudo-sound term is much easier to calculate relative to Lighthill’s stress tensor. According to this dilatation theory which also corresponds to the acoustic/viscous splitting technique derived by Hardin and Pope, Hurdle, Ribner and Crawley studied the aerodynamic noise generating from a jet engine. Combining large eddy simulation and dilatation theory, Flemming used a hybrid approach to study the combustion noise of a turbulent flame. Hiramoto et al. investigated the sound generated in a separated shear flow by flow visualization and fluctuating static pressure measurements, and the results showed that the dilatation theory’s source term and the vortical structure are closely correlated. Escobar et al. presented a study on vortex sound propagation by using finite element method and compared dilatation theory with Lighthill’s acoustic analogy theory. The results showed that the dilatation theory can get good solutions easily. Based on the dilatation theory, Papageorgiou and Tsangaris developed a numerical discretization scheme for acoustic wave equation and solved some benchmark problems.

Computational Aeroacoustics (CAA) is not the same as Computational Fluid Dynamics (CFD). Needs of accurate and efficient numerical solvers in CAA motivated the development of low-dispersion and low-dissipation schemes. Varieties of finite difference schemes occupy a dominant position in CAA and some of them can provide certain accuracy. However, the spectral element method (SEM) can provide high resolution and good flexibility with a low number of elements. Numerically, SEM has the advantage of low dispersion and diffusion alongside exponential convergence in the polynomial order. Therefore, the SEM has been widely used in wave propagation studies, CFD and CAA.

The motivation for simulating the sound generated by a co-rotating vortex pair in this work is that various vortices occupy only a very small portion in a flow but play a key role in organizing the flow, as “the sinews and muscles of the fluid motion” and “the sinews of turbulence”. Vortices are also “the voice of fluid motion” because they, at low Mach numbers, are the only source of aeroacoustic sound and noise. The other reason is that it has analytic solutions. Moreover, many other workers have used it to verify the validity of the proposed numerical schemes for CAA.

In the present work, we present an analysis methodology, described in Section 2, which aims to supply another computational tool for CAA. The inhomogeneous convected wave equation based on the dilatation theory was solved numerically by using the high resolution SEM. In order to demonstrate the capabilities and limits of this method, we have studied a benchmark 2D vortex sound propagation problem in detail.

The paper is organized as follows. In Section 2 we provide a description of the inhomogeneous convected wave equation with dilatation theory. The governing equation obtained will then be discretized by SEM in space and Newmark-$\beta$ method in time. In Section 3, the sound generated by a co-rotating vortex pair is investigated in detail. Finally, Section 4 contains the conclusions of our work.

2. GOVERNING EQUATION

As well known, the original Lighthill’s acoustic analogy completely ignores the mean flow-sound interaction effects. Then Phillips and Lilley make a correction for the Lighthill’s equation. Phillips’ equation takes into account partially the interaction of the mean flow with the sound. Thus, the equa-
tion is valid for a moving medium, with some accuracy. Lilley’s equation takes the effects of the static flow into account in a better way than the Phillips’ equation. In Lilley’s equation, all the “propagation effects” that occur in a transversely sheared mean flow are inside the left hand side of the equation. Therefore, Lilley’s equation has been extensively used to examine the mean flow-sound interaction and for calculation of subsonic jet noise. However, the complexity of solutions and nonlinearity of equations has also been presented. Hence, in the present paper, we consider the Lighthill’s acoustic analogy theory in a mean flow velocity \( U \) which is constant and parallel to the \( x \) direction. For the simplicity of illustration, the Lighthill’s equation can be written as

\[
\frac{1}{c_0^2} \frac{D^2 p'}{Dt^2} - \frac{\partial^2 p'}{\partial x_i^2} = \frac{\partial^2 T_{ij}}{\partial x_i x_j},
\]

where

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + U \frac{\partial}{\partial x};
\]

and \( c_0, U \) are the speed of sound and mean flow, respectively, \( p' \) is the pressure fluctuation, \( T_{ij} \) represents the Lighthill’s stress tensor.

Some assumptions are made here to obtain Eq. (1):

1. The speed of sound is a constant.
2. There are no mass, heat, force or momentum source distributions.
3. The refraction effects are not included in the scope of this paper.
4. The sources were assumed to be acoustically compact.

For the inviscid flow with a low Mach number and high Reynolds number in the form

\[
T_{ij} \approx \rho_0 u_i u_j;
\]

and \( u_i \) is the fluid velocity of the \( i \)-direction. Introducing Mach number \( Ma = U/c_0 \), Eq. (1) can be written as

\[
\frac{1}{c_0^2} \frac{D^2 p'}{Dt^2} + \frac{2}{c_0} Ma \frac{\partial}{\partial x} (\frac{\partial p'}{\partial t}) - \left( 1 - Ma^2 \right) \left( \frac{\partial^2 p'}{\partial x^2} + \frac{\partial^2 p'}{\partial y^2} \right) = \frac{\partial^2 T_{ij}}{\partial x_i x_j}. \tag{4}
\]

The pressure fluctuations will therefore satisfy the above inhomogeneous convected wave equation which is an exact consequence of the continuity and momentum equations. However, if the flow is incompressible, the pressure fluctuations can be split into the pseudo-sound and acoustic part

\[
p' = p^\text{inc} + p^\rho. \tag{5}
\]

The pseudo-sound pressure is also called hydrodynamic pressure fluctuations or incompressible pressure fluctuations. The acoustic perturbations are what we term sound, as they are characterized by their ability to propagate (not the generation) into the hearing region. The pseudo-sound pressure perturbations are the consequence of fluid flow simply changing in the source region, it exhibits no wave propagation. The \( p^\text{inc} \) field dominating within and near the turbulence at subsonic speeds, constitutes what is known as the acoustic near field, and it has virtually the characteristics of the pressure field in an incompressible flow being dominated by inertial rather than compressional effects, and hence the name pseudo-sound. Further, it is overridden by the acoustic radiation field \( p^\rho \), which decays more slowly with distance. Figure 1 shows the diagram of decay for acoustic and pseudo-sound pressure in 3-D problem. And it has also been suggested by Ribner, Crawford and Ris- torcelli. The horizontal axis represents the distance from the sound source, \( r/m \).

Finally, by taking the divergence of momentum equation and combined with the time derivative of the continuity equation, we can get a specific equation for the pseudo-sound pressure \( p^\text{inc} \)

\[
- \nabla^2 p^\text{inc} = \rho_0 \frac{\partial^2 u_i u_j}{\partial x_i x_j} \tag{6}
\]

and substitute back in Eq. (1) and use Eq. (5) to get

\[
\frac{1}{c_0^2} \frac{D^2 p^\rho}{Dt^2} - \frac{\partial^2 p^\rho}{\partial x_i^2} = - \frac{1}{c_0^2} \frac{D^2 p^\text{inc}}{Dt^2}. \tag{7}
\]

The above equation can be rewritten as

\[
\frac{\partial^2 p^\rho}{\partial t^2} + 2c_0 Ma \frac{\partial}{\partial x} \left( \frac{\partial p^\rho}{\partial t} \right) - c_0^2 \left( 1 - Ma^2 \right) \left( \frac{\partial^2 p^\rho}{\partial x^2} + \frac{\partial^2 p^\rho}{\partial y^2} \right) = - \frac{\partial^2 p^\text{inc}}{\partial t^2}. \tag{8}
\]

Equation (8) provides another view of getting acoustic pressure by using only the second derivative of pseudo-sound pressure. This formulation corresponds to what derived by Hardin and Pope. The right hand side of Eq. (8) can be regarded as the forcing term of the inhomogeneous convected wave equation for the acoustic pressure. And like Lighthill’s result, the above equation is an exact consequence of the continuity and momentum equations. The initial condition is given by

\[
p^\rho (x, y, 0) = 0, \quad \frac{\partial p^\rho}{\partial t} (x, y, 0) = 0. \tag{9}
\]
2.1. Nonreflecting Boundary Condition

One of the typical problems in the numerical simulation of acoustic wave propagation is the treatment of boundary conditions. The nonreflecting boundary conditions are introduced to suppress the spurious waves which are generated by truncating the unbounded domain, and this would lead to a significant reduction in computational cost, especially for the three-dimensional large-scale numerical simulation.

In this study, the C-E-M nonreflecting boundary condition is modified to be appropriate for the convective wave equation.33-35 The expression appears as what shown below

\[
\frac{1}{c_0(1 + Ma)} \frac{\partial p^a}{\partial t} + \frac{\partial p^a}{\partial n} = 0; \quad (10)
\]

where \( n \) represents the outward unit normal vector out of the computation region at the boundaries. With a minus sign placed in front of \( Ma \) on the upstream boundary and a plus sign on the downstream boundary, the \( Ma \) is zero on the boundary perpendicular to the flow direction.

2.2. Spatial Discretization

Both integral formula and variational formula can be applied to the governing equation. Because they are computationally less expensive than volume discretization methods where a whole discretization of the acoustic domain is required, the integral methods remain widely used in CAA studies, but in such a situation, integral formula would require extensive experience of a hard-wall Green’s function that is not known for complex geometries. On the other hand, the integral formula must explicitly take into account the interactions between the solid surfaces and the induced noise which is named as surface sources. On the contrary, the variational formula is able to take into account the effects implicitly. And therefore, in the present paper, we adopt the SEM to discretize the governing equation in space.

After introducing the Sobolev space (\( d \) denotes the space dimension)

\[
H^1(\Omega) := \left\{ v \in L^2(\Omega) : \frac{\partial v}{\partial x_j} \in L^2(\Omega), \forall j = 1, \ldots, d \right\};
\]

and its subspace

\[
H^1_0(\Omega) := \left\{ v \in H^1(\Omega) : v|_{\Gamma_n} = 0 \right\}. \quad (12)
\]

Then, the weak formulation of Eq. (8) and Eq. (9) reads as follows: find \( p^a(t) \in H^1(\Omega) \) such that, for any \( v \in H^1_0(\Omega)\),

\[
\int_{\Omega} \left( \frac{\partial^2 p^a}{\partial t^2} + 2c_0 Ma \frac{\partial p^a}{\partial x} \left( \frac{\partial p^a}{\partial t} \right) - c_0^2 \left( (1 - Ma^2) \frac{\partial^2 p^a}{\partial x^2} + \frac{\partial^2 p^a}{\partial y^2} \right) \right) v \, d\Omega = \int_{\Omega} - \frac{\partial^2 p^a}{\partial t^2} v \, d\Omega. \quad (13)
\]

The variational formulation of above equation with the nonreflecting boundary condition Eq. (10) yields as follows

\[
\int_{\Omega} \frac{\partial^2 p^a}{\partial t^2} v \, d\Omega + 2c_0 Ma \int_{\Omega} \frac{\partial p^a}{\partial x} \left( \frac{\partial p^a}{\partial t} \right) v \, d\Omega + c_0^2 \int_{\Omega} \left[ (1 - Ma^2) \frac{\partial^2 p^a}{\partial x^2} + \frac{\partial^2 p^a}{\partial y^2} \right] v \, d\Omega - c_0 \int_{\Gamma_{\text{up}}} (1 + Ma) \frac{\partial p^a}{\partial x} v \, ds + c_0 \int_{\Gamma_{\text{down}}} (1 + Ma) \frac{\partial p^a}{\partial x} v \, ds + c_0 \int_{\Gamma_{\text{wall}}} (1 + Ma) \frac{\partial p^a}{\partial x} v \, ds + c_0 \int_{\Gamma_{\text{wall}}} (1 + Ma) v \, ds = \int_{\Omega} - \frac{\partial^2 p^a}{\partial t^2} v \, d\Omega. \quad (14)
\]

where \( \Gamma_{\text{up}} \), \( \Gamma_{\text{down}} \) and \( \Gamma_{\text{y}} \) is the upstream boundary, downstream boundary and the boundary in the \( y \) direction, respectively.

In the Chebyshev spectral element method, the computational domain \( \Omega \) is decomposed into \( N_d = N_m \times N_n \) non-overlapping subdomains, where \( N_m \) the element number in the \( x \) direction, \( N_n \) the element number in the \( y \) direction. Each spectral element is mapped into a standard element \([-1, 1] \) by

\[
\xi = \frac{2}{L_x}(x - x_i) - 1 \quad \text{or} \quad x = \frac{1}{2} L_x(\xi + 1) + x_i; \quad (15)
\]

\[
\eta = \frac{2}{L_y}(y - y_i) - 1 \quad \text{or} \quad y = \frac{1}{2} L_y(\eta + 1) + y_i; \quad (16)
\]

where \( L_x = x_{m+1} - x_m, L_y = y_{n+1} - y_m \) are the lengths of the \( i \)th element in \( x \) and \( y \) directions, respectively. Hence, the trial functions and the test functions can be written as

\[
p^a(i, \xi, \eta) = \sum_{j=0}^{N_i^x} \sum_{k=0}^{N_i^y} h_{\xi j}^i(\xi) h_{\eta k}^i(\eta) u_{jk}^i; \quad (17)
\]

\[
v^i(\xi, \eta) = \sum_{p=0}^{N_i^x} \sum_{q=0}^{N_i^y} h_p^i(\xi) h_q^i(\eta) v_{pq}^i; \quad (18)
\]

where \( N_{i^x}, N_{i^y} \) is the number of nodes in each element in \( x \) and \( y \) directions, respectively. Interpolation functions can be expressed as

\[
h_{\xi j}^i(\xi) = \frac{2}{N_i^x} \sum_{m=1}^{N_i^x} \frac{1}{c_m} T_m(\xi) T_m(\xi); \quad (19)
\]

\[
h_{\eta k}^i(\eta) = \frac{2}{N_i^y} \sum_{n=0}^{N_i^y} \frac{1}{c_n} T_n(\eta) T_n(\eta); \quad (20)
\]

\[
h_p^i(\xi) = \frac{2}{N_i^x} \sum_{l=0}^{N_i^x} \frac{1}{c_{p l}} T_l(\xi) T_l(\xi); \quad (21)
\]

\[
h_q^i(\eta) = \frac{2}{N_i^y} \sum_{r=0}^{N_i^y} \frac{1}{c_{q r}} T_r(\eta) T_r(\eta); \quad (22)
\]

where \( T_m, T_n, T_l, T_r \) are Chebyshev polynomials. The interpolation functions satisfy the cardinal interpolation property

\[
h_{\xi j}^i(\xi_k) = \delta_{jk}, \quad h_{\xi j}^i(\eta_l) = \delta_{pq}; \quad (23)
\]

where \( \delta_{jk}, \delta_{pq} \) are Kronecker’s deltas representing the identity matrix. The parameter \( c_m \) is defined by

\[
c_m = \begin{cases} 
2 & m = 0, N_i^x, \\
1 & m \neq 0, N_i^x.
\end{cases} \quad (24)
\]
Finally, the variational problem discretized by the means of SEM is equivalent to solving the following Differential Algebraic Equations

\[ \mathbf{M} \ddot{p}^a(t) + \mathbf{C} \dot{p}^a(t) + \mathbf{K} p^a(t) = \mathbf{S}; \]  

where \( \mathbf{M} \), \( \mathbf{C} \), \( \mathbf{K} \) and \( \mathbf{S} \) are the global mass matrix, global damping matrix, global stiffness matrix and global loading matrix.

2.3. Time Integration Scheme

Newmark- \( \beta \) scheme is used to avoid the stability restriction. It is an implicit method of direct integration of the equations, for which the relationships between the acoustic pressure \( p^a \), the first time derivative \( \dot{p}^a \) and the second time derivative \( \ddot{p}^a \) in the interval of \( t \sim t + \Delta t \) are as follows:

\[ \begin{align*}
\ddot{p}^a_{t+\Delta t} &= \ddot{p}^a_t + (1-\gamma)\ddot{p}^a_{t} \Delta t + \gamma \ddot{p}^a_{t+\Delta t} \Delta t;
\dot{p}^a_{t+\Delta t} &= \dot{p}^a_t + \dot{p}^a_{t} \Delta t + \left( \frac{1}{2} - \beta \right) \dot{p}^a_{t} \Delta t^2 + \beta \dot{p}^a_{t+\Delta t} \Delta t^2;
\end{align*} \]  

The choice of \( \gamma \) and \( \beta \) in the equations above will influence the stability and accuracy of the method.\(^{36}\) After utilising the above relationships, Eq. (25) reduces to a system of algebraic equations with constant coefficients for each given time step \( \Delta t \), the obtained form can be written as

\[ \begin{align*}
\left( \mathbf{K} + \frac{1}{\beta \Delta t^2} \mathbf{M} + \frac{\gamma}{\beta \Delta t} \mathbf{C} \right) \ddot{p}^{a}_{t+\Delta t} &= \mathbf{S}_{t+\Delta t} + \mathbf{M} \frac{1}{\beta \Delta t^2} \ddot{p}^a_{t} + \mathbf{C} \left( \frac{\gamma}{\beta \Delta t} \dot{p}^a_{t} + \left( \frac{\gamma}{2 \beta} - 1 \right) \dot{p}^a_{t} \Delta t \right) + \mathbf{C} \left( \frac{\gamma}{\beta \Delta t} \dot{p}^a_{t} + \left( \frac{\gamma}{2 \beta} - 1 \right) \dot{p}^a_{t} \Delta t \right); \intertext{and the initial step of is written as follows:}
\ddot{p}^a_0 &= \mathbf{M}^{-1} \left( \mathbf{S}_0 - \mathbf{K} \ddot{p}^{a}_0 - \mathbf{C} \ddot{p}^{a}_0 \right). \tag{29}
\end{align*} \]

3. RESULTS AND DISCUSSION

In this paper, the acoustic field generated by a co-rotating vortex pair is calculated in detail, and can be used as a benchmark problem to verify computational aeroacoustics numerical schemes. This vortex sound problem is a good test for the algorithm because an analytical closed form solution exists for both the incompressible flow field and the acoustic field. At the same time, it also represents the basic acoustic field generated by turbulent shear flows and can be used to understand the mechanism of sound generation.

The schematic of the co-rotating vortices is presented in Fig. 2. The two point-vortices separated by a fixed distance \( 2r_0 \) rotate around each other along a circular path of radius \( r_0 \) and have an equal circulation intensity \( \Gamma \). The angular rotational speed is \( \omega = \Gamma / (4 \pi r_0^3) \), and the period of rotation is \( T = 8 \pi r_0 / \Gamma \). Each vortex induces on the other a velocity \( v_{\text{rel}} = \Gamma / (4 \pi r_0^3) \), thus the rotating Mach number is \( M_v = v_{\text{rel}} / c_0 \).

The flow field is assumed to be viscous and incompressible. Thus, it can be determined numerically by the evaluation of a complex potential function \( \phi(z, t) \)

\[ \phi(z, t) = \frac{\Gamma}{2 \pi i} \ln z^2 \left( 1 - \frac{b^2}{z^2} \right); \tag{30} \]

where \( z = x + iy = re^{i \theta}, b = r_0 e^{-\omega t} \).

To make acoustic computations, flow variables are required as input to the acoustic equations. The hydrodynamic velocity can be obtained by differentiating Eq. (30) with respect to \( z \). From the unsteady Bernoulli’s equation, the hydrodynamic pressure \( p^{\text{inc}} \) can be found as follow

\[ u_x - i u_y = \frac{\partial \phi(z, t)}{\partial z} = \frac{\Gamma}{2 \pi} \frac{z}{z^2 - \beta^2}; \tag{31} \]

\[ p^{\text{inc}} = \rho_0 \frac{\partial}{\partial t} \left( \text{Re} \left( \phi(z, t) \right) \right) - \frac{1}{2} \rho_0 \left( u_x^2 + u_y^2 \right); \tag{32} \]

where \( \text{Re} \) denotes the real part of a complex quantity.

The inherent unsteadiness of the flow field of the co-rotating vortex pair generates sound. The analytical solution by Müller and Obermeier\(^{27}\) is used to validate the numerical simulation. The fluctuating pressure is given by

\[ \begin{align*}
\ddot{p} &= \frac{\rho_0 \Gamma^4}{64 \pi^3 r_0 c_0} \left( J_2(kr) \sin(\psi) - Y_2(kr) \cos(\psi) \right); \tag{33}
\end{align*} \]

where wave number \( k = 2 \omega / c_0 \) and \( J_2(kr), Y_2(kr) \) are the second-order Bessel function of the first and second kind, respectively.

The acoustic computations are performed in the domain \( L \times L \) with the uniform square grid system. The flow domain, the same as the acoustic source region, corresponds to the inner square domain with dimensions 0.5L \* 0.5L. Zero initial values are used for all acoustic fluctuations. The circulation intensity and the rotating Mach number are the only parameters that determine the frequency and amplitude of the solution. The parameters used for the following simulation are \( \Gamma = 1.00531 \text{ m}^2 \text{s}^{-1}, r_0 = 1 \text{ m}, c_0 = 1 \text{ ms}^{-1}, Ma = 0 \), \( L = 400 \text{ m}, N_m = N_n = 55, N_x = N_y = 2 \), this results in a wave length \( \lambda \approx 39 \text{ m}, M_v = 0.08 \) and the element size \( \Delta x = \Delta y = 3.64 \text{ m}, \) corresponding to about 11 points per wave length (PW), time step \( \Delta t = 0.1 \text{ s} \).

Figure 3 shows a 3-D graphical view of the acoustic pressure analytical solution. As what can be seen from the graph.
above, the double spiral pattern clearly illustrates the rotating quadrupole nature of the radiated waves, and the acoustic pressure becomes singular at the coordinate origin and has large gradients close to the vortex centers. To avoid the numerical singularity at the center of the vortices, the vortex core model is used, and the other method involves placing the mesh points far enough from the vortex centers. The original point vortex, Rankine vortex and Scully vortex tangential velocity variation against the radial distance are plotted in Fig. 4. A cut-off practice was adopted in this study. The effect of considering different cut-off distances at the coordinate origin is illustrated in Fig. 5. The four different distances from the origin are: $r/r_0 = 1.5, 4.0, 6.0$ and $10.0$, respectively. Figure 5 shows that the amplitude of acoustic pressure is directly associated with the cut-off distance. And with the decreasing of cut-off distances, the numerical solutions are getting better and better. The main reason for this phenomenon is that the bigger cut-off distance has reduced the source terms more. However, for grid points located closer than the point vortex separation distance $r_0$, convergence will not obtain on account of large velocity gradients. In addition, on account of the fact that the element size we select is not fine enough, thus for grid points located at distances $r/r_0 \leq 1.5$, no source term (the right-hand side of Eq. (8)) was computed. The source term distribution acquired from the computation at time $t = 100$ s is presented in Fig. 6. As can be seen from the Fig. 6, the source region is large enough to avoid obvious truncation of the source term. The amplitudes on the boundary are about $2.5\%$ of their peak in the source domain.

A grid resolution study is investigated. Figure 7a shows the comparison of the numerical and analytical acoustic pressure solutions along the positive horizontal $x$-axis obtained with three different grid spacings. Good agreement is observed except near the center of the vortices due to the source term’s cut-off. It is shown that using a grid spacing of $\Delta x = 3.92$ m, with about 10 PPW, provides as good a result as those obtained for the case of $\Delta x = 2.82$ m, with about 14 PPW, except near the center of the vortices. Hence grid independency is achieved with $\Delta x = 3.92$ m. Additionally, the effect of temporal resolution is given in Fig. 7b. It is easy to see that the accuracy...
Figure 7. Comparison of the numerical and analytical acoustic pressure solutions along the positive horizontal x-axis (a) different grid spacing and (b) different time step.

Figure 8. Time evolution of predicted acoustic pressure contours in the stationary medium (a) 60 s, (b) 120 s, (c) 180 s, and (d) 240 s.
enhanced with the shortening of the time step, which means the smaller time step, the better prediction of the amplitudes and phase.

The time evolution of predicted acoustic pressure distributions in the stationary medium is presented in Fig. 8. It can be indicated that the nonreflecting boundary condition worked well for wave propagation in unbounded domains.

The pressure signal is recorded at the monitor point \((0, 58.2)\) in order to analyze the radiating frequency, its evolution is reported in Fig. 9. It can be indicated that there is only some slight difference between the amplitudes of the numerical solutions and the analytical one, which could be owing to the neglect of the source terms near the vortex centers.

The effect of considering different rotating Mach number is illustrated in Fig. 10. The parameters used for this simulation are \(L = 200\) m, \(\Delta x = \Delta y = 3.33\) m, \(\Delta t = 0.1\) s, \(M_r = 0.03, 0.08\) and \(0.13\), respectively. It shows that the intensity of the sound source is directly associated with the \(M_r\). Moreover, a higher Mach number means shorter wavelengths with a lower spatial resolution of the waves when the space discretization is not changed. In this case, a finer mesh discretization is needed to resolve this problem.

To evaluate the frequency content of the sound generated by the vortex pair, a fast Fourier transform algorithm was done. Acoustic results of the three different rotating Mach number given above are compared in Fig. 11 in terms of sound pressure levels (SPL) radiated at the monitor point \((0, 58.2)\). The agreement is excellent at the fundamental frequency. But in other frequencies, the amplitudes of the numerical solutions are mostly lower than the analytical solutions, which could also be due to the neglect of the source terms near the vortex centers. In addition, the fundamental frequency is increased as the \(M_r\) increases. Higher harmonics are seen in numerical solutions, but they don’t appear in analytical ones.

There are numerous situations, however, where the surrounding medium is more nearly in a state of motion. Therefore, we now consider the situations with a uniform flow and a sheared mean flow and discuss the behavior of the solution for each case. All the other parameters remain the same while the Mach number is changed into \(M_r = 0.1\) and \(0.3\) in uni-
form mean flow situations. The acoustic pressure contours generated by the vortex pair in the uniform mean flow at time 100 s is shown in Fig. 12. It can be indicated that the Doppler effects are well captured, which means that the amplitude of the waves increases and their wavelength decreases in the upstream regions. Meanwhile, it is opposite in the downstream regions. Additionally, with the increase of the Mach number, the Doppler effects become increasingly obvious.

Finally, the effects of non-uniform mean flows on acoustic wave propagation are investigated. The shear profile, shown in Fig. 13, is defined by the following hyperbolic tangent expression of the longitudinal mean velocity

\[
u(y) = \Delta U \tanh(2y/\delta);
\]

where \(\Delta U\) and \(\delta\) are, respectively, the peak velocity and the shear layer thickness. In the present test, they are chosen to be \(\Delta U = 0.1c_0\) and \(0.3c_0\), \(\delta = 10\) m. The same computations as in the previous case are performed. Figure 14 shows the acoustic pressure contours generated by the vortex pair in the sheared mean flow at time 100 s. In comparison with Fig. 12, wave fronts are ovalized due to mean flow convection effects.

4. CONCLUSION

In the present study, the capabilities of the spectral element method for the accurate simulation of the acoustic field generated by a co-rotating vortex pair using the inhomogeneous convected wave equation combined with Ribner’s dilatation theory are investigated. It was observed that the application of the second time derivative of the pseudo-pressure as the source of the inhomogeneous convected wave equation can better simulate the acoustic field propagation. The simulation of consider-
The acoustic pressure amplitude decreased slightly due to the cut off of the sources near the vortex centers, in this case the employment of a vortex core model could provide better results. Additionally, spectral analysis was also considered for evaluating the frequency content of the sound generated by the vortex pair. Moreover, sound propagation in uniform flows and sheared mean flows were simulated and analyzed. Future work will investigate the flow-induced noise problems with the spectral element method and dilatation theory.

ACKNOWLEDGEMENT

This work was supported by the National Basic Research Program of China [No. 2012CB026004].

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Rolling Bearing Fault Detection in the Initial Stage of Degradation Based upon Optimized NLM and TKEO

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(Received 17 January 2017; accepted 22 September 2017)

During the operation of rolling bearings, vibration signals contain abundant state information, which exhibits strong nonstationarity and nonlinearity. It is always arduous to detect the initial damage point during the lifetime. Non-local means (NLM) algorithm can suppress noise and highlight the components of the fault impact, but the problem lies in the determination of parameters which directly affect the result. In this paper, we proposed a signal processing method combined NLM optimized by Fruit fly Optimization Algorithm (FOA) and Teager Kaiser energy operator (TKEO) to detect the initial stage degradation of bearings. First of all, the proposed optimal NLM algorithm is used to denoise the bearing vibration signals which are gathered in the initial stage of bearing degradation. Then, the TKEO algorithm is applied to suppress the non-impulsive components and the periodic impulsive characteristics of the denoised signals are enhanced simultaneously. Furthermore, the analysis of the frequency components in the Teager energy spectrum is conducted to detect whether the bearings are abnormal or not. Experimental and comparative analyses are presented to validate the proposed method in the end.

1. INTRODUCTION

Rolling bearing is one of the most widely used rotating components at present. It plays an important role as “mechanical joints” in electromechanical equipment and weapon equipment. Unfortunately, rolling bearing is usually vulnerable due to the disadvantageous operation environment, so it is important to find out the hidden damage in it.1 Condition Based Maintenance (CBM) is a kind of popular maintenance method in recent years. This method can detect abnormalities and figure out whether there is tendency to malfunction according to the real-time state monitoring data so as to take further maintenance and preventive measures.2 If the damage of the rolling bearing can be detected as soon as possible, there will be enough time to work out reasonable maintenance program to further ensure the safe and reliable operation of the equipment.

The vibration signal of the rolling bearing contains abundant state information, which shows strong non-stationary and non-linear characteristics. And it is influenced by the complex vibration transmission path, serious noise disturbance, the coupling of multi-vibrational source excitation and response and other factors.3 Especially in the early stage of bearing degradation, the shock signal caused by structural damage is difficult to detect.4 Fast Kurtogram (FK) method processes signals through a pass-band filter, which is simple and convenient to be applied in vibration signal denoising.5 However, it should be realized that FK has a limited range of possible centre frequencies particularly when the bandwidth is wide.6 Meanwhile, there is a limitation to the accuracy improvement of extracting transient characteristics from noisy signals and recognizing machinery fault with the kurtogram method.7 As a new denoising method, non-local means (NLM) algorithm has the advantages of simple calculation, no iteration and positive performance. Therefore, it has gradually become a hotspot research topic in image processing8 and biomedical.9 However, this algorithm is still in its infancy in the processing of one-dimensional vibration signals. Combining NLM with Empirical Mode Decomposition (EMD), Mien et al.10 firstly applied NLM in the field of vibration signals and the bearing fault diagnosis was realized. Lv et al.11 diagnosed rolling bearing fault successfully by fast NLM algorithm with the envelope spectrum analysis. Zhu et al.12 combined NLM and Local Mean
Decomposition (LMD) to achieve the fault feature extraction of rolling bearing, which is better than just using LMD. Nevertheless, the NLM parameters are usually determined by human experience. The determination of the parameters directly affects signal denoising, which further influences the results of fault detection in the early stage of bearing degradation.

The local search performance of genetic algorithm (GA) is not satisfactory. In the late stage of parameter optimization, the convergence speed of the algorithm is relatively slow when it is close to the global optimal value and that increases the time of obtaining the global optimal parameter. Fruit fly Optimization Algorithm (FOA) is a kind of global optimization swarm intelligent algorithm proposed by PAN in 2011. This algorithm has the advantages of few parameters, fast calculation speed, strong ability of global optimization and easy implementation, and it has been popularized and applied in a multitude of fields. In this paper, for the purpose of maximizing the kurtosis of denoising signals, FOA is applied to select the parameters of NLM to achieve the optimal smoothing effect of vibration signals in the process of noise reduction.

Teager Kaiser energy operator (TKEO) is a nonlinear difference operator. It is suitable for signal demodulation analysis in the impact detection because of its high time resolution, fast demodulation speed, and the positive ability of enhancing signal transient characteristics. However, TKEO demodulation method only applies to the single-component signals for amplitude modulation and frequency modulation. During the bearing operation, there are always other components associated with rotating frequency and other strong noise components in the vibration signals. Therefore, TKEO has no obvious effect on the demodulation of the vibration signals with low SNR.

In order to solve the above problems, a fault detection method based on NLM optimized by FOA and TKEO in the initial stage of rolling bearing degradation is proposed in this paper. First of all, the root mean square (RMS) index is used to find the initial stage vibration signal of the whole life data. Maximization of the kurtosis of the denoised signal is taken as the optimization target. The filter parameters and structure half width parameters of NLM are optimized through FOA. The vibration signal is denoised by optimized NLM. The impact components are further enhanced through TKEO. Finally, the bearing fault detection is finished by observing the frequency components of Teager energy spectrum, and the type of fault is further determined.

The paper is organized as follows: Section 2 presents FOA search for optimal parameters in details, NLM algorithm and the optimized NLM algorithm by FOA and GA. Section 3 presents TKEO in details. Section 4 presents a proposed method based upon optimized NLM and TKEO for rolling bearing fault detection in the initial stage of degradation. A positive discussion and analysis of the experimental results are also obtained in this section by comparing the proposed method with TKEO demodulation directly on origin data. Finally, our conclusions are provided in Section 5.

2. OPTIMIZED NLM ALGORITHM BY FOA

2.1. NLM Algorithm

NLM algorithm is a kind of image processing method based on block similarity theory proposed by A BUADES et al. This method is developed from the neighbourhood filtering. The core idea is to find as much as possible similarity to the weighted average in a large range called the search window. There is a large amount of redundant information in nature, which contains a number of similar components. The different modules will be repeated, and the theory was found based on the fact. The vibration signal of the rolling bearing contains abundant state information. It is bound to repeat a certain period of shock signal when the bearing is damaged in operation. The noise superimposed on the structural similarity block is random. Therefore, we can remove the noise by weighted average. NLM algorithm in one dimensional signal is as follows:

Assuming that \( x(t) \) is a real signal, \( n(t) \) is the noise signal, the actual signal can be expressed as:

\[
y(t) = x(t) + n(t).
\]

In the non local mean filtering algorithm, \( X \) is used to estimate the original signal \( x \), which is obtained by the weighted average of all the similar structures in the search domain. Therefore, the estimated value can be expressed as:

\[
X(s) = \frac{1}{Z(t)} \sum_{t \in N(s)} \omega(s,t)y(t).
\]

Where, \( N(s) \) represents the search area with the sample point \( s \) as the centre. \( Z(t) \) represents the summation of all blocks of similar structure. \( \omega(s,t) \) represents similarity. \( y(t) \) represents actual signal containing noise. In formula (2):

\[
Z(t) = \sum_{t \in N(s)} \omega(s,t);
\]

\[
\omega(s,t) = \exp\left(\frac{\sum_{\delta \in \Delta} (y(s+\delta) - y(t+\delta))^2}{2L\Delta^2}\lambda^2\right); \quad (4)
\]

\[
0 \leq \omega(s,t) \leq 1; \quad (5a)
\]

\[
\sum_{t} \omega(s,t) = 1; \quad (5b)
\]

where \( \lambda \) represents filter parameter. It controls the attenuation rate of \( \omega(s,t) \), and determines the smoothness of the denoised signal. \( \Delta = [s-p, s+p] \) represents the target structure block taking \( s \) as the centre. \( L = [t-p, t+p] \) represents the similar structure block taking \( t \) as the centre. Weight between two similar blocks is measured with \( \omega^2(s,t) \) which denotes the sum of the squared point-to-point dissimilarity.

The target recovery structure block is assumed to be \( A \). Three key parameters of NLM are the half width of structure

International Journal of Acoustics and Vibration, Vol. 23, No. 2, 2018
block \( P \), the half width of search area \( K \) and the filter parameter \( \lambda \). The meaning of each parameter is demonstrated in Fig. 1. The target structure block A and the similar structure block B are matched by a search window, which has the same length, and the similarity is calculated by \( \omega(s, t) \). The similar structure block \( B \) is found in the search domain to estimate the target block \( A \) centre point \( s \).

2.2. Fruit Fly Optimization Algorithm

First, the position of the fruit fly population is initialized. According to the coordinate position of the fruit fly population, each of the flies in the initial population searches the direction and distance of food through their smell sense. The specific location of the food can not be learned. Consequently, the distance between the fruit fly and the origin need to be calculated. The reciprocal of the distance was used as the food concentration decision value of the fruit fly individual. The food concentration in each individual location in the fly population is calculated, and the maximum value of the food concentration and the position coordinates of the fruit fly are preserved according to the concentration. The fly population flies to the coordinates with vision. The coordinate of the fruit fly individual is initialized again in the new position, and the above process is repeated until the location of the food is found. The flow chart of FOA is manifested in Fig. 2.

The detailed steps of the algorithm are as follows:\textsuperscript{14}

1. The fruit fly population is assumed to be consisting of \( N \) fruit fly individual. Above all, the location of fruit fly population \((x, y)\) is initialized. The location of fruit fly individual is further initialized according to \((x, y)\). The individuals search food with smell sense.

\[
\begin{align*}
    x_i &= x + L_k \\
    y_i &= y + L_k
\end{align*}
\]

Where, \( L_k \) is the random step value in the step interval \([-L, L]\). \( L \) is the maximum step size that the fruit fly individual can fly in search of food with smell.

2. The distance between individual \( i(1, \ldots, N) \) and food is calculated. The decision value of taste concentration \( Dc_i \) is further calculated through the distance.

\[
    d_i = \sqrt{x_i^2 + y_i^2};
\]

\[
    Dc_i = \frac{1}{d_i};
\]

3. The taste concentration obtained by individuals smell sense is calculated with smell concentration decision function based upon \( Dc_i \).

\[
sm_i = \text{fitness function}(Dc_i);
\]

4. The individual of the highest taste concentration is found.

\[
[Tsm \text{ index }] = \max(sm);
\]

where, \( Tsm \) is obtained highest concentration. \text{index} is the order number of fruit fly individual obtaining highest concentration. \( sm \) is a set of fruit fly population taste concentration. Whether the taste concentration is better than the previous generation is determined. If Yes, the step (5) is executed. Otherwise, repeat steps (2) to (4).

5. The location of the best taste concentration and obtained individual is preserved, and fruit fly population fly to the position with vision.

\[
smT = Tsm;
\]

\[
\begin{align*}
    xT &= x(\text{index}) \\
    yT &= y(\text{index})
\end{align*}
\]

6. Whether to meet the preset conditions is determined (the fixed fitness value according to the actual problem or the maximum number of iterations). If Yes, the best location of food is found. Otherwise, return step (2).

2.3. Optimized NLM Algorithm by FOA

It is crucial to determine the parameter when NLM algorithm is applied to denoised signal. A greater search area can
obtain a better result. However, increasing the search area will cost much time. It is more formidable to determine \( p \) and \( \lambda \). Ville and Kocher aimed at simulation on research Sure method.\(^{21,22}\) It can achieve best result that \( \lambda \) is determined as 0.5\( \sigma \). \( \sigma \) is noise standard deviation. Nevertheless, \( \sigma \) is more formidable to be determined in the actual signal, and it can only be determined by the way of estimation. FOA is a kind of swarm intelligence algorithm for global optimization. In order to achieve the best denoising effect, the optimal \( p \) and \( \lambda \) can be determined by FOA. The accuracy of the initial stage of degradation fault detection is further ensured.

Kurtosis is the dimensionless parameter in time domain analysis, which describes maximum impulsiveness of a waveform. Its mathematical expression is as follows:

\[
K = \frac{E(x - \mu)^4}{\sigma^4},
\]

where, \( \mu \) is the mean value of signal \( x \). \( \sigma \) is the standard deviation of signal \( x \).

Kurtosis is mainly used to measure the degree that the vibration signal amplitude deviates from the normal distribution. The kurtosis is about 3 when the signal is approximate normal distribution, while the kurtosis increased obviously as the signal has more impact component.\(^{23}\)

Based upon the above analysis, the maximum kurtosis is taken as optimized target in denoised signal, and the filter parameters and structure half width parameters of NLM are optimized through FOA. The best denoising effect is achieved and more impact components are highlighted. The optimization steps are as follows:

1. The search area of NLM is determined as following principle: If the signal length is short (\( N < 4000 \)), \( N(S) \) is taken as the signal length to make the search area covering the whole signal range. The more weighted average similar blocks is performed to obtain, which makes good use of redundant information of the signal itself. If the signal length is long (\( N > 4000 \)), \( N(S) \) is determined as: \( 1/2 \ast N \leq N(s) \leq 2/3 \ast N \), considering the problem of calculating time.\(^{11}\)

2. The maximum kurtosis is taken as optimized target in denoised signal. The filter parameters \( \lambda \) and structure half width parameters \( p \) of NLM are optimized through FOA.

3. The optimized parameters are put into NLM, and then the original vibration signal denoising is completed with FOA-NLM.

### 2.4. Optimized NLM Algorithm by GA

\( N(S) \) is also determined as: \( 1/2 \ast N \leq N(s) \leq 2/3 \ast N \), which is the same as optimized through FOA. The free parameters \( p \) and \( \lambda \) greatly affect the denoising effect of NLM algorithm. GA is used to search for better combination of the parameters in NLM, and it can obtain the optimal solution after a series of iterative computations. The optimization steps are as follows:

1. Initialization: Randomly generate an initial population of chromosomes which represent the values of parameters \( p \) and \( \lambda \).
2. Calculating the fitness function: The maximum kurtosis is taken as fitness function in denoised signal.
3. GA operators: Selection, crossover and mutation operators generate the offspring of the existing population in GA. Offspring replaces the old population and forms a new population in the next generation by the three operations, the evolutionary process proceeds until stop conditions are satisfied.
4. Suppressing noise: The optimized parameters by GA are put into NLM, and then the original vibration signal denoising is completed by GA-NLM.

### 3. TEAGER KAISER ENERGY OPERATOR

Signal energy is traditionally defined as the square of the amplitude of the signal. However, the impact component may be submerged when the impact amplitude is small. TKEO estimates the total energy required for signal source to generate a dynamic signal through the nonlinear combination of the instantaneous value of the signal and its derivative. Compared with the traditional energy definition, the product between the square of frequency and signal amplitude replace the square of the amplitude. The frequency of the transient shock is higher. Accordingly, TKEO can effectively enhance the transient shock component. TKEO has the advantage of simple calculation and it is suitable for processing AM-FM signals which has a slow change of instantaneous frequency and relative high SNR.\(^{24,25}\)

For a AM-FM signal \( x(t), a(t) = a(t) \cos[\phi(t)] \). The TKEO is defined as follows:\(^{25}\)

\[
\psi[x(t)] = [\dot{x}(t)]^2 - x(t)\ddot{x}(t);
\]

where, \( \dot{x}(t) = \frac{dx(t)}{dt}, \ddot{x}(t) = \frac{d^2x(t)}{dt^2} \).

It can be demonstrated that the discrete version of the TKEO is:

\[
\psi[n] = [x(n)]^2 - x(n - 1)x(n + 1).
\]

Instantaneous frequency \( \omega(n) \) is the difference function of \( \phi(n) \):

\[
\omega(n) = \phi(n) - \phi(n - 1);
\]

\( x(n) = a(n) \cos[\phi(n)] \) substitute in Eq. (15):

\[
\psi[x(n)] = a(n)^2 \cos^2 \phi(n) - a(n - 1) \cos[\phi(n - 1)]a(n + 1) \cos[\phi(n + 1)].
\]

Suppose \( y(n) = x(n) - x(n - 1) \)

\[
\psi[y(n)] = 4a(n)^2 \sin^2 \frac{\omega(n)}{2} \sin^2 \omega(n).
\]

The instantaneous frequency and instantaneous amplitude are both calculated:

\[
\omega(n) = \arccos \left(1 - \frac{\psi[y(n)]}{2\psi[x(n)]}\right);
\]

\[
|a(n)| = \frac{\psi[x(n)]}{\sqrt{1 - \frac{\psi[y(n)]}{2\psi[x(n)]}}}. \tag{20}
\]
4. ROLLING BEARING FAULT DETECTION IN THE INITIAL STAGE OF DEGRADATION BASED UPON OPTIMIZED NLM AND TKEO

4.1. Rolling Bearing Fault Detection Method in the Initial Stage of Degradation

The procedure of the proposed method that achieves rolling bearing fault detection in the initial stage of degradation is shown as follows:

1. Initial stage of degradation signal is determined with RMS index of the run-to-failure. Maximization of the kurtosis of the denoised signal is taken as the optimization target. The filter parameters and structure half width parameters of NLM are optimized through FOA.

2. Original vibration signal is denoised with optimized NLM.

3. The denoised signal is processed with TKEO. The transient shock components are enhanced and non impact components are suppressed.

4. Fault detection is finished through spectrum analysis for the processed signal, and the fault type is further determined based on the frequency component in the Teager energy spectrum.

4.2. Rolling Bearing Inner Race Fault Detection in the Initial Stage of Degradation

In this paper, the bearing whole life tester provided by Hangzhou Bearing Test & Research Center (HBRC), and the experimental platform (ABLT-1A) is manifested in Fig. 3. A load of 6.6 kN is appended on the bearings to accelerate the bearing to failure. The type of bearing used in the experiment is 6204, which is applied to mechanical equipment widely, and its structure parameters are displayed in Table 1. The rotation speed is kept at 1500 rpm. The whole lifetime of the test-bearing is 9800 min. Inner race failure occurred at the end of the test to failure experiment. YD-1 acceleration sensors are adopted in data acquisition module. Data were collected every 10 minutes, and 20480 points are collected every time. The data sampling rate is 25.6 kHz. A total of 980 sets data are collected.

Inner race failure occurred in the experiment. Inner race fault frequency is calculated:

\[ f = \frac{zf_r}{2} \left(1 + \frac{D_b}{D_c} \cos \alpha \right) \]  \hspace{1cm} (21)

where, \(z\) represents ball number. \(f_r\) represents rotating frequency. \(D_b\) represents Ball diameter. \(D_c\) represents Pitch diameter. \(\alpha\) represents contact angle. The bearing structure parameters in Table 1 are entered into Eq. (21). Inner race fault frequency is obtained as 123.7 Hz.

The RMS of whole lifetime vibration data is demonstrated in Fig. 4 to search for the degradation initial stage vibration. RMS begins to increase at 6970 min as we can see from the

Fig. 4. The RMS of whole lifetime.

The search area of NLM \(N(S) = 10^4\) is determined. Original vibration signal at 6970 min is denoised with optimized
Table 1. Structure parameters of rolling bearing.

<table>
<thead>
<tr>
<th>Type</th>
<th>Pitch diameter</th>
<th>Ball diameter</th>
<th>Ball number</th>
<th>Contact angle</th>
<th>Rated radial load</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>6204</td>
<td>335 mm</td>
<td>7.938 mm</td>
<td>8</td>
<td>0°</td>
<td>6.6 kN</td>
<td>0.11 kg</td>
</tr>
</tbody>
</table>

Table 2. Parameters of the optimizing algorithms.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>The maximum number of iterations</th>
<th>Population size</th>
<th>Initial coordinates</th>
<th>Step value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOA</td>
<td>200</td>
<td>20</td>
<td>( x = 5 \cdot \text{rand}(1,1) )</td>
<td>20</td>
</tr>
<tr>
<td>GA</td>
<td>200</td>
<td>20</td>
<td>Crossover probability: 0.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mutation probability: 0.1</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5. The time domain and squared envelope spectrum at 6970 min.

Figure 6. The fitness curve of parameter optimization.

Figure 7. Timeplot after FOA-NLM.

Figure 8. Timeplot after optimized GA-NLM is also demonstrated in Fig. 9. It is arduous to find out the more pleasurable optimization method only from the time domain waveform. The denoised signal is processed with TKEO, and TKEO frequency spectrum after FOA-NLM is shown in Fig. 8. The frequency and the doubling can be detected from the figure. There are some deviations between the fault frequency and the theoretical value on trial, which is mainly caused by the instability of the speed. It can not affect the detection results, and the results demonstrate the effectiveness of the proposed method in this paper.

In order to verify the proposed method more favourable compared with the FK method. The corresponding kurtogram is manifested in Fig. 11 and the maximum kurtosis is \( K_4 \), indicated by the black rectangle in the figure. The filter signal with...
Table 3. Comparison of FOA and GA optimization algorithms results.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Optimized results ((p, \lambda))</th>
<th>Time in optimization algorithms (s)</th>
<th>Convergent iterative number</th>
<th>The kurtosis of denoised signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOA</td>
<td>((20, 0.035))</td>
<td>2.8841</td>
<td>14</td>
<td>5.3693</td>
</tr>
<tr>
<td>GA</td>
<td>((49, 0.142))</td>
<td>19.8294</td>
<td>26</td>
<td>3.8702</td>
</tr>
</tbody>
</table>

Table 4. Parameters of the rolling bearing.

<table>
<thead>
<tr>
<th>Type</th>
<th>Pitch diameter</th>
<th>Ball diameter</th>
<th>Ball number</th>
<th>Contact angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZA2115</td>
<td>71.5 mm</td>
<td>8.4 mm</td>
<td>16</td>
<td>15.17°</td>
</tr>
</tbody>
</table>

Figure 8. TKEO frequency spectrum after FOA-NLM.

Figure 9. Timeplot after GA-NLM.

Figure 10. TKEO frequency spectrum after GA-NLM.

Figure 11. Kurtogram for the experimental bearing at 6970 min.

Figure 12. Filter signal timeplot.

4.3. Rolling Bearing Outer Race Fault Detection in the Initial Stage of Degradation

Cincinnati University Intelligent Maintenance Systems (IMS) experimental data is selected to study rolling bearing outer race fault detection in the initial stage of degradation. Four bearings are installed on one shaft as shown in Fig. 15. The parameters of rolling bearing are displayed in Tab. 4. The

ity to noise. In the early stage of degradation, the fault signal is extremely weak. The instantaneous frequency and amplitude are estimated through TKEO. Nevertheless, fault can not be detected as a result of fault frequency submerged in noise. This is consistent with the conclusions given in reference. Accordingly, the necessity of the proposed method is verified.
rotation speed is kept at 2000 rmp. A radial load of 26671 N is applied onto the shaft and bearing by a spring mechanism, and all bearings are force lubricated. The acceleration sensors are installed on bearing housing (the horizontal and vertical direction). Data collection is facilitated by NI DAQ Card 6062E.

Three data sets are included in the experimental data. The second data is chosen for outer race fault detection in the initial stage of degradation. At the end of the test-to-failure experiment, outer race failure occurred in bearing 1. The whole lifetime of the test bearing is 164 h. Data are collected every 10 minutes, and 20480 points are collected every time. The data sampling rate is 20 kHz. A total of 984 sets data are collected. The outer race fault frequency can be obtained as 236 Hz.

The RMS of whole lifetime vibration data is demonstrated in Fig. 16 to search for the degradation initial stage vibration. RMS shows larger fluctuations at 7000 min, and the fault can be detected. The RMS begins to increase at 5290 min. Consequently, the data at this time is set as the vibration signal of the initial stage of degradation. Its time domain figure and squared envelope spectrum is shown as Fig. 17. Whether fault has occurred is not determined from the squared envelope spectrum. Fault frequency is submerged in noise, and fault can not be detected effectively.

Parameters of the optimizing algorithms are the same as in Section 4.2. The fitness curve of parameter optimization is shown in Fig. 18. Optimized results are demonstrated in Table 5. The results show that the kurtosis of the denoised signal is obviously increased. The impact component has been improved accordingly.

Original vibration signal at 5290 min is denoised with optimized NLM. Time domain waveform after optimized NLM is manifested in Fig. 19. The denoised signal is processed with TKEO, and TKEO frequency spectrum after optimized NLM is shown in Fig. 20. The fault frequency and the doubling can be detected from the figure, and the results demonstrate the effectiveness of the proposed method in this paper.

Time domain waveform after optimized GA-NLM is also demonstrated in Fig. 21. It is arduous to find out the more pleasurable optimization method only from the time domain waveform. The denoised signal is processed with TKEO, and TKEO frequency spectrum after GA-NLM is shown in Fig. 22. The fault frequency is submerged in noise and we hardly discover fault information and it is very little if anything detects the fault. Consequently, we can determine that the denoising effect is less impressive by GA-NLM.

The corresponding kurtogram is manifested in Fig. 23 and the maximum kurtosis is \( K_{1,5} \), indicated by the black rectangle in the figure. The filter signal with the maximum kurtosis is displayed in Fig. 24, and it is further processed by TKEO in Fig. 25. Although it can be detected by fault frequency, the detection effect is not as outstanding as the proposed method. Meanwhile, we hardly discover fault information in inner race fault detection in Fig. 13. This shows that the performance of FK filter is not stable, for which the six parameters of a filter can affect the detection result.

In order to explain the necessity of this method, TKEO demodulation method that is directly carried out in origin signal is taken as comparison. The spectrum of TKEO demodulation directly on the 5290min data is shown in Fig. 26, which is similar to Fig. 16(GA). The denoising effect is less impressive by GA-NLM, for which GA falls into local optimal solution. In the early stage of degradation, the fault signal is extremely weak, and TKEO is its sensitivity to noise. It is arduous to find out fault information and the fault is either little or nothing is
Table 5. Comparison of FOA and GA optimization algorithms results.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Optimized results ((p, \lambda))</th>
<th>Time in optimization algorithms (s)</th>
<th>Convergent iterative number</th>
<th>The kurtosis of denoised signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOA</td>
<td>((42, 0.115))</td>
<td>3.4859</td>
<td>25</td>
<td>7.8365</td>
</tr>
<tr>
<td>GA</td>
<td>((78, 0.462))</td>
<td>25.8754</td>
<td>43</td>
<td>5.2483</td>
</tr>
</tbody>
</table>

![Figure 17. The time domain and squared envelope spectrum at 5290 min.](image)

![Figure 18. The fitness curve of parameter optimization.](image)

![Figure 19. Timeplot after FOA-NLM.](image)

5. CONCLUSIONS

1. The parameter selection of NLM directly affects the denoising effect, and FOA can optimize the parameters of NLM so as to avoid the blindness of parameter selection.

2. The performance of FK filter is not stable, for the reason that the parameters of a filter can affect the detection result. The denoising effect is not as outstanding as FOA-NLM method.

3. The impact composition in the early degradation state of the rolling bearing is extremely weak. TKEO demodulation is difficult to detect the fault with signals whose SNR is low. The optimized NLM can achieve positive denoising effect and highlight the impact components, and TKEO can effectively detect the fault components after optimized NLM processing.

4. The proposed method which is able to detect the fault components in the initial stage of bearing degradation is applied in two kinds of experiment data including the inner race fault one and the outer race fault one. The obtained experimental results demonstrate the effectiveness of the proposed method.

ACKNOWLEDGEMENTS

The authors are grateful to Cincinnati University Intelligent Maintenance Systems. This project is supported by National Natural Science Foundation of China (Grant No. 51541506).

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Figure 26. The spectrum of TKEO demodulation directly on the 5290 min data.


Dynamic Characteristics of the Flange Joint with a Snap in Aero-Engine

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(Received 17 February 2017; accepted 2 February 2018)

As an important component in aero-engine, the stiffness of the flange joint has a direct impact on vibration characteristics. The paper studies the stiffness characteristics of the flange joint which includes a snap by nonlinear transient analysis in ANSYS. The angle of rotation-load curves under harmonic transverse load are obtained. It is discovered that there are different routes of curves in the process of uploading and unloading, and that the curves form a closed cycle, which is called hysteresis loop. Then, the paper also analyses the hysteresis’s generation mechanism and the influence of joint parameters — preload of bolts and interference of the snap — on the shape and area of hysteresis loop. Finally, a model, which is comprised of the three-dimension finite element (namely, the joint) and beam element, is built to study the influence of hysteresis on vibration. The results show that the hysteresis of joint can be aroused under the circumstance of certain vibration amplitude and that the hysteresis can rapidly accelerate the amplitude attenuation by the mean of energy dissipation. The research results can be applied to vibration suppression design of aero-engine.

NOMENCLATURE

ICE Infinitesimal Contact Element.
Fcr Critical Force.

1. INTRODUCTION

The flange joint with a snap is an important kind of joint structure, which is widely used in the rotor and stator of aero engine for its advantages: easy installation, stable performance, and good centring. Initially, the joint of mostly is ignored and the two parts which are combined by joint were considered as one unit; thus, the stiffness characteristics of joint were neglected. Later studies, however, discover that there is obvious stiffness loss in joint, which has great influence on vibration characteristics. As a result, many scholars start to pay attention on its stiffness characteristics and simulation.1–3 Cankurt calculated the stiffness of the joint through genetic algorithm.4 Wang studied the stiffness loss of the joint in aero-engine and its influence on vibration of rotor.5 Yao presented a new dynamic modelling method which is called the improved thin-layer element method to simulate the stiffness characteristics of the joint.6 Qin studied the Bolt loosening at rotating joint interface and its influence on rotor dynamics.7 These researches do not take into consideration the influence of a snap on the stiffness of joint, and assume the stiffness is constant. In fact, the stiffness and contact status of the joint do vary with the loading’s fluctuation. Thus, scholars studied nonlinear stiffness and energy dissipation of joint.8–11

Since there are several bolts distributing in the joint, the characteristic of bolted joints has a great influence on the stiffness of the joint. Existing researches point out that hysteresis appears when the bolt was under the transverse harmonic load.12 Bograd studied the hysteresis for the structure with several bolts under the transverse load.13 Qin studied axial stiffness of the interface joined by clamp band and discovered the characteristic of hysteresis.14 Van-Long attained the hysteresis of the flange joint without a snap through the experiment.15 Scholars focused their study on the simulation of the hysteresis.16, 17 Oldfield, Matthew used the Jenkins element to simulate the hysteresis when single bolt was under the transverse load.18, 19 Hysteresis of flange joint is caused by the shear slipping between the two parts which are combined by the bolt. But the shear slipping between the two pieces of flange is restricted by the snap and no research has ever proved whether the hysteresis exits in such a case or not.

Shuguo established the three-dimension finite element model of joint with a snap to study the stiffness characteristic under the transverse load.20 It was found that the bending stiffness decreases suddenly once the load reaches a certain value in the process of loading and that the values of bending stiffness before and after the decrease are both constants. This phenomenon was verified by his experimental result, but no analysis of generation mechanism was conducted. In this paper, the bending stiffness of joint is studied in the process of not only loading but also unloading. The sudden decrease of stiffness is obtained again through numerical calculation. Furthermore, the joint’s hysteresis, which is related to the sudden decrease of stiffness, is discovered, and the generation mechanism of the sudden decrease of stiffness and the hysteresis are studied. The paper will provide reference for vibration suppression design and analysis.

2. STRUCTURE AND MODEL

The main characteristics of flange joint, cylinder, nut, head of bolt, contact interface, and the snap are shown in Fig. 1a, whose structure is the same size as in the reference.20 The snap contact interface is set as interference fit for the purpose of strict centring. The yellow lines signify contact interfaces, and contact between the screw and the bolt hole are ignored. The complete finite element model is shown in Fig. 1b; the solid parts of that model are modeled by three-dimensional brick elements called SOILD185 in ANSYS, and the contact interfaces are modelled by CONTAC174 and TARGE170. KEY-OPT CNOF is set as a positive value to simulate the interfe-
The transverse load is applied along Y direction of this node can present the rotation of the right edge of the transverse load. Due to the rigid region, the angle of rotation of rotation is extracted from the node which is applied on the transverse vibration of the structure. The data of angle of the flange joint because the stiffness has a great influence to study the angular bending stiffness (stiffness for short) in order to take nonlinear factors into consideration. The full transient dynamic analysis is carried out to ensure that the inertial force of the model has little influence on the result. The frequency of the harmonic transverse load is set as 1 Hz in which the harmonic transverse load experiences two cycles. The time integration of ANSYS is turned on in the second stage from 1 s to 3 s, and PRETS179 is used to simulate the preload of the bolt. A node which is established in the axis of the cylinder for applying cyclic loading, along with all nodes on the right edge of the cylinder is utilized to create a rigid region. Figure 1c shows the local details of the finite element mesh. The screw thread and the contact interface between nut and screw are ignored which make little difference to the stiffness of bolt. And the nut and screw are assumed as a whole.

All nodes on the left edge of the cylinder are constraint fixed. The transverse load which is applied to finite element model can be divided into two stages as is shown in Fig. 2. 0 s to 1 s indicates the first stage, in which the transverse load remains zero and the preload of bolt is applied on the element PRETS179 by use of the command of sload. Meanwhile, the time integration of ANSYS is turned off so that the preload can be applied as prestressing force of the structure. Time integration of ANSYS is turned on in the second stage from 1 s to 3 s, in which the harmonic transverse load experiences two cycles. The frequency of the harmonic transverse load is set as 1 Hz to ensure that the inertial force of the model has little influence on the result. The full transient dynamic analysis is carried out in order to take nonlinear factors into consideration.

This paper is to focus the research object on the angle of rotation to study the angular bending stiffness (stiffness for short) of the flange joint because the stiffness has a great influence on the transverse vibration of the structure. The data of angle of rotation is extracted from the node which is applied on the transverse load. Due to the rigid region, the angle of rotation of this node can present the rotation of the right edge of the flange joint. The transverse load is applied along Y direction and the axis of flange cylinder is along Z direction, thus the angle of rotation taken into consideration is rotation around X direction. Figure 3 presents the angle of rotation - load curve under the amplitude of harmonic load of 20,000 N. The curve is clearly a hysteresis loop like a willow leaf. The two cycles in the curve are less distinct. In the initial stage of loading, the stiffness decreases suddenly as soon as the load reaches the value of 6,000 N. The stiffness before decrease is defined as initial stiffness, and the one after is defined as changed stiffness.

The angle of rotation-load curve, which is nonlinear, is divided into 7 linear stages, as shown in Fig. 3. Stage a is the process of initial loading, in which the angle of rotation increases linearly along with the increase of load. The sudden change of stiffness occurs at the junction of stage a and b. This phenomenon is consistent with the experimental result in the reference, therefore, the numerical calculation in this paper is proved credible. The angle of rotation and the load reach their maximum at the end of stage b. In stage c, which is the beginning of the unloading process, the stiffness of the joint is the same as the initial stiffness in stage a. Stage d is also the process of unloading and stage e is the process of loading in the opposite direction to the initial loading. The curves in stage d and e are approximately linear. Stage f is the beginning of the unloading process the same as stage c, and stage g is the unloading process the same as stage d. After Stage g comes the second cycle, which starts from stage b but not a. The entire curve excluding stage a is symmetrical about the origin. The stiffness values of stage a, c, and e are almost identical to the initial stiffness. The stiffness values of stage b, d, c, and g are similar to the changed stiffness. These different kinds of stiffness values, the initial stiffness and the changed stiffness, display the shape of hysteresis loop.

### 3. MECHANISM OF HYSTERESIS

The stiffness of the joint consists of two parts; one is the stiffness of ring flange $k_1$, and the other is the stiffness of the snap $k_2$. Therefore, the total stiffness is

$$k = k_1 + k_2;$$ (1)

When the joint is bending, the value of $k_1$ remains constant on the condition of ignoring the material nonlinearity and large deformation; however, the value of $k_2$ varies with the con-
tact status of the snap’s interface. Thus, the total stiffness $k$ changes.

Assume that at the interface of the snap exist an infinitesimal contact element (ICE), as shown in Fig. 4a and Fig. 4b.

External forces of ICE are the pressure $P$ and the tensile force $F$ caused by the bending of joint. $\Delta x$ represents the sum of elastic deformation and slipping distance under the load of tensile force $F$. If tensile force $F < \mu P$ ($\mu$ is the coefficient of slipping friction of the interface), the ICE will be in sticking condition, on which there is no slipping in the interface and $\Delta x$ will only consist of elastic deformation. In this case, the required tensile force is:

$$F = GL\Delta x; \quad (2)$$

where, $G$ is the shear modulus and $L$ is the width of ICE as shown in Fig. 4b.

According to Coulomb’s law, once the tensile force reaches the condition:

$$F = GL\Delta x > \mu P; \quad (3)$$

The interface begins to slip. If the slipping is at low speed, the tensile force can be written as

$$F = \mu P. \quad (4)$$

The result can be obtained from Eq. (3) that:

1. When the interface begins to slip, the bigger $G$ is, the smaller $\Delta x$ is.
2. The smaller $P$ and $\mu$ are, the smaller the tensile force $F$ is, which starts the slipping.

It can be seen from Eq. (3) that the tensile force $F$ remains constant no matter how $\Delta x$ changes, once the interface begins to slip.

Assume that at the upside and downside of the snap exist two ideal ICEs, which represent the contact status of the majority parts of interface at each side, as shown in Fig. 4a. In this section, the sticking, which always means deformation of ICE, and slipping of ideal ICEs are analysed based on the different characteristic of the joint’s deformation in the 7 stages (as shown in Fig. 4c).

In stage $a$, the upsides of two pieces of flange plates are subject to compression under the upward load; the downsides of the flange plates are subject to tension and tend to be separated. There is no deformation and slipping in the upside ideal ICE, because two pieces of flange plates are close to each other; and only elastic deformation is occurred in the downside ideal ICE because of the smaller load in the initial loading stage $a$. With the load increasing in stage $b$, the downside ideal ICE begins to slip.

Stage $c$ is the stage of unloading, in which the elastic deformation of the downside ideal ICE occurred in stage $a$ is restored firstly, and elastic deformation is occurred subsequently in the opposite direction.

In stage $d$, the contact statuses of upside and downside of the flange change. The upsides of the two pieces of flange plates begin to be subject to tension and the elastic deformation of upside ideal ICE occurs.

After stage $e$, the upside ideal ICE experiences three stages in sequence: slipping (stage $e$), restoring elastic deformation and elastic deformation in the opposite direction (stage $f$), and slipping again (stage $g$).

In stage $g$, the contact statuses of upside and downside flange change again. After stage $g$, the next cycle begins from stage $b$, but not stage $a$, which is the initial loading stage existing only in the first cycle.

In summary, there is only elastic deformation of ideal ICE in stage $a$, $c$, and $f$; and the ideal ICEs experience slipping in stage $b$, $d$, $e$, and $g$. Though there is elastic deformation of ICE in the stage $d$ and $g$, slipping is the major characteristic in the two stages.

The contact statuses of the interface vary in different positions of the snap. Thus, the ideal ICE does not represent any specific contact statuses of the whole interface, but only shows the general trend of slipping and elastic deformation of the upside and downside part of the snap.

In order to prove valid analysis of ideal ICE above, the accumulated slipping distance of all contact elements in the snap are extracted from the computational results of ANSYS. Figure 5 shows the variation of average slip distance along with the angle of rotation. The slope of curve represents the rate of change of slipping distance. The higher rate means that more elements experience slipping as every element has almost the same speed. According to the speed of slipping, the curve can be divided into 7 stages, which is corresponding to the 7 sages in Fig. 4c. As shown in Fig. 5, the slopes of curve in stage $a$, $c$, and $f$ are almost the same and relatively small. It means that more part of the interface is in the status of sticking than the one in stage $b$, $d$, $e$, and $g$, thus there is less slipping but more elastic deformation. The slopes of curve in stage $b$, $d$, $e$, and $g$ are almost the same and bigger than in stage $a$, $c$, and $f$. It means that the more part of the interface experiences slipping.

Consider an arbitrary position $i$ on the snap as shown in Fig. 6, and its displacement (including deformation and slipping) and load when the joint is bending are analysed.

When the angle of rotation of joint is $\varphi$, the displacement of ICE $i$ can be written as

$$\Delta x_i = \tan \varphi (r \sin \theta_i + r) \approx \varphi (r \sin \theta_i + r); \quad (5)$$

where $r$ is the radius of the snap.
According to Eq. (2) and (3), when ICE \( i \) is in the status of sticking, the tensile force is

\[
F_i = G L \Delta x_i. \tag{6}
\]

When ICE \( i \) is in the status of slipping, the tensile force is

\[
F_i = \mu P_i. \tag{7}
\]

Taking all ICEs of the snap in account, the bending moment which causes the angle of rotation \( \phi \) can be written as

\[
M = \sum_{i=1}^{n} F_i (r \sin \theta_i + r); \tag{8}
\]

where \( n \) is the number of ICEs of the snap.

Assume that the amount of ICEs in the status of stick is \( m \), the bending moment \( M \) can be deducted by combining the three Eq. (6), (7), and (8) as

\[
M = \sum_{i=1}^{m} G L \varphi (r \sin \theta_i + r)^2 + \sum_{i=m+1}^{n} \mu P_i (r \sin \theta_i + r). \tag{9}
\]

The bending stiffness \( k_2 \) can be obtained by calculating the derivative of the bending moment

\[
k_2 = \frac{dM}{d\varphi} = \sum_{i=1}^{m} GL (r \sin \theta_i + r)^2. \tag{10}
\]

It can be seen from Eq. (10) that there is a positive correlation between bending stiffness \( k_2 \) and the value of \( m \). That is, the higher the value of \( m \) is, the bigger bending stiffness \( k_2 \) is; and the lower the value of \( m \) is, the smaller bending stiffness \( k_2 \) is.

According to the data in Fig. 5, there is less slipping but more elastic deformation in stage \( a, c, \) and \( f \). Thus, the total stiffness is bigger than the one in stages \( b, d, e, \) and \( g \), in which more part of the interface experiences slipping than in stage \( a, c, \) and \( f \). The difference of stiffness in distinct stages causes the angle-loading curve to be a hysteresis loop in one cycle.

4. INFLUENCE OF THE JOINT’S PARAMETERS ON HYSTERESIS LOOP

The harmonic load of 5 kinds of amplitude (5 kN, 10 kN, 15 kN, 20 kN, 25 kN) is respectively applied to the finite element model with the interference fit of 0.06 mm to obtain the angle-loading curve, as shown in Fig. 7.

In Fig. 3, it is clearly shown that once the stiffness changes in the stage of initial loading, the curve is a hysteresis loop. In Fig. 7, four curves are hysteresis loops but one curve is not because the amplitude of load 5 kN is too small to make the stiffness change. In the four hysteresis curves, the value of the load which is required to make the stiffness change is almost the same, which is called \( F_{CR} \) in this paper. When the amplitude of load is 5 kN, the entire curve is a line, with nothing nonlinear. The comparison of the four loop curves reveals that the bigger the amplitude of load is, the bigger the area of hysteresis loop is. For the same \( F_{CR} \) in the four curves, the widths of hysteresis loops are the same. But the length of hysteresis loop increases with the growth of the amplitude of load, since the influence of material nonlinearity is ignored.

The 5 models with different contact characteristic in the snap are used to study the effect of interference on the hysteresis of joint, as shown in Fig. 8. It is clearly shown that the hysteresis loop is getting shorter and thicker when the interference increases. That is because the interference directly affects the pressure of contact interface, making it difficult to slip. Thus, the \( F_{CR} \) is positive correlated to the interference. At the same time, bigger interference leads to higher value of stiffness, which results in the less angle of rotation. When the interference is 0 mm, there is no initial contact pressure in the snap. But tiny interference will appear with the deformation of joint under the transverse load. It is not easy to distinguish the change of stiffness due to the small contact pressure in the
snap and the hysteresis loop is very thin. The curve of the model with no snap is completely a line, and the stiffness of the model with no snap is less than both the initial stiffness and the changed stiffness in the other four models. Thus, the snap has the effect of increasing the bending stiffness because of its interference in the contact interface.

Figure 9 shows the effect of preload on the hysteresis of the joint. The 4 cases of preload vary from 2200 N to 5000 N, in which the 4 curves almost coincide. This illustrates that the preload has little effect on the stiffness and hysteresis.

5. EFFECT OF HYSTERESIS ON VIBRATION

In the displacement-loading curve, the area of hysteresis loop presents the energy dissipation of the structure’s vibration in one cycle. Thus, the structure with hysteresis will consume a large amount of energy during vibration for its high frequency of vibration.

5.1. Rotor Model with Flange Joint

In order to enhance the computing efficiency and retaining the characteristic of hysteresis, the joint of the rotor was modelled by three-dimensional finite element model, and the other parts are model by beam element based on Timoshenko theory(BEAM188). The two kinds of model are connected through constraint equations, as shown in Fig. 10a.

In this charter, the effect of hysteresis on the attenuation of vibration is studied. For the general application for both rotor and stator, the gyroscopic effect of rotor is ignored. The left node of model is constrained in three directions (UX, UY and UZ), and the right node is constrained in two directions (UX and UY). Step-impulsive loading was applied to a node between the joint and the disk to simulate impulsive load. The full transient dynamic analysis is used to calculate during 0–1 s. 0–0.07 s is used to apply preload and 0.07 s–0.1 s is used to apply impulsive load. 0.1–1 s is used to calculate the response of rotor. The time step is 5s–4s, which is so short to capture the vibration frequency. Figure 10b is the vibration diagram of first order bending vibration which is excited by transverse load.

5.2. Time-domain Response

The vibration response of the node to which the impulsive load is applied is analysed to study the attenuation of amplitude. The slope of envelope curve of the vibration response curve signifies the speed of attenuation. The two curves of vibration responses under different load are shown in Fig. 11. The free vibration is started at 0.1 s, and the initial amplitudes under load of 10 kN and 40 kN are 1.7 mm and 6.8 mm respectively. With the increase of time, the amplitudes of the two sets of calculated data attenuate obviously. But big distinction can be found in the speed of attenuation between the two sets of calculated data. The speed of attenuation before 0.2 s under the load of 10 kN is faster than the one after 0.2 s. And the speed before 0.4 s under the load of 40 kN is faster than the one after 0.4 s. It is evident from Fig. 11 that once the amplitude is attenuated to 0.3 mm, the speed of attenuation noticeably slows down. For this model, the amplitude of 0.3 mm is its critical amplitude in this paper. If the amplitude is bigger than the critical amplitude, the joint vibrates with the characteristic of hysteresis, which increase the dissipation of energy. Thus the amplitude will be attenuated to the critical amplitude rapidly. When the amplitude is equal to the critical amplitude, the stiffness of joint is linear and the joint vibrates with no characteristic of hysteresis. Thus the speed of attenuation noticeably slows down as the lack of energy dissipation of hysteresis. It is also clearly observed that the bigger amplitude is, the faster speed of attenuation is. Because the bigger amplitude leads to the bigger longitude of the hysteresis loop, it results in more dissipation of energy in one cycle.

Figure 8. Angle of rotation-load curve of models with the different snap.

Figure 9. Angle of rotation-loading curve of models with different preload.

Figure 10. Rotor model with flange joint a) Finite element model, b) Vibration diagram.
The model which is without a snap is studied to compare the effect of the snap on the attenuation of amplitude. The two curves of free vibration responses under the load of 10 kN, which each other belong to the model with a snap and the model without a snap are shown in Fig. 12. Through comparing the changes of two curves, it is found that the initial amplitude of the model without a snap is obviously bigger than the one of the model with a snap because the stiffness of the joint with a snap is bigger and the speed of attenuation of the model without a snap is obviously slower for the lack of dissipation of energy. The model without a snap vibrates in the condition of big amplitude for the whole time.

The loads of 10 kN and 1.5 kN are applied to the model with the interference of 0.06 mm respectively to obtain its free vibration response. The two curves of the free vibration response are shown in Fig. 13. The initial amplitude of the set of 1.5 kN is obviously smaller than the one of 15 kN. In the whole process, the speed of attenuation remains low, in which there is no obvious change of the speed of attenuation. The amplitude of the set of 1.5 kN is so small that the whole process is in the stage of initial loading and there is no hysteresis in the whole vibration to consume energy.

The analyses above show that the speed of attenuation which is caused by the structural damping is very slow, but the amplitude of the model with a snap will be attenuated to the critical amplitude rapidly by the way of dissipation of energy. If the hysteresis characteristic is applied to the design of vibration suppression in aero-engine, compared to squeeze film damper, the stiffness of joint is linear when the amplitude is below a certain value. Once the amplitude is above the certain value, the hysteresis appears and makes the amplitude fall down rapidly below the certain value.

5.3. Frequency Domain Response

The frequency response curves of four sets of calculated data are shown in Fig. 14. It can be found that the response frequency in Fig. 14d is smaller obviously than the other three sets because the lack of the snap causes the smaller stiffness. The Bottom of the response peaks in Fig. 14b and Fig. 14c, in which the hysteresis happens, are thicker than the two response peaks in Fig. 14a and Fig. 14d; There are more complex frequency components closed to the peak frequency in the former response peaks. The hysteresis is caused by the difference of stiffness in distinct stages which also cause this complex frequency components. The two sets of peak frequency in Fig. 14b and Fig. 14c are a little smaller than the one in Fig. 14a, this is because the model without hysteresis only experiences the stage , and the model with hysteresis mainly experience the stage , , , and whose stiffness is smaller than the one in stage as shown in Fig. 3.
6. CONCLUSIONS

Based on FEM nonlinear numerical simulation, the nonlinear dynamic characteristic of the joint is discovered in this paper.

1. When the joint with a snap is under the certain transverse harmony load, the angle of rotation-loading curve presents the hysteresis characteristic, which is caused by the slipping of the contact interface in the snap.

2. The bigger the amplitude of load is, the bigger the area of hysteresis loop is. The bigger the value of interference is, the shorter and thicker of hysteresis loop is. The decrease in friction of the snap, which facilitates slipping, makes it easier for the hysteresis to appear.

3. When the model’s amplitude is below a certain value (called critical amplitude in this paper), the bending stiffness keeps a constant. But when the amplitude is above the certain value, the hysteresis appears and makes the amplitude fall back quickly below the value.

4. When the hysteresis appears, the peak frequency is a little smaller and there are more complex frequency components around the peak frequency.

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Vibration Analysis of a Spring Supported FG Beam Under Harmonic Force

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(Received 19 March 2017; accepted 2 February 2018)

The harmonic response and resonance frequency behavior of the spring supported FG beam against the lateral motion for the various values of support stiffness at the ends are analyzed by using the force transmissibility parameter with the finite-element method and Lagrange’s equations. The steady-state responses to a sinusoidal varying force are determined for the support reactions in a frequency domain. The problem is solved within the framework of the Timoshenko beam theory. For the convergence study, some of the results are compared with previous works’ values, and a good agreement can be seen. The study concludes that the support stiffness, volume fraction coefficient and the ratio of young module level all strongly affect the natural frequencies and the force transmissibility. For some values of spring stiffness of support, the rigid body motion of the beam occurs and reduces the effect of volume fraction coefficient on the natural frequencies.

1. INTRODUCTION

Pure metal usage is limited in engineering application since the application may be in need of a material which has almost opposite properties. For instance, there may be a material requirement for an engineering application, like a material having both a good strength and a temperature resistance. The alloys serve that purpose to a certain extent but they have some limitations varying by their production method, like the thermodynamic equilibrium limit, the difficulty of alloying the two different materials having melting points quite a change. The composite structures can also combine two materials having different properties, but they have a disadvantage too. When two materials constituting the composite structure have a different coefficient of expansion, separation at the interface of the composite can be seen in high temperature.

In order to overcome the problems above, especially the problem of composite structure, in a space plane project, Japan scientists came up with an idea of functionally graded material (FGM) in 1984 during. Since material and mechanical properties of FGM vary gradually from one surface to another, it cannot be seen as a separation. Despite the difficulty in production of FG material, the wide range of application areas and above-mentioned advantages drive the researchers to study FGM structures like beams, plates and shells in the last decade. Sankar studied the elasticity solution of the functionally graded beams subjected to transverse loading with the assumption of Euler–Bernoulli beam theory.1 The material properties are assumed to vary exponentially through the thickness. He showed that the beam theory is valid for long, slender beams with slowly varying transverse loading. Aydogdu and Taskin investigated the free vibration of simply supported functionally graded beams by using various beam theories.2 The beam theories that they used to investigate the free vibration of the simply supported FG beam are Euler-Bernoulli beam theory, parabolic shear deformation beam theory (PSDBT) and exponential shear deformation beam theory (ESDBT). Material property variation through the thickness is considered as the power law and exponential law.

Simsek and Kocaturk investigated the free and forced vibration of a functionally graded simply-supported beam subjected to a concentrated moving harmonic load.3 The material properties are assumed to vary through the thickness according to the power-law method. The Euler-Bernoulli beam theory is used while driving equations. Su et al. developed the dynamic stiffness method to investigate the free vibration behavior of functionally graded beams.4 They derived the governing differential equations of motion and natural boundary conditions for free vibration by using Hamilton’s principle. A parametric study is carried out to demonstrate the effects of the length to thickness ratio and the variation of the power law index parameter. Suddoung et al. investigated the free vibration response of the stepped beams made from functionally graded materials for the various types of elastically end constraints.5 The differential transformation method (DTM) is employed to obtain natural frequencies and mode shapes of stepped beam. The step ratio, step location, boundary conditions, spring constants and material volume fraction values were taken into investigation parametrically. Lj proposed a unified approach for analyzing the static and dynamic behavior of functionally graded beams with the rotary inertia and shear deformation.6 Primarily, Timoshenko beam theory is considered. Euler-Bernoulli beam theory is obtained by reducing from the Timoshenko beam theory. The material properties are assumed to vary through the thickness according to the power-law. Yu and Zhong presented a general two-dimensional solution for a cantilever functionally graded beam with arbitrary graded variations of material property distribution in terms of the Airy stress function.7 Kapuria et al. proposed a finite element model for the static and free vibration analysis of the layered functionally graded beams by using third order zigzag theory.8 They estimated the effective modulus of elasticity. It is also validated by experiments for
two different functionally graded beam systems under different boundary conditions. Piovan and Sampaio performed the study of the vibrations of functionally graded sliding beams by considering the Euler-Bernoulli beam theory with finite element modeling.9 The authors developed this study for functionally graded rotating beams. Sina et al. developed a new beam theory to analyze the free vibration of functionally graded beams by using FSDBT and Hamilton’s principle.10 The material properties are assumed to vary through the thickness according to the power-law method. Alshorbagy et al. investigated the free vibration analysis of functionally graded beams by using numerical finite elements method.11 The functionally graded beam is based on the Euler-Bernoulli beam theory and the equations of the motion are derived by using virtual work principle. The material properties are assumed to vary through the thickness and longitudinal direction according to the power-law. The effects of different boundary conditions are investigated. Demir and Oz investigated the resonance frequency behavior of a functionally graded beam under viscoelastic boundary conditions with the finite element method, by using Euler–Bernoulli beam theory.12 The material properties of the beam are supposed to vary through the thickness according to the power-law distribution. In order to attain different boundary conditions, various stiffness and damping coefficients are applied to viscoelastic support elements. Wattanasakulpong and Unghbakorn investigated linear and nonlinear vibration responses of the functionally graded beams with the elastically restrained ends by taking into account the porosities which occur during fabrication. The differential transformation method (DTM) is used to solve linear and nonlinear vibration responses of FG beams. Duy et al. investigated free vibration response of a FG beam having an elastic foundation and spring supports.14 Young’s modulus, mass density and width of the beam were supposed to vary in thickness and axial directions respectively following the exponential law. Shvartsman and Majak studied the buckling of axially functionally graded (FG) Euler–Bernoulli beams with elastically restrained ends.15 Calim examined the free and forced vibrations of AFG Timoshenko beams on the elastic/viscoelastic foundation by solving different kinds of problems.16 The vertical displacement of a beam is examined for various foundation parameters. Ravishankar et al. investigated the effects of different angular velocities and different aspect ratios of rotating and non-rotating hybrid composite beams, on the free vibration analysis of FGB.17 In their study, the free vibration analysis of rotating and non-rotating fiber metal laminate (FML) beams, hybrid composite beams (HCB), and functionally graded beams (FGB) are investigated. FML beams are high-performance hybrid structures based on alternating stacked arrangements of fiber-reinforced plastic (FRP) plies and metal alloy layers. Hybrid composite beams are materials that are made by adding two different fibers. Functionally graded beams are new materials that are designed to achieve a functional performance with gradually variable properties in one or more directions. The effects of different metal alloys, composite fibers, and different aspect ratios and angular velocities on the free vibration analysis of FML beams are studied. The effects of different angular velocities and different aspect ratios of rotating and non-rotating hybrid composite beams are also investigated. Finally, the effects of different angular velocities and different material distributions, namely the power law, exponential distribution, and Mori Tanaka’s scheme on the free vibration analysis of FGB, are also investigated.

In this paper, the free and harmonic vibration analysis of a spring supported functionally graded beam is done by using finite element method. Timoshenko beam theory is considered for the given model. The material properties of the beam are assumed to vary through the thickness according to the power-law form. Various stiffness values are examined for the spring support. These values provide the solution for free-free (κ = 0, the support stiffness) and simply-supported (κ = ∞) boundary conditions as well. In the numerical examples, the steady state responses to a sinusoidal varying force are determined for the support reactions in the frequency domain.

Lagrange equations are used in the finite element modelling. By using the Lagrange equations, the problem is reduced to a system of algebraic equations. The convergence study is based on the numerical values obtained for various sizes of an element. The accuracy of the results is established by comparison with previously published exact results of beams based on Timoshenko beam theory obtained for the special cases of the investigated problem. Results given in this paper may be useful for further investigations in this field. The support stiffness, volume fraction coefficient and ratio of Young module level effect on the frequency parameters and force transmissibility are investigated. For the low spring values, rigid body motion occurs in the spring-beam system, therefore volume fraction coefficient is limited on the natural frequencies.

2. THEORY AND FORMULATIONS

The spring point supported functionally graded beam of length L, thickness h, and width b, is considered as seen in Fig. 1. The dynamical behavior of the beam is governed by the Timoshenko beam theory and all the transverse deflections occur in the same plane, defined by the x and z axes. The origin of axis is chosen at the left end of the beam as shown in Fig. 1.

\[ E(z), G(z), \rho(z), \nu(z) \] are the material properties and elastic modulus, shear modulus, density and Poisson’s ratio respectively. \[ E(z), G(z), \rho(z), \nu(z) \] of the functionally graded beam are assumed to vary through the thickness according to the power law distribution in Eq. (1); \( P_L \) and \( P_T \) are the corresponding material properties of the upper and lower surfaces of the beam and \( n \) is the power-law exponent. The variation of

Figure 1. A spring supported FG beam.
Young’s modulus and the mass density in the thickness direction can be seen from Fig. 2.

\begin{equation}
P(z) = (P_U - P_L) \left( \frac{z}{h} + \frac{1}{2} \right)^n + P_L. \tag{1}
\end{equation}

Lower surface of the beam is steel. Upper surface of the beam varies in conjunction with the change of \(E_{\text{ratio}}\). In other words, different material properties for the upper surface can be obtained by assigning different values to \(E_{\text{ratio}}\). The material distribution changes continuously from the upper surface to the lower surface with respect to thickness and power-law exponent. Material properties \(E\) and \(\rho\) of the steel are 210 GPa and 7800 kg/m\(^3\) respectively. \(E_U\): Young’s modulus of upper surface material, \(E_L\): Young’s modulus of lower surface material, \(\rho_U\): density of upper surface material, \(\rho_L\): density of lower surface material.

The transverse displacement, \(w(x, z, t)\), rotation of cross-section, \(\theta(x, t)\), and axial displacement \(u(x, z, t)\) of any point within the framework of Timoshenko beam theory are given as:

\begin{equation}
\begin{aligned}
u(x, z, t) &= u_0(x, t) - z \cdot \theta(x, t); \\
w(x, z, t) &= w_0(x, t). \tag{3}
\end{aligned}
\end{equation}

The strains and stresses of the beam are:

\begin{equation}
\begin{aligned}
\varepsilon_{xx} &= \frac{\partial u_0}{\partial x} - z \frac{\partial \theta}{\partial x}; \\
\gamma_{xz} &= \frac{\partial w_0}{\partial x} + \frac{\partial u_0}{\partial z} = \frac{\partial w_0}{\partial x} - \theta; \\
\sigma_x &= E(z) \cdot \varepsilon_x; \\
\tau_{xz} &= \kappa_c \cdot G(z) \cdot \gamma_{xz}. \tag{7}
\end{aligned}
\end{equation}

The elastic strain energy of a finite element obeys Hooke’s law, can be written as follows after some arrangement:

\begin{equation}
\begin{aligned}
U &= \frac{1}{2} \int_0^L \int_A (\sigma_{xx} \varepsilon_{xx} + \tau_{xz} \gamma_{xz}) dAdx = \\
&= \frac{1}{2} \int_0^L \int_A (\varepsilon_{xx}^2 E(z) + \gamma_{xz}^2 \kappa_c G(z)) dAdx; \tag{8}
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
U &= \frac{1}{2} \int_0^L \left[ A_{xx} \left( \frac{\partial u_0}{\partial x} \right)^2 - 2B_{xx} \left( \frac{\partial u_0}{\partial x} \frac{\partial \theta}{\partial x} \right) \\
&\quad + D_{xx} \left( \frac{\partial \theta}{\partial x} \right)^2 + A_{xz} \left( \frac{\partial w_0}{\partial x} - \theta \right)^2 \right] dx; \tag{9}
\end{aligned}
\end{equation}

where \(A_{xx}\) axial, \(D_{xx}\) bending, \(A_{xz}\) shear and \(B_{xx}\) coupling rigidity terms.

\begin{equation}
\begin{aligned}
(A_{xx}, B_{xx}, D_{xx}) &= \int_A E(z)(1, z, z^2) dA; \tag{10}
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
A_{xz} &= \int_A \kappa_c G(z) dA = \kappa_c (G_U - G_L) \frac{bh}{n+1} + \kappa_c G_L bh. \tag{11}
\end{aligned}
\end{equation}

Shear stress correction coefficient \(\kappa_c\) is assumed \(\kappa_c = \frac{5}{6}\) for rectangular section.

The kinetic energy of the finite element beam due to bending without the rotary inertia effects and axial displacement is given as:

\begin{equation}
\begin{aligned}
T &= \frac{1}{2} \int_0^L \int_A \rho(z) \left( \dot{u}^2 + \dot{w}^2 \right) dAdx = \\
&= \int_0^L \int_A \rho(z) \left[ \left( \frac{\partial u_0}{\partial t} - z \frac{\partial \theta}{\partial t} \right)^2 + \left( \frac{\partial w_0}{\partial t} \right)^2 \right] dAdx; \tag{12}
\end{aligned}
\end{equation}
where \( \dot{u} \) and \( \ddot{w} \) are the time derivatives of the axial and transverse displacement respectively Eq. (13) can be written as follows:

\[
T = \frac{1}{2} \int_0^L \left[ I_A \left( \frac{\partial u_0}{\partial t} \right)^2 - 2I_B \left( \frac{\partial u_0}{\partial t} \frac{\partial \theta}{\partial t} \right) + I_D \left( \frac{\partial \theta}{\partial t} \right)^2 + I_A \left( \frac{\partial u_0}{\partial t} \right)^2 \right] dx; \quad (13)
\]

where \( I_A, I_B, I_D \) are inertial terms as follow:

\[
(I_A, I_B, I_D) = \int_A \rho(z)(1, z, z^2) dA; \quad (14)
\]

\[
\beta = \frac{D_{xx}}{A_{xx}}. \quad (15)
\]

Figure 3 shows the six degrees of freedom for a two nodes finite element beam model. It has a length of \( L \). The nodal displacements are given in Eq. (16):

\[
\{ q \} = [u_0(t), w_0(t), \theta_0(t), u_j(t), w_j(t), \theta_j(t)]^T. \quad (16)
\]

The transverse, rotation and axial displacements of the element \( u_0(x,t), \theta(x,t) \) and \( w_0(x,t) \) are expressed in discrete form for using the nodal displacements by shape functions in matrix notation:

\[
u_0(x,t) = [N]_{u_0} \{ q \}; \quad (17)\]

\[
w_0(x,t) = [N]_{w_0} \{ q \}; \quad (18)\]

\[
\theta(x,t) = [N]_{\theta} \{ q \}; \quad (19)\]

\([N]_{u_0} \): The shape functions for the axial displacement and element (see Eq. (20)); \([N]_{w_0} \): The shape functions for bending displacement (see Eq. (21)); \([N]_{\theta} \): The shape functions for the rotation of cross-section displacement (see Eq. (22)).

The discretized displacement Eqs. (17)–(19) are substituted in Eqs. (9)–(13) to obtain:

\[
U = \frac{1}{2} \int_0^L \left[ A_{xx} ([N]_{u_0} \{ q \}) \right]^2 - 2B_{xx} ([N]_{u_0} \{ \dot{q} \}) \left( [N]_{w_0} \{ q \} \right) + 2D_{xx} ([N]_{w_0} \{ \dot{q} \}) \left( [N]_{w_0} \{ q \} \right) \right] dx; \quad (23)
\]

\[
T = \frac{1}{2} \int_0^L \left[ I_A ([N]_{u_0} \{ \dot{q} \})^2 - 2I_B ([N]_{u_0} \{ \dot{q} \}) \left( [N]_{w_0} \{ q \} \right) + I_D ([N]_{w_0} \{ \dot{q} \}) \left( [N]_{w_0} \{ q \} \right)^2 \right] dx. \quad (24)
\]

Expansion of Eqs. (23)–(24) are written as follow:

\[
U = \frac{1}{2} \left[ T \right]^T \{ q \}; \quad (25)
\]

\[
T = \frac{1}{2} \left[ \{ \ddot{q} \} \right]^T [M] \{ \ddot{q} \}. \quad (26)
\]

Terms in Eqs. (25) and (26) between brackets consist of the stiffness and mass matrices of an element. The functional of the finite element is:

\[
I = T - U. \quad (27)
\]

By using the Lagrange equations:

\[
\frac{\partial I}{\partial \ddot{q}_k} - \frac{d}{dt} \left( \frac{\partial I}{\partial \dot{q}_k} \right) = 0; \quad k = 1, 2, 3, 4, 5, 6. \quad (28)
\]

Equation (28) yields to the following equation for the considered element in matrices form:

\[
[K_e] \{ \ddot{q} \} + [M_e] \{ \dot{q} \} = 0. \quad (29)
\]

The matrices \([K_e], [M_e]\) are 6\times6 element stiffness and mass matrices respectively.

Having developed formulations for the mass matrix and stiffness matrix of a finite element, the global equations for a finite element model of a structure can be assembled and the functional of the problem in terms of the kinetic energy of the beam, potential energy of the beam, potential energy of the external forces and couples, potential energy of the supports at any time can be written as follows:

\[
T = \frac{1}{2} \{ \ddot{q} \}^T [M] \{ \ddot{q} \}. \quad (30)
\]
\[
[N]_{w_0} = \begin{bmatrix}
0 & 1 - \frac{x(12\beta - 2\tau)^2 + 3Lx^2}{L(2L^2 + 12\beta)} & x(L-x)(L^2 - xL + 12\beta) & 0 & \frac{x(12\beta - 2\tau)^2 + 3Lx^2}{L(2L^2 + 12\beta)} - x(6\beta + Lx)(L-x)
\end{bmatrix};
\]
\[
[N]_{q} = \begin{bmatrix}
0 & -6x(L-x)(L^2 + 12\beta) & L^2 - x^2 & 3x(L-x)(L^2 + 12\beta) & 0 & 6x(L-x)(L^2 + 12\beta) L(L^2 + 12\beta)
\end{bmatrix}.
\]

| Table 1. Non-dimensional frequencies of a simply supported beam. |
|---------------------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| \(L/h = 5\) & n = 0 & n = 0.2 & n = 0.5 & n = 1 & n = 2 & n = 5 & n = 10 & n = \infty |
| Difference\% & 0.00% & 0.00% & 0.00% & 0.00% & 0.00% & 0.00% & 0.00% |
| \(L/h = 20\) & n = 0 & n = 0.2 & n = 0.5 & n = 1 & n = 2 & n = 5 & n = 10 & n = \infty |

\[ U = \frac{1}{2} \{q\}^T [K] \{q\}; \quad (31) \]
\[ V = -\{q\}^T \{F\}; \quad (32) \]
\[ V_s = \frac{1}{2} k_1 \{q(t)\}^2 + \frac{1}{2} k_2 \{q_{p-1}(t)\}^2; \quad (33) \]
\[ p = 3m + 3; \quad n = m + 1; \quad (34) \]

where \(m\) is the number of the finite elements, \(n\) is the number of the nodes of the system, \(p\) is the total number of degrees of freedom of the model, \(k_1, k_2\) represent the stiffness of the first and second support respectively.

Functional for the whole system can be expressed as:
\[ I = T - (U + V + V_s); \quad (35) \]
then, using the Lagrange equations for the whole system:
\[ \frac{\partial I}{\partial q_k} - \frac{d}{dt} \frac{\partial I}{\partial \dot{q}_k} = 0; \quad k = 1, p; \quad (36) \]
yields the following equation for the whole system:
\[ [K] \{q\} + [K_s] \{q\} + [M] \{\ddot{q}\} = \{F(t)\}; \quad (37) \]
where the matrices \([K], [K_s], [M]\) are \(p \times p\) system stiffness matrix, support stiffness matrix and the mass matrix respectively. The only nonzero terms of the \([K_s]\) matrix are \(K_{s22}, K_{s(p-1)(p-1)}\).

Considering that the concentrated force affects the midpoint of the beam, it is expressed as:
\[ \{F(t)\} = \{Q\} \cdot e^{i\omega t}; \quad Q(3\pi + 2) \neq 0. \quad (38) \]
The time-dependent nodal displacements can be expressed as follows:
\[ \{q(t)\} = \{\tilde{q}\} e^{i\omega t}; \quad (39) \]
in Eq. (39), the elements of \(\{\tilde{q}\}\) are complex variables containing a phase angle. By taking into account Eqs. (37)–(39) can be expressed in the following matrix form:
\[ [K] \{\tilde{q}\} + [K_s] \{\tilde{q}\} - \omega^2 [M] \{\tilde{q}\} = \{Q\}. \quad (40) \]
Equation (40) can be written in the following form:
\[ ([K] + [K_s] - \omega^2 [M]) \{\tilde{q}\} = \{Q\}. \quad (41) \]
The maximum total magnitude of the reaction forces of the supports is given as follow:
\[ P = (k_1) \tilde{q}_1 + (k_2) \tilde{q}_{p-1}; \quad (42) \]
therefore the force transmissibility at the supports is determined by:
\[ T_R = \frac{P}{Q}; \quad (43) \]
\(T_R\): Force transmissibility.

3. NUMERICAL RESULTS

The transmissibility and natural frequency parameters of a functionally graded beam, spring supported at the ends is calculated numerically. The ratio of the beam of length \(L\) to thickness \(h\) \((L/h)\) is assumed 20 for brevity. The parameters \(k_s\) are taken as having the same respective values at the two supports denoted by \(k_s = k_1 = k_2\). \(\kappa\) dimensionless stiffness coefficient and \(\lambda\) frequency parameter are defined as follows:
\[ \kappa = \frac{k_s L^3}{EI}; \quad \lambda = \omega L^2 \sqrt{\frac{\rho A}{EI}}. \quad (44) \]
Equation (41) can be written by using Eq. (44) in the following dimensionless form:
\[ \{[K] + \kappa [K_s] - \lambda^2 [M]\} \{\tilde{q}\} = \{Q\}. \quad (45) \]
Table 3. Second non-dimensional frequency parameters $\lambda_2$ of simply supported FG beam for different material distribution and different E ratio.

<table>
<thead>
<tr>
<th>$E_r$</th>
<th>$n = 0$</th>
<th>$n = 0.2$</th>
<th>$n = 0.5$</th>
<th>$n = 1$</th>
<th>$n = 2$</th>
<th>$n = 5$</th>
<th>$n = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>4.406245</td>
<td>4.714416</td>
<td>4.883838</td>
<td>5.369031</td>
<td>5.572323</td>
<td>5.685525</td>
<td>5.973866</td>
</tr>
<tr>
<td>0.5</td>
<td>5.239939</td>
<td>5.375873</td>
<td>5.471799</td>
<td>5.745394</td>
<td>5.848865</td>
<td>5.906397</td>
<td>6.067487</td>
</tr>
<tr>
<td>4</td>
<td>8.812491</td>
<td>8.607338</td>
<td>8.427495</td>
<td>7.592957</td>
<td>7.243986</td>
<td>7.116811</td>
<td>6.848703</td>
</tr>
</tbody>
</table>

Table 4. Third non-dimensional frequency parameters $\lambda_3$ of simply supported FG beam for different material distribution and different E ratio.

<table>
<thead>
<tr>
<th>$E_r$</th>
<th>$n = 0$</th>
<th>$n = 0.2$</th>
<th>$n = 0.5$</th>
<th>$n = 1$</th>
<th>$n = 2$</th>
<th>$n = 5$</th>
<th>$n = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>6.544575</td>
<td>6.996</td>
<td>7.252524</td>
<td>7.972999</td>
<td>8.283845</td>
<td>8.452594</td>
<td>8.878737</td>
</tr>
<tr>
<td>0.5</td>
<td>7.782856</td>
<td>7.98103</td>
<td>8.125884</td>
<td>8.535158</td>
<td>8.691026</td>
<td>8.777235</td>
<td>9.015712</td>
</tr>
</tbody>
</table>

Figure 4. The force transmissibility of the FG beam for the various $\kappa$, $E_{ratio} = 0.25, 0.5, 1, 2, 3, 4$ and $n = 0$.

The eigenvalues $\lambda$ are found from the condition that the determinant of linear homogeneous equations given by Eq. (45) by neglecting the force.

Young module ratio can be written as follow:

$$E_{ratio} = \frac{E_U}{E_L};$$

and the mass ratio is considered as constant:

$$\rho_{ratio} = \frac{\rho_U}{\rho_L} = 1.$$
frequencies with different $E_{\text{ratio}}$ and power-law exponent ‘$n$’ for simply supported beam can be observed. The mass ratio is considered as in Eq. (47).

The first three natural frequencies are also obtained for the harmonic analysis of the spring supported beam situation. Because of the symmetry of the structure and external force in the harmonic analysis of the spring supported beam, only symmetrical vibrations with respect to the midpoint of the beam arise. The symbol $S$ represents symmetrical vibration with respect to the midpoint of the beam. Although the $S - 1$ mode existing in the case of simply supported beam does not occur in the completely free beam, it occurs for every value of stiffness parameter, which is different from zero. Therefore, the stiffness parameters vary from 1 to $10^5$. While increasing the stiffness parameter $\kappa$, the frequency parameters become the values of a simply supported beam. For the lower values of stiffness parameter $\kappa$, since the two supports have the same respective values, the beam vibrates in vertical rigid body modes (Z direction) in addition to elastic modes. This condition can be defined as “While the coefficient of the spring of the support increases, the vibration mode goes from rigid to elastic”. Namely, for the low values of spring coefficient ($\kappa = 1$ and $\kappa = 10$), beam vibrates predominantly in rigid body mode, and for the high values of spring coefficient (simply supported condition), beam vibrates predominantly in elastic body mode. This situation affects the influence of the $E_{\text{ratio}}$ and power-law exponent ‘$n$’ on the frequency parameters.

According to Tables 2–4, natural frequency behavior of simply supported FG beam is very consistent in each mode. When $E_{\text{ratio}} \leq 1$, dimensionless natural frequency parameters increase while $n$ increases from 0 to 10 (Tables 2–4). However, an opposite situation is observed when $E_{\text{ratio}} > 1$. Dimensionless natural frequency parameters decrease while $n$ increases (Tables 2–4). Thus, both conditions approach homogeneous beam condition. It can be observed in Tables 2–4 that the variation of $E_{\text{ratio}}$ is more effective on the frequency parameters than the variation of the $n$ for simply supported FG beams.

It can be observed in any figure of Figs. 4–8 that the frequency parameters and the difference between frequencies of each $n$, increase with increasing $\kappa$ when $\kappa > 1$. Since rigid modes are dominant for $1 < \kappa < 10$, $n$ doesn’t cause a significant change in frequencies for these boundary conditions. This can be observed from the differences between frequencies of each $n$ (Figs. 4–8).

When considering elastic supported FG beams, effect of $E_{\text{ratio}}$ is important at small $n$ values (Figs. 4–8). When $E_{\text{ratio}} < 1$, dimensionless natural frequencies of FG beam increase with increasing $n$. When $E_{\text{ratio}} > 1$, dimensionless natural frequency values decrease with increasing $n$ (Figs. 4–8).

At high $n$ values, difference between each $E_{\text{ratio}}$ values is smaller than those of small $n$ values (Figs. 4–8).

With increasing $E_{\text{ratio}}$ the differences between natural fre-
quencies are increasing except for $\kappa = 1$ and $\kappa = 10$. The natural frequencies increase with increasing $E_{ratio}$ and the effect of increasing $E_{ratio}$ increases with increasing $\kappa$ (any figure of Figs. 4–8).

The effect of increasing power-law exponent “$n$” is lighter on the first resonance frequency $(5 - 1)$ than the others, especially at the $\kappa = 1$ and $\kappa = 10$ spring coefficient (Figs. 4–8).

With getting the higher frequencies; the frequency differences of the values of $\kappa = 1, 10, 100$ decrease. This can be seen in Figs. 4–8.

With getting the higher frequencies; the frequency differences of the values of $\kappa = 1000, \infty$ increase. This can be seen in Figs. 4–8.

Because of the influence of the rigid body mode ($\kappa = 1, 10$), the effect of the power exponent coefficient is limited. With the increasing coefficient of the stiffness of spring, the effect of the power exponent coefficient increases. So the maximum effect can be seen at the simply supported condition where $\kappa = \infty$. This can be seen in Figs. 4–8.

There is an optimum value of force transmissibility at $\kappa = 1$, the intersection point of these lines exist at $\kappa = 1$, through which all the response curves pass except first natural frequencies, regardless of the power-law exponent “$n$”. This can be seen in Figs. 4–8.

The variation of the $E_{ratio}$ is more effective on the natural frequency than the variation in power exponent ($n$). This can be seen in Figs. 4–8.

4. CONCLUSION

The harmonic vibration analysis of functionally graded spring supported beam is investigated by means of finite element method. Various stiffness values are examined for the spring-support. The equations of motion are derived using Lagrange equations under the assumptions of the Timoshenko beam theory. The material properties of the beam vary through the thickness of the beam according to the power-law volume fraction function.

According to the numerical results:

- Only the symmetrical vibrations with respect to the mid-point of the beam arise because of the symmetry of the structure and external force in the harmonic analysis of the spring supported beam.
- The effect of the power exponent coefficient is limited for the lower stiffness coefficient, because of the influence of the rigid body mode. The effect of the power exponent coefficient increases with the increasing coefficient of the stiffness of spring.
- There is an optimum value of force transmissibility at $\kappa = 1$, The optimum value is the point through which...
all the response curves pass, regardless of the power-law exponent “n” except first natural frequencies.

- The variation of the \( E_{\text{ratio}} \) is more effective on the natural frequency than the variation in power exponent (n).

- The study concludes that the support stiffness, volume fraction coefficient and ratio of young module level all strongly affect the natural frequencies and the force transmissibility except the rigid body modes.

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Roughness Evaluation Approach for Nonstationary Vehicle Noise Based on Wavelet Packet and Neural Network Techniques

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(Received 6 April 2017; accepted 2 February 2018)

Based on the wavelet packet decomposition (WPD) and artificial neural network (ANN) methods, this paper presents a new technique for auditory roughness evaluation (ARE) of nonstationary vehicle noise, named WPD-ANN-ARE model. According to sound transfer and perception by the human auditory system, the noise roughness of a sample vehicle under different conditions of constant speed, acceleration, and braking are calculated. After comparisons by the time-frequency analysis techniques in common use, a WPD model with approximately twenty-one critical bands, which is specially designed by considering the auditory perception characteristics of human, is proposed for envelope feature extraction of vehicle noise. Taking the WPD extracted features as inputs and the calculated roughnesses as outputs, a back-propagation ANN with one hidden layer is trained and established for ARE of nonstationary vehicle noises. The verification results show that errors of the time-varying roughness calculated from the WPD-ANN-ARE are below 8.56 percent, which suggest a very good accuracy of the newly proposed ARE model. In applications, the WPD-ANN-ARE can be directly used in ARE of vehicle noises. And the modelling approach presented in this paper may be extended to other sound related fields for sound quality evaluation (SQE) in engineering.

1. INTRODUCTION

Currently, traffic noise is a major source of noise pollution especially in urban areas, which has attracted more public attention.1 Because of the physiological and psychological effects of noise on drivers, this would increase the risk of traffic accidents.1 Considerable research effort has been put in sound quality evaluation (SQE) of vehicle noise in the past few decades.2–4 The sound quality of a vehicle, as one of the important indices in ride comfort evaluation, has become a key consideration of car buyers and manufacturers.

For auditory characteristics, it is different in the physics and psychoacoustics, due to the human hearing process. Description of the sound quality is multidimensional. Some psychoacoustic indices, such as A-weighted SPL, loudness, sharpness, roughness, fluctuation strength, tonality, annoyance, pleasantness, etc., were proposed to quantitatively relate sound stimuli to human sensations.5–7 In the SQE engineering, it has been found that the A-weighted SPL is not a perfect method because the sound masking effects are ignored. Based on the equal-loudness-contours, Zwicker proposed a methodology for calculating the specific loudness, and thereby the loudness and sharpness models. The Zwicker loudness model has been used in the standard ISO 532B,8 and the sharpness model has not yet been internationally standardized but has been accepted by the German national standard DIN 45692.9

Auditory roughness is a complex effect which quantifies the perception of rapid amplitude modulation of a sound signal. The unit of measure is the asper.5 One asper is defined as the roughness produced by a 1000 Hz tone of 60 dB which is 100% amplitude modulated at 70 Hz. As an important psychoacoustic parameter, the roughness has been widely studied and discussed. Helmholtz first presented the concept of auditory roughness.10 Hearing tests showed that the amplitude modulated sounds with modulation frequencies from 20 Hz to 70 Hz may generate roughness.11 Subsequently, many mathematical models for quantitative calculation of auditory roughness were proposed. Some of them were developed by considering the modulation frequency and index, the excitation envelope and level differential, and the autocorrelations in critical bands of a sound.7,12–14 The Aures model has been regarded as a flexible method in auditory roughness calculation.2 Thus, the improved versions of the Aures model were developed by considering the effects of phase differences and carrier frequencies in the critical bands.15,16 Hoeldrich and Pflueger17 developed a roughness model to evaluate vehicle interior noise using parameters that could be adjusted for different modulation parameters. The traditional roughness models mainly differ in the methods used to transform the calculated excitations in the critical bands into the roughness. In practice, these methods are difficult to apply because the excitation level differentials in the critical bands are not easy to obtain from real noise signals.

The psychoacoustical indices have been widely considered in quality evaluation of industrial products. In view of the vehicle noise SQE, Hashimoto18 studied a psychoacoustical booming index for both stationary and nonstationary interior noises, which were quantified using a weighting function from the subjective response to sensation level. The rumbling noise of a vehicle below 300 Hz was investigated by Lee,19 who pointed out that the human feeling on the rumbling noise mainly depended on the roughness and loudness indices. Due to the nonlinear perception feature of human auditory system, it is sometimes impossible to find an exact physical model to de-
scribe the perception response for all people. Thus, following the human hearing process, some SQE methods based on virtual human “ear” and “brain” have been recently proposed for SQE of vehicle noises. The time-frequency analysis algorithms, such as the short-time Fourier transform (STFT), discrete wavelet transform (DWT), wavelet packet analysis (WPA) and the Wigner-Ville distributions (WVD), were introduced into SQE engineering for sound feature extraction (virtual “ear”) of the impact and nonstationary vehicle noises.\textsuperscript{20–22} Originally used for signal processing, wavelet techniques have been developed and successfully applied in structure analysis, diagnosis of crack faults and wave propagation in structures.\textsuperscript{39–44} Due to good time-frequency characteristics, the wavelet-based algorithms are usually considered for SQE of both stationary and nonstationary noises.

It has been found that, the wavelet band filters with proper sampling rates can be well matched to the critical filter bank in the human auditory system. To classify sound patterns, some techniques based on the artificial neural network (ANN) and the support vector machine (SVM) methods (virtual “brain”) were used for predicting the loudness, sharpness and annoyance indices of vehicle noise.\textsuperscript{23–25} Compared with the loudness and sharpness indices, the roughness of a sound is more difficult in the feature extraction and perception modelling, because there exists hardly any correlation between the subjective impressions of test persons and available roughness parameters.\textsuperscript{17} To recognize the patterns of auditory roughness, the ANN used in the loudness and sharpness predictions needs to be reconstructed and modified. From the previous literature, it has been found that roughness, which has a big contribution to the perceived annoyance of vehicle noise, cannot be neglected.\textsuperscript{26} Currently, there is not any virtual “ear-brain” based approach for auditory roughness prediction. This paper attempts to solve the above key issues and develop a novel model for roughness evaluation of vehicle noise.

Based on the above discussions, we concluded that the in-situ methods for roughness calculations remain controversial and cannot garner enough public acceptances to be used in standards. Thus, it is both necessary and useful to develop a new methodology to calculate the auditory roughness. In this paper, a new roughness model is presented by combining the WPD and ANN techniques and evaluates the vehicle interior noise. The modulation index of a sound signal is used instead of the statistical parameters. The WPD-ANN-ARE model is developed and successfully applied in structure analysis, diagnosis of crack faults and wave propagation in structures.\textsuperscript{24} From the previous literature, it has been found that roughness, which has a big contribution to the perceived annoyance of vehicle noise, cannot be neglected.\textsuperscript{26} Currently, there is not any virtual “ear-brain” based approach for auditory roughness prediction. This paper attempts to solve the above key issues and develop a novel model for roughness evaluation of vehicle noise.

2. ESTABLISHMENT OF VEHICLE NOISE DATABASE

Sample vehicle noises with a 10 seconds signal frame were measured with the LMS.testlab data acquisition system at the sampling rate of 44100 Hz. The experimental conditions were elaborately arranged, following the measurement method for vehicle interior noise in the standard ISO 5128.\textsuperscript{28} A dry asphalt four-lane two-way test road was selected. Around the test site, there was no sound reflecting object within 50 meters. The weather was clear with a temperature of 26°C with no wind when the test were carried on. Meanwhile, the windows were closed; the air conditioner and all electronic equipment in the vehicles were turned off. Two models of Volkswagen, Lavida and Golf, Sedan with 1.6L four-cylinder gasoline engine, were used in the experiments. To meet the requirements of signal number in ARE modelling, eleven vehicle operating conditions were set as constant speed of 30, 40, 50, 60, 70, 80, 90 and 100 km/h, emergency braking to a stop from 60 and 80 km/h, and full throttle acceleration from 50 km/h to 120 km/h. Type 4189-A-021 microphones were mounted near to the passenger’s right and left ears, according to the GB/T 18697 standard. Under the working conditions, the noise signals were measured and saved in “mat” and “wav” formats respectively. The signal-to-noise ratios of the measured signals were above 20 dB, due to well-controlled background noises from the test site and the measurement system. The noise sample at each point with 5 seconds signal frame was selected from the three times measured results by hearing tests. As a preparation for the ARE modelling, a database of the measured interior noises is established. There are 132 noise signal frames (each vehicle: 11 working conditions by 6 channels) of the sample vehicles included in the database, which is used for training and verification of the WPD-ANN-ARE model in the following text.

Figure 1 gives the vehicle noises measured under the working conditions of acceleration from 50 km/h to 120 km/h, braking from 80 km/h to 0 km/h and constant speed 70 km/h. It can be seen from their spectra that, regardless of the working conditions, the vehicle noise energy is mainly distributed in a low-frequency range below 400 Hz. The maximum energy components of the signals are below 50 Hz, which may be caused by vibration of the vehicle body. The energy components between 80 and 150 Hz are from the second order vibration of the engine. The interior noise energies above 1000 Hz are very small and can almost be neglected.

3. AUDITORY ROUGHNESS CALCULATION

3.1. Human Auditory-Perception Process

In psychoacoustics, it has been found that human auditory perception is generated by vibrations from the stapes, which stimulate different parts of the basilar membrane in the inner ear, and thereby the auditory nerve. The auditory perception
process with nonlinear filtering properties in the frequency domain can be described by the critical bands. For a given frequency, the critical band is the smallest band of frequencies around it which activate the same part of the basilar membrane. The critical bandwidth represents the human ear’s resolving power for simultaneous tones or partials. In this paper, an empirical formula is used to convert one-third octave centre frequency \( f_c \) (kHz) into critical band rate \( z \) (Bark),

\[
z(f_c) = \begin{cases} 
  11.82 \arctan(1.21 f_c) & f_c \leq 1.5 \\
  5 \ln(f_c/1.5) + 12.61 & f_c > 1.5 
\end{cases}
\]  

\( (1) \)

The critical bandwidths of the Bark scales are defined as \( \Delta f_c = 100 \text{ Hz} \) (if \( f_c < 0.5 \text{ kHz} \)), otherwise \( \Delta f_c = 1/5 f_c \).

In the SQE field, psychoacoustic parameters such as the weighted SPLs, loudness, sharpness, roughness, fluctuation strength, tonality and articulation index, have been discussed in the past few decades. It has been found that, for a vehicle noise, total effects of the loudness, sharpness and roughness on sound quality are above 90 percent. This paper focuses only on the study of the roughness index for vehicle noise evaluation.

### 3.2. Roughness Index Calculation

In this paper, an improved Aures model, which considered the effects of transfer functions of the human ear, modulation index, modulation and carrier frequencies of a signal, is slightly modified and used for auditory roughness calculation. A procedure for roughness computation programming is given in Figure 2. Following the steps in Figure 2, the total roughness of a noise signal can be obtained.

**Step 1: Signal partition and frequency weighting.** According to the sound masking effects in the time domain, a noise signal is first cut into a set of successive frames with length of 50 ms for short-time ARE. The fast Fourier transform (FFT) is performed for calculating the one-third octave SPLs of each frame by using the Blackman window. Considering the structural effects of the human ear, the sound transmission coefficients are weighted on the signal SPLs, before the signals enter into the inner ear.

**Step 2: Calculation of sound excitation level in the Bark domain.** The intensity quantity of a sound is based on critical bandwidths and is therefore a subjective representation of frequency. Sound intensity within a critical band can be expressed as

\[
I_G(f) = \int f + 0.5 \Delta f_G(f) \frac{dI}{df} df. 
\]  

\( (2) \)

Accordingly, sound intensity level (SIL) \( L_G \) (dB) may be defined as

\[
L_G = 10 \log \frac{I_G(f)}{I_0}; 
\]  

\( (3) \)

where, \( f \) is the sound frequency, \( I_0 \) is the intensity of a reference sound, \( I_0 = 10^{-12} \text{ W/m}^2 \). The maximum SIL is \( L_G \) in a critical band. The frequency components in each frame are transformed into excitation patterns by overlapping the critical band filters. The calculated sound excitation levels in 24 channels (bands) are transferred into specific excitation time signal \( e(t) \) by the inverse fast Fourier transform (IFFT).

**Step 3: Calculation of modulation indices.** The cubic spline interpolation method is used for extracting the amplitude envelopes of \( e_i(t) \). The obtained signal is fed to the weighting function \( H_i \) of modulation frequency. The modulated signal \( em_i(t) \) is obtained. The modulation index in the \( i \)th channel \( m_i \) can be defined as

\[
m_i = \frac{rms_{em_i(t)}}{|e_i(t)|}; 
\]  

\( (4) \)

where, \( rms_{em_i(t)} \) is the root mean square (RMS) of \( em_i(t) \), and \( |e_i(t)| \) is RMS value of the signal \( e_i(t) \).

**Step 4: Specific and total roughness calculations.** A phase impact factor \( c_i \) representing effects of the \((i-1)\) and \((i+1)\) channels on the \( i \)th channel is defined as

\[
c_i = c_{i-1} \times c_{i+1}; 
\]  

\( (5) \)

where, \( c_{i-1} \) and \( c_{i+1} \) are the correlation coefficients of \( em_i(t) \) with those in the \((i-1)\)th and \((i+1)\)th channels. Thus, the specific and total values of roughness can be calculated by and in the \( i \)th channel may be shown as

\[
R_i = (g_i \times c_i \times m_i)^2; 
\]  

\( (6) \)

\[
R = 0.25 \sum_{i=1}^{24} R_i; 
\]  

\( (7) \)

where, \( R_i \) is the specific roughness in the \( i \)th channel, \( g_i \) is the weighting coefficient of carrier frequency, and \( R \) is the total roughness.

### 4. FEATURE EXTRACTION

Vehicle noises are typically nonstationary signals, such as the braking or the acceleration noises. Their features should be represented in time and frequency domains. In this paper, the extracted feature matrices of the vehicle noises will be used for ANN training and verification, which needs the matrix spaces to be as small as possible. The frequency spectral...
analysis and continuous time-frequency representations, such as the continuous wavelet transform (WT), WVD and its improved versions, cannot satisfy this requirement. The WPD and Hilbert-Huang Transform (HHT) approaches, which have been frequently mentioned in the fault diagnosis and sound quality fields, are considered and compared for feature extraction of the low-frequency acceleration vehicle noise in this paper.

4.1. Wavelet Packet Decomposition

The wavelet transform is the process of decomposing a signal using wavelets. A family of orthogonal functions as

$$\Psi_{a,b}(t) = |a|^{-1/2}\psi((t-b)/a), a, b \in R, a \neq 0$$

are generated from a wavelet function $\psi(t)$ by dilation and translation operations, which are governed by the scale factor $a$ and shift factor $b$. Setting $a = a_0^j$ and $b = a_0^{-j}kb_0$ ($j, k \in Z, a_0 > 1, b_0 > 0$), the wavelet function becomes $\psi_{j,k}(t) = a_0^{-j/2}\psi(a_0^j(t-nb_0))$. If $a_0 = 2, b_0 = 1$, the discrete wavelet transform (DWT) and its reconstructed version of a signal $x(t)$ is $L^2(R)$ are defined as,

$$W_x(a,b) = W_x(2^{-j}, 2^{-j}k) = 2^j \int_{-\infty}^{+\infty} \psi(2^j(t-k))x(t)dt;$$

$$x(t) = \sum_j \sum_k W_x(2^{-j}, 2^{-j}k)\psi(2^j(t-k)).$$

Based on the DWT, the WPD was derived using the definitions of the scaling function $\phi(t)$ and the wavelet function $\psi(t)$. Let $u_0(t) = \phi(t), u_0(t) = \psi(t)$, and define:

$$u_{2n}(t) = \sqrt{2} \sum_k h_k u_n(2t-k);$$

$$u_{2n+1}(t) = \sqrt{2} \sum_k g_k u_n(2t-k).$$

These recursive equations specify a wavelet packet \{u_n(t)\}, where $h_k$ and $g_k$ satisfy the equations $\sum h_{n-2k}h_{n-2m} = \delta_k,m, (\sum h_n = \sqrt{2})$, $g_n = (-1)^n h_{1-n}$ and \{u_{j,m,n}(t) = 2^{-j/2}u_n(2^j(t-m))\}, where $j, m$, and $n$ are the scale, translation, and oscillation parameters $j, m \in Z, n \in Z_+$. The decomposed coefficients of the signal $x(t)$ is $C_{j,n} = \{C_{j,m,n}\}_{m \in Z}$, where $C_{j,m,n} = \langle x, u_{j,m,n} \rangle$. Thus, WPD of the signal can be obtained by,

$$C_{j,2n} = \sum_{k=-\infty}^{\infty} h_{2m-k}C_{j+1,n};$$

$$C_{j,2n+1} = \sum_{k=-\infty}^{\infty} g_{2m-k}C_{j+1,n}.$$

Conversely, the wavelet packet reconstruction is expressed as the following,

$$C_{j+1,n} = \sum_{k=-\infty}^{\infty} h_{2m-k}C_{j,k} + \sum_{k=-\infty}^{\infty} g_{2m-k}C_{j,k+1,n}.$$

4.2. Hilbert-Huang Transform

The HHT, which is combined by the empirical mode decomposition (EMD) and the Hilbert spectral analysis, is an adaptive time-frequency analysis method for analysing data in nonlinear and nonstationary processes. The HHT kernel is the EMD approach with which a signal can be decomposed into intrinsic mode functions (IMFs). The IMFs yield instantaneous frequencies as functions of time that give an identification of signal components, thereby a signal representation of the HHT in the time and frequency domains. To obtain the instantaneous frequency characteristics of a signal, the IMFs are defined as functions having the same numbers of zero-crossing and extrema and the symmetric envelopes (with respect to time axis) defined by the local maxima and minima. To extract the IMFs from a complex signal in engineering, the EMD needs to be performed. After the EMD, the signal $x(t)$ can be expressed as,

$$x(t) = \sum_{i=1}^{n} c_i + r_n;$$

where, $c_i$ is the ith decomposed IMF of the signal $x(t), r_n$ is the residual signal, which occupies very little energy of the signal, can be ignored. Taking the Hilbert transform on both sides of Eq. 16, the Hilbert spectrum $H(\omega, t)$ may be determined by,

$$H(\omega, t) = Re \sum_{i=1}^{n} a_i(t)e^{j \int \omega_i(t)dt};$$

where, $Re$ is the operator for the real part, $a_i(t)$ and $\omega_i(t)$ are the functions of the amplitude and the instantaneous frequency, respectively. $H(\omega, t)$ can describe the signal amplitude varying on a time-frequency plane. The HHT marginal spectrum $h(\omega)$ can be defined as,

$$h(\omega) = \int_{0}^{T} H(\omega, t)dt;$$

where, $T$ is the length of the signal $x(t), h(\omega)$ reflects the signal amplitude changing with frequency. The instantaneous frequency of the IMFs can localize the signal characteristics in the time-frequency domain.

4.3. Comparison of WPD and HHT

As an example, the 50 km/h to 120 km/h acceleration noise signal was selected to compare the WPD and HHT. As seen in Figure 1, the energy of vehicle noise was concentrated in the low-frequency range below 400 Hz. Thus, for a clearer comparison, the sample signal was first preprocessed by a resampling frequency of 882 Hz, and then filtered by using a high-pass filter with a cutoff frequency of 20 Hz to match the threshold of the human ear. A 2-level WPD and a 9-level HHT were performed on the preprocessed acceleration noise signal, respectively, using the specifically written Matlab programs. In the WPD, the Daubechies wavelet db35 was adopted since its characteristics are closer to the critical bands. In the HHT, the IMFs of the signal were calculated by using the EMD. The decomposed results are shown in Figs. 3 and 4.
H. Guo, et al.: ROUGHNESS EVALUATION APPROACH FOR NONSTATIONARY VEHICLE NOISE BASED ON WAVELET PACKET AND...

Figure 3. The WPD component signals of a vehicle acceleration noise and their spectra.

Figure 4. The IMF components calculated by EMD of a vehicle acceleration noise and their spectra.

the 2-level WPD, the original signal was decomposed into four components with approximate averaged frequency bandwidths of 100 Hz, which is very close to the critical bands of human hearing in the low frequencies below 400 Hz. After the HHT, eight IMFs and a residual component of the noise signal were obtained by EMD. The first four IMFs with nearly all energy of the original sound signal and their spectra are shown in Figure 4. It can be seen that the frequency bandwidths of the first, second, third and the fourth IMFs are around 100 to 300 Hz, 50 to 150 Hz, 20 to 50 Hz and 20 to 30 Hz, respectively. Frequency overlaps have occurred among the IMF components. It means that the HHT cannot be used to define the critical bands according to the auditory characteristics of human. The EMD is an adaptive method according to the signal features, which led to the random frequency ranges of the IMFs. It can be seen from Figure 3 that, comparing with the HHT, the WPD can decompose a signal into multiple levels and each reconstructed sub-signal has a definite relative frequency band. This provides the possibility to specially design for matching the critical bands in the human auditory system. Therefore, the WPD is applied in feature extraction of the nonstationary vehicle noise in this paper.

Figure 5. A specially designed wavelet tree for decomposition of vehicle noise signal.

Table 1. Node combinations in the WPD for frequency range approximation to critical bands.

<table>
<thead>
<tr>
<th>Critical band rate $z$ (Bark)</th>
<th>Selected nodes in the wavelet tree of WPD</th>
<th>Approximate frequency ranges of the WPD (Hz)</th>
<th>Corresponding frequency ranges in one-third octave analysis (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(8,0)</td>
<td>20–86</td>
<td>22–90</td>
</tr>
<tr>
<td>2</td>
<td>(8,1)</td>
<td>86–172</td>
<td>90–180</td>
</tr>
<tr>
<td>3</td>
<td>(8,2)</td>
<td>172–258</td>
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<tr>
<td>4</td>
<td>(8,3)</td>
<td>258–344</td>
<td>280–355</td>
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<td>344–430</td>
<td>355–447</td>
</tr>
<tr>
<td>6</td>
<td>(8,5),(9,12)</td>
<td>430–559</td>
<td>447–562</td>
</tr>
<tr>
<td>7</td>
<td>(9,13),(8,7)</td>
<td>559–689</td>
<td>562–708</td>
</tr>
<tr>
<td>8</td>
<td>(7,4)</td>
<td>689–861</td>
<td>708–891</td>
</tr>
<tr>
<td>9</td>
<td>(7,5),(8,12)</td>
<td>861–1119</td>
<td>891–1120</td>
</tr>
<tr>
<td>10</td>
<td>(8,13),(7,7)</td>
<td>1119–1372</td>
<td>1120–1410</td>
</tr>
<tr>
<td>11</td>
<td>(6,4)</td>
<td>1378–1722</td>
<td>1410–1780</td>
</tr>
<tr>
<td>12</td>
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<tr>
<td>13</td>
<td>(7,13),(6,7)</td>
<td>2239–2842</td>
<td>2240–2820</td>
</tr>
<tr>
<td>14</td>
<td>(5,4)</td>
<td>2842–3445</td>
<td>2820–3550</td>
</tr>
<tr>
<td>15</td>
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<td>3550–4470</td>
</tr>
<tr>
<td>16</td>
<td>(6,13),(5,7)</td>
<td>4478–5512</td>
<td>4470–5620</td>
</tr>
<tr>
<td>17</td>
<td>(4,4)</td>
<td>5512–6890</td>
<td>5620–7080</td>
</tr>
<tr>
<td>18</td>
<td>(4,5),(5,12)</td>
<td>6890–8957</td>
<td>7080–8910</td>
</tr>
<tr>
<td>19</td>
<td>(5,13),(4,7)</td>
<td>8957–11025</td>
<td>8910–11200</td>
</tr>
<tr>
<td>20</td>
<td>(3,4),(6,40)</td>
<td>11025–14126</td>
<td>11200–14100</td>
</tr>
<tr>
<td>21</td>
<td>(6,41),(5,21) (4,11),(4,12)</td>
<td>14126–17916</td>
<td>14100–17800</td>
</tr>
</tbody>
</table>

4.4. Feature Extraction for Roughness Modelling

Correct feature extraction of vehicle noise directly affects the accuracy of a sound quality prediction. The feature representation of a sound signal often involves a significant amount of redundancy. To maintain the small size of the network inputs in the following section, it is necessary to reduce the redundant information in the sound feature data in both the time and frequency domains. Considering the sound filtering and masking effects of human auditory system, an incomplete 9-level WPD model with 21 approximate critical bands was established in the frequency domain and a 50 ms time interval was selected in the time domain. According to the sampling rate of the signal and the Zwicker model in which the one-third octaves were used instead of the critical bands, the incomplete wavelet tree was specially designed and shown in Figure 5. Except for the nodes (4, 13) and (3, 7), the end nodes of all WPD levels in the wavelet tree were used to construct the approximate critical bands. The WPD end-node combinations and their corresponding frequency ranges are listed in Table 1.

It should be mentioned that, to simplify the wavelet tree, the measured signals were first resampled at a rate of $S =$
22050 Hz and passed through a high-pass filter with a cutoff frequency of 20 Hz, because hearing threshold of human is from 20 Hz to 20 kHz. The 9-level WPD with Daubechies wavelet ‘db35’ with filter length of 70 was performed by the Mallat algorithm. A set of wavelet band-pass filters, combined by low- and high-pass filters with different sampling rates, was created. Thus, if the WPD nodes were properly combined, the wavelet band-pass filters can be matched to the one-third octaves in the Zwicker model. For the N-level WPD, a wavelet filter bank with $2^N - 1$ band-pass filters may be created by the Mallat algorithm, which is combined by $2^N$ pairs of low- and high-pass filters. Through the Mallat algorithm, the ‘db35’ quadrature mirror filters $H, h$ and $G, g$ were used for signal decomposition and reconstruction, where $H, G$ and $h, g$ denote the low- and high-pass filters, respectively. For a sub-signal decomposition at node $(n, m)$ in the wavelet tree, the frequency $(2n+1)S/2^{n+1}$ Hz was defined as the upper cutoff frequency of $H$, as well as the lower cutoff frequency of $G$. Inversely, the sub-signal should be reconstructed by the filters $h$ and $g$. The approximate frequency ranges in Table 1 were defined by the cutoff frequencies of the wavelet band-pass filters. For example, the 21th critical band was combined by the nodes (6, 41), (5, 21), (4, 11) and (4, 12). The lower and upper frequencies of the wavelet band-pass filter, which consists of the high-pass filter at node (5, 20) and the low-pass filter at node (3, 6) in Figure 5, are very close to those of the corresponding one-third octave in the Zwicker model, as shown in Table 1. In the implementation, the resampled signals were fed to the WPD filter bank, which should actually be completed by applying the 9-level WPD to the resampled interior noise. The noise component in the 21th critical band could be finally obtained by summing the reconstructed sub-signals at the related four nodes. The Bark scale serial number is defined as 1 to 24. The scales (critical bands) 22, 23, and 24 in high frequencies were ignored, since vehicle noise is in a lower frequency range. Considering the sound masking effects of human auditory system in the time domain, very short duration backward masking may be neglected, and the forward masking gradually attenuates in the form of an exponential which approaches zero at 50 ms. Accordingly, in this paper, the interval 50 ms was selected and applied to reduce the redundant information in the time domain sound signal.

Auditory roughness is defined as the human perception to the low-frequency envelope fluctuations of a sound signal. There is a bandpass relationship between the roughness and modulation sound frequency. Thus, the modulation index $m$ was selected as a representation of roughness feature of sounds. A scheme of roughness feature extraction designed for vehicle noises is shown in Figure 6. Firstly, a vehicle noise was decomposed by using the WPD in Figure 5, and the decomposed 21 sub-signals were reconstructed and combined according to the definitions in Table 1. The 21 sub-signals were numbered by $i$, where $i$ equals 1, 2, 3 . . . 21. In the time dimension, the sub-signals were divided into $R$ frames with a 50 ms interval. In the $j$th time interval, following Step 2 in the procedure of roughness calculation in Figure 2, the cubic spline interpolation was used to extract the signal envelopes $e_{ij}(t)$. As an example, the envelope of the first sub-signal of a vehicle noise is shown in Figure 7. The extracted signal envelopes $e_{ij}(t)$ were further filtered by the weighting functions $H_i$ of modulation frequency shown in Figure 8, thereby $em_ij(t)$. Using Eq. 4, the modulation index $m_{ij}$ in the $i$th band can be calculated. It was finally obtained a roughness feature matrix with a size of 21 by $R$ for each of the noise signals. An example roughness feature matrix is given in Table 2. It can be seen from the extracted matrix that the roughness feature quantities of a vehicle noise fluctuate with both the time and frequency, and mainly distributes in the low Bark scales.

5. WPD-ANN-ARE MODELING AND VERIFICATION

5.1. Artificial Neural Network with Back-Propagation Algorithm

The ANN, a mathematical model inspired by biological neural networks, is composed of many interconnected artificial neurons operating in parallel. The multilayered feedback network is commonly used in engineering.

Figure 6. A designed scheme for roughness feature extraction of vehicle noise.

Figure 7. The envelope of the first sub-signal extracted by using the cubic spline interpolation method.

Figure 8. The weighting filters of modulation frequency in different critical bands.
Table 2. The extracted roughness feature matrix of a vehicle noise in time-frequency domain.

<table>
<thead>
<tr>
<th>z (Bark)</th>
<th>Time</th>
<th>0–50 ms</th>
<th>50–100 ms</th>
<th>100–150 ms</th>
<th>150–200 ms</th>
<th>200–250 ms</th>
<th>250–300 ms</th>
<th>300–350 ms</th>
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<tbody>
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<td>1</td>
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<td>0.237358</td>
<td>0.588959</td>
<td>0.519934</td>
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</table>

Figure 9. A three-layer BP neural network with a hidden layer.

The ability to reproduce arbitrary nonlinear functions of input makes them suitable for complex pattern recognition tasks. In this work, a feed-forward ANN with three-layer perceptron was adopted to project the input signal features to the output roughness patterns of vehicle noises, as shown in Figure 9. The inputs are multiplied by related weights and summed, and then passed through a sigmoid function. The sample x_k is fed to the network and produces an output y. The input pattern x_k is propagated through the network in the following way:

\[
y_i^{(2)} = f_1 \left( \sum_{j=1}^{M} w_{ij}^{(2)} y_j^{(1)} \right)
= f_1 \left( \sum_{j=1}^{M} w_{ij}^{(2)} f_2 \left( \sum_{k=1}^{N} w_{ik}^{(1)} x_k \right) \right);
\]

where, y_i^{(1)} and y_i^{(2)} are the outputs of the hidden and output units, and M and N are the numbers of input and hidden units; w_{ij}^{(n)} is the weight corresponding to the jth source, ith target, and nth layer; f_1 and f_2 are the transfer functions for the hidden and output layers, respectively. The pure linear (purelin) and hyperbolic tangent (tansig) transfer functions were selected in this paper, which are defined as,

\[
f_1(u) = \alpha, \quad f_2(u) = \frac{2}{1 + e^{-2u}} - 1.
\]

Thus, the tansig and purelin transfer functions were selected according to the empirical formula.\(^{35}\)

The back-propagation (BP) algorithm is a gradient-search approach that attempts to minimize the squared error of the output by adjusting the weights in the network backwards. The mean squared error (MSE) of BP can be described as,

\[
E = \frac{1}{2|L|} \sum_{l=1}^{L} \sum_{\{x\} \in L} [d(\{x\}) - y(\{x\})]^2;
\]

where, d(x) is the desired network output for sample x, and |L| is the cardinality of a learning set. To minimize E, the change in weight for output unit i from hidden unit j can be derived by the gradient,

\[
\Delta w_{ij}^{(2)} = \frac{\partial E}{\partial w_{ij}^{(2)}} = \sum_{\{x\} \in L} \left[ d_i - y_i^{(2)} \right] f(y_i^{(2)}) (1 - f(y_i^{(2)})) y_j^{(1)}.
\]

By the chain rule, the weight change for hidden units can be given by,

\[
\Delta w_{jk}^{(1)} = \sum_{\{x\} \in L} \frac{\partial E}{\partial y_j^{(1)}} \frac{\partial y_j^{(1)}}{\partial w_{jk}^{(1)}}
= \sum_{\{x\} \in L} \delta_i^{(2)} w_{ij}^{(2)} f(y_i^{(1)}) \{x_k\}
\]

in which, \(\delta^{(2)}\) is back-propagated to the hidden layers, \(\delta^{(2)} = [d_i - y_i^{(2)}] f(y_i^{(2)})\). Using the momentum adaptive-learning-rate algorithm,\(^{35}\) the weight can be updated by \(w_{new} = (1 - \alpha) w_{old} - \eta \Delta w\), where \(\eta\) and \(\alpha\) denote the learning rate and the momentum term, respectively.
that H layers, and C where, I of training tests was carried out by assuming formula, H the dimension of the input matrix is extended to 21. In this paper, 33 left-ear noise signals measured under the eleven working conditions are taken from the vehicle noise database, i.e., n = 33. The network output is set to the total roughness at that corresponding moment, which has been calculated in Section 3, thus the output node number is 1. Accordingly, the output matrix has a dimension of 1 × 33. Node number Hn of the hidden layer is estimated by the following empirical formula,37

\[ H_n = \sqrt{I_n + O_n + C}; \]  

(24)

where, In and On are the node numbers in the input and output layers, and C is a constant, C = 1–10. One may calculate that Hn equals to 4–16. To find an optimal Hn value, a set of training tests was carried out by assuming Hn values from 4 to 16. The averaged RMS errors of ten tests are shown in Figure 10. It can be seen that, with the Hn increasing, the errors of predicted results are fluctuant and reach a minimum value at Hn = 10. Thus, the node number of the hidden layer was set to ten in this present work. In addition, the Levenberg-Marquardt (LM) algorithm was adopted and used in the ANN training,37 due to rapid convergence in solving nonlinear least squares problems.

5.2. Architecture of WPD-ANN-ARE Model

An established three-layer ANN model for ARE is shown in Figure 9. However, some parameters for performing the ANN, such as neuron (node) numbers of the input, hidden and output layers, and the network training algorithm, need to be specified. The extracted roughness feature vector within a 50 ms interval of each vehicle noise is taken as inputs, thus the input node number is 21. For training a network with n signals, the dimension of the input matrix is extended to 21 × n. In this paper, the dimension of the input matrix is extended to 21 × 33. Node number Hn of the hidden layer is estimated by the following empirical formula,37

\[ H_n = \sqrt{I_n + O_n + C}; \]  

(24)

where, In and On are the node numbers in the input and output layers, and C is a constant, C = 1–10. One may calculate that Hn equals to 4–16. To find an optimal Hn value, a set of training tests was carried out by assuming Hn values from 4 to 16. The averaged RMS errors of ten tests are shown in Figure 10. It can be seen that, with the Hn increasing, the errors of predicted results are fluctuant and reach a minimum value at Hn = 10. Thus, the node number of the hidden layer was set to ten in this present work. In addition, the Levenberg-Marquardt (LM) algorithm was adopted and used in the ANN training,37 due to rapid convergence in solving nonlinear least squares problems.

5.3. WPD-ANN-ARE Training and Verification

The ANN training procedure is conducted by a Matlab program. To check the robustness of the ANN structure, the normalized noise samples are divided into three sets for network training, validation, and testing.38 The training set is used for learning, which is to fit the ANN weights. The validation set is used to tune the ANN architecture. The test set is used only to assess the performance of the fully ANN. The training and validation sets are defined by 33 left- and right-ear noise signals, and the test set consists of 33 signals that are randomly taken from the vehicle noise database. The robust performance of the WPD-ANN-ARE model is shown in Figure 11. It can be seen that the MSEs of the training, validation and test sets are rapidly decreased within fourteen epochs. The network tends to stabilize at the sixth epoch, and the MSE values reach 0.09 at the eighth epoch. This implies that both the ANN structure and the selected parameters are effective and feasible for ARE of vehicle noise. To verify the WPD-ANN-ARE, furthermore, an original noise signal was randomly selected from the vehicle noise database and fed to the new model for roughness prediction. Then, the comparison was made between the new model and the Aures model, as shown in Figure 12. The selected vehicle noise was measured at the driver position under the emergency braking condition (stopping from 80 km/h), which is one of the representative conditions. It can be seen that the WPD-ANN-ARE result is very close to that from the Aures model, especially for the roughness values above 0.3 asper. The mutation of the auditory roughness is predicted by the new model, which shows good characteristic in tracking the extreme values in the time domain, thus able to capture the main sensations of auditory roughness over time. In view of the results, the MSE of the new model result is 0.0427, and the maximum error of the predicted total roughnesses of the noise signals is 8.56 percent, which can meet the engineering requirements (less than 10 percent). The above comparisons suggest a good accuracy of the WPD-ANN-ARE model in roughness evaluation of vehicle noise. It should be mentioned that there is no any standardized method for ARE till now. The Aures model is the relative authority and has been widely accepted by the international academic community which is the reason why it is adopted as a reference for WPD-ANN-ARE model verification in this paper.

6. CONCLUSIONS

This paper presents a new technique named WPD-ANN-ARE model, which is developed by combining the WPD and ANN, for ARE of vehicle noise. Considering the characteristics of human auditory perception, a WPD-based model with 21 critical bands is built to extract the noise features. Taking the extracted roughness feature matrices defined by modulation index as inputs and the calculated roughness from the psychoacoustic model as outputs, a three-layer ANN model is designed and trained by the BP algorithm for ARE of noises. The results show that the WPD-based model is effective for feature extraction of nonstationary vehicle noises, and the WPD-
ANN-ARE results are in agreement with those from the conventional roughness model. This implies that the WPD-ANN-ARE model is accurate enough to map a nonstationary vehicle noise to its roughness, and may be regarded as a good substitute for complex psychoacoustic models in vehicle SQE engineering. In applications, the newly proposed technique can be used to estimate sound quality of vehicles, and may be extended to other noise signals for ARE in engineering.

7. ACKNOWLEDGEMENTS

This work was supported by National Natural Science Foundation of China (No. 51675324), partly supported by Natural Science Foundation of Shanghai (No. 14ZR1418600).

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1. INTRODUCTION

The main methods used by aerospace industries to inspect damage are visual inspection and coin tapping.\(^1\) Visual inspection, which is quick and low cost, is an obvious approach to assess surface damage. A tap test is also an inexpensive inspection technique, which has in recent years been automated for greater accuracy.\(^2\)

A tap test is one of the simplest non-destructive inspection methods which serves as a first line of defense when looking for flaws in composite structures.\(^3\) Tap test works well for thin laminates, honeycomb structures and other sandwich panels, but it is not so effective on thicker parts.\(^4,5\) When testing thick composite structures, deeply buried defects are inaudible to human ear.\(^5\) Tap test only requires a coin (or similar object) and a good sense of hearing. As the surface is impacted with the coin, it emits a sound which is dependent upon the local stiffness. When a tap occurs over a damaged area, where the local stiffness is lower, the sound is dampened, producing a "dead" tone compared to a tap over an undamaged region which produces a higher tone.\(^6\) With this simple method, operators are able to quickly identify major flaws requiring further inspection of the structure.

The tap test can use two kinds of physical measures such as impact force and acoustic emission. The impact force can be acquired by using a force transducer, and sound pressure is detected via a microphone. The tap test finds flaws only in the exact region of tapping, and it is necessary to tap multiple regions to examine a wider area. The tap test uses the difference between the measured impact force or sound pressure histories of an undamaged structure and a locally damaged structure. For structurally radiated sound, the acoustic field is directly coupled to the structural response.

The finite element method (FEM) used to analyze the impact behavior of the composite laminate has been known to require a long computation time due to the nonlinear nature of the contact condition. For instance, Shivakumar et al. did not use FEM to compute the impact behavior of laminate, but instead used a spring mass model to predict the impact force history.\(^7\) In their study, the contact energy due to indentation as well as the transverse shear and bending energy of the laminate are considered, and they reported that the contact energy can be neglected in the thin plate. However, their study was confined to circular plates with transversely isotropic properties. Choi suggested a spring mass model using linearized contact law.\(^8,9\) In that study, it was reported that the linearized contact law approach could be applied to a low-velocity impact response analysis problem by using commercial FEM software. Kim proposed an equivalent mass model to model a hammer-shaped impactor. From the test results, it was observed that the equivalent mass model provided an appropriate solution when the impactor shape was complex.\(^10\)

Many researchers have investigated the radiated sound from a vibrating structure by using a numerical method.\(^11,12\) If the acoustic loading is assumed to be negligible and the dynamic behavior of the structure and radiated sound are de-coupled, the radiated sound can be analyzed by using a Rayleigh surface integral. The radiated sound pressure has been compared to tap test data. The tap test serves as a good initial examination to identify major flaws, yet it lacks the sensitivity related to the other effects such as boundary condition. Minak et al. investigated the effect of clamped and simple support boundary conditions on circular carbon fiber reinforced polymer (CFRP) laminates subject to low velocity impact. The results showed that the clamped boundary condition increased the stiffness leading to increase delamination.\(^13\)

Kim investigated the effect of in-plane load on the tap test.\(^10\) Most aircraft structures are under some degree of load while on the ground. The upper skin of the main wing is subjected to in-plane tensile load, while the lower skin is subjected to compressive load. Therefore, the effect of in-plane loads should be considered when the tap test is performed. From the results of that study, it could be seen that the initial in-plane tensile load increased the stiffness of the laminates. In contrast, the stiffness of the laminates decreased under an in-plane compressive load. Therefore, the contact force and contents of high-frequency sound increase under an in-plane tensile load.
while the contact force and contents of a high-frequency sound decrease under an in-plane compressive load.

Composite sandwich plates are widely used in the aerospace industry because of their high specific strength and stiffness. However, a composite sandwich exhibits a relatively low impact damage resistance by the low fracture toughness of matrix in the laminate face sheet. The impact damage of a composite sandwich-panel is difficult to detect because of the nature of sandwich structure, and such damage can cause a significant reduction in the load capacity of the composite sandwich.

In this study, a numerical simulation is used to investigate the effect of boundary condition on composite laminates and damage in the sandwich structure. The boundary condition effects are simulated by employing artificial springs to restrain edge displacement and a de-bonding model is used to simulate the damage in the sandwich structure.

2. NUMERICAL SIMULATION OF TAP TEST

The tapping event can be regarded as an impact problem. In this paper, the spring mass model was applied to compute the impact behaviour of the composite laminate. If the transverse displacement of an impacted plate is very small, compared to the thickness of the plate, one may consider the plate as a plane source in an infinite baffle. When reflection and diffraction of sound at the boundary of the plate are ignored, the sound pressure radiated from the plate can be obtained from Rayleigh’s surface integral.\(^{15}\)

2.1. Validation of Numerical Model

2.1.1. Impact Response Analysis

The difference of displacement between the impactor and deformation of laminate at the impacted location means indentation. The indentation is converted to contact force using contact law or experimental indentation law. Choi showed that a linearized contact law approach can be applied to a low-velocity impact response analysis problem with the use of commercial FEM software.\(^{9}\) In this paper, MSC/NASTRAN is used. Figure 1 shows the FEM model for impact response analysis. The mass of the impactor is located at the end of the spring element, and the other end of the spring element is attached to the laminate at the impacted location. After impact analysis, we can extract the impact force history acting at the spring. In the FEM model, a four-node plate element was used and transient analysis was carried out with the initial condition of initial velocity of the lumped mass being given as impact velocity. This spring mass model consists of one spring representing the stiffness of contact law and plate elements representing the composite laminate. The hammer-shaped impactor is simplified by having the concentrated mass use the spring mass model. The equivalent concentrated mass is determined by the following procedure:\(^{10}\)

\[
mgR_c \times (1 - \cos \theta) = \frac{1}{2} I_0 \dot{\theta}^2; \quad v_1 = \dot{\theta} \times R_c; \quad (1)
\]

where \(R_c\) is the mass centre of the impactor, \(\theta\) is the angle rotated from the neutral position, \(I_0\) is the mass moment of the inertia of the impactor with respect to rotation centre, \(v_1\) is the impact velocity of the impact position and \(\dot{\theta}\) is the angular velocity of the impactor. The equivalent impactor mass is computed as follows:

\[
\frac{1}{2} I_0 \dot{\theta}^2 = \frac{1}{2} m_e v_1^2; \quad (2)
\]

where \(m_e\) is the equivalent mass of the impactor. In this model, model parameters are mass and mass moment of inertia of impactor.

2.1.2. Acoustic Sound Analysis

The acoustic sound radiated from a vibrating plate can be obtained by evaluating the Rayleigh surface integral where each elemental area on the plate’s surface is regarded as a point source of an outgoing wave and its contribution is summed.

![Figure 1](image1.png)

Figure 1. Spring mass model using commercial FEM software.

![Figure 2](image2.png)

Figure 2. Coordinate system used for evaluating acoustic pressure.

<table>
<thead>
<tr>
<th>Table 1. Material properties.</th>
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<tbody>
<tr>
<td><strong>Material properties of lamina</strong></td>
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<tr>
<td><strong>G(<em>{12}) = G(</em>{13}) = G(_{23}) = 3.74 GPa</strong></td>
</tr>
<tr>
<td><strong>(\nu_{12} = 0.3)</strong></td>
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<tr>
<td><strong>(\rho = 1600.0 \text{ kg/m}^3)</strong></td>
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<tr>
<td><strong>Thickness = 0.14 mm</strong></td>
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<table>
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<tr>
<th><strong>Material properties of impactor</strong></th>
<th><strong>E = 70.0 GPa</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(\nu = 0.3)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>(\rho = 2700.0 \text{ kg/m}^3)</strong></td>
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</table>
Figure 3. Tap test system: (a) Side view, (b) Front view.

Figure 4. Configuration of impactor.

Figure 5. Comparison of impact force between analysis and test.

Figure 6. Comparison of sound pressure history between analysis and test.

Figure 7. Boundary condition using rotational spring.
with an appropriate time delay. Referring to Fig. 2, the acoustic pressure $P(r, t)$ at observation point $r_0$ with a Cartesian coordinate $(x_0, y_0, z_0)$ at time $t$ induced by the vibration of the plate is analysed by using the Rayleigh surface integral, which is shown in Eq. (3):

$$
P(r, t) = \frac{\rho_d}{2\pi} \int_S \frac{1}{|r - r_0|} \frac{\partial^2}{\partial t^2} W(r_0, t - \frac{|r - r_0|}{c_a}) \, dS.
$$

(3)

where $\rho_d$ and $c_a$ are the mass density and wave velocity of the acoustic medium, $\partial^2 W/\partial t^2$ is the acceleration of the plate element, which was computed from impact response analysis, and applied with an appropriate time delay. The properties of air as the acoustic medium are density $\rho_d = 1.21$ kg/m$^3$ and speed of sound $c_a = 340$ m/s.

### 2.1.3. Validation of Numerical Model

The experimental investigation was performed on a laminate specimen. The dimension of the laminate was $19 \times 19$ cm$^2$, and the boundary condition of the plate had four edges clamped. In this paper, in order to make the fixed boundary condition during contact duration, a bolt clamped device was used to fix the laminate specimen. The laminate had a lay-up of $[0/\pm45/0/\pm45/0]$. The material properties of the lamina are shown in Table 1. A pendulum type tap test system was set up as shown in Fig. 3. A piezoelectric force transducer sensor was used to measure the impact force history. The force transducer was located at the mid position of the impactor. A microphone was used to measure the sound pressure. The microphone was located 15.0 cm behind the centre of the laminate. Figure 4 shows the shape of the impactors. A tapping event is occurred when an impactor was rotated upward.
from the vertical position and then released. Figure 5 shows the impact force history given by the analysis and test when an impactor was rotated 15.0 degrees from the vertical line and then released. In this case, the impact velocity is 0.337 m/s and the equivalent mass is 0.956 kg. This figure shows that the impact force history computed by the spring mass model provided accurate results. Figure 6 shows the sound pressure given by the analysis and test. Some differences may have occurred in sound pressure due to possible imperfections of the experimental impact and support conditions. These calculations did not consider the energy from the fixture and the reflection of sound.

### 2.2. Influence of Boundary Condition

In general, composite laminates are used to fabricate structural parts. These products are supported by elastic restraints or connected to other members, which can also be treated as elastic supports for restraining the plates. Therefore, the effect of an elastically restrained boundary should be considered when the tap test is performed. To predict the effect of support properties on the sound radiated from the plate and impact force histories, a rotational spring is used at the edges of the plate. Figure 7 shows the elastically supported boundary condition used for the impact response and acoustic sound analysis. Figures 8–11 show the impact force histories of a rotationally supported plate when the rotational stiffness was 1.0 Nm, 10.0 Nm, 100.0 Nm, and 1000.0 Nm, respectively. The plate had dimensions of 19 × 19 cm² and the laminate had a lay-up [0/45/0/−45/0/−45/0/45/90]s. The motion of the four edges was assumed to be restrained by a rotational spring, \( k_r \). It can be noted from these figures that the maximum contact force increased and the contact duration decreased as the rotational stiffness increased. In this case, the impact velocity is 0.337 m/s and the equivalent mass is 0.956 kg. Figures 12–15 show the sound pressure histories of the rotationally supported plate when the rotational stiffness was 1.0 Nm, 10.0 Nm, 100.0 Nm, and 1000.0 Nm, respectively. The sound pressure was calculated at 15.0 cm above the centre of the plate. To investigate the effect of the rotational stiffness in a frequency domain, a comparison of spectra is shown in Figs. 16–19. These were achieved by carrying out a Fourier transform of the sound pressure histories. From the results, it is known that the contents of frequency increased as the rotational stiffness increased.
2.3. Damage Detection in Sandwich Structure

Figure 20 shows a spring mass model of a sandwich structure. In this model, a sandwich plate was constructed as an assembly of a face modelled with four-node plate elements and a core modelled with eight-node solid elements. Figure 21 shows the debonding model of the sandwich plate. The nodes of the debonding area are detached and not connected to each other when the sandwich structure is impacted by the lumped mass. The analysis model of the sandwich plate is $19 \times 19 \times 1.5$ cm$^3$, and the boundary condition of the plate is four edges clamped. The face of the sandwich plate has a lay-up of $[0/45/-45/0]_s$, and the size of debonding is $0.3 \times 0.3$ cm$^2$. Figure 22 shows the comparison of the impact force histories of undamaged and debonded sandwich plates. In this case, the impact velocity is 0.337 m/s and the equivalent mass of the impactor is 0.092 kg. It can be observed from the result that the maximum contact force decreased and the contact duration increased due to debonding. A comparison of sound pressure histories computed by analysis both with and without debonding is shown in Fig. 23. The sound pressure was calculated at 15.0 cm above the centre of the plate, and Fig. 24 shows the spectra of the sound pressure history. From the results, it is observed that the impact on the damaged area did not excite the higher structural modes as strongly as the impact on the undamaged area. Therefore, the sound produced does not contain higher frequencies and the structure sound is “duller”.

2.4. Application of Tap Test to Rotor Blade

Figure 25 shows the FE model of composite rotor blade. The span length is 9.45 m, cord length is 0.46 m and crack length is 0.15 m. In this model, four-node plate element was used. The total number of elements are 51,644 and nodes are 50,614. The laminate had a lay-up $[0/45/0/-45/0/-45/0/45/0]_{2s}$. The fixed boundary condition is applied to root of rotor blade. Figure 26 shows the comparison of the impact force histories of undamaged and cracked models. In this case, the impact velocity is 0.337 m/s and the equivalent mass of the impactor is 0.956 kg. It can be observed from the result that the maximum contact force decreased and the contact duration increased due to crack.
3. CONCLUSION

The purpose of this study was to investigate the physical basis of tapping sound. In this paper, the elastically restrained composite laminates were modelled by a spring element, and the effects of a rotational stiffness were reviewed. It can be noted from the results that the maximum contact force increased and the contact duration decreased as the rotational stiffness increased. The frequency contents of sound pressure increased as the rotational stiffness increased. The spring mass model was also used to simulate a tap test on a sandwich structure, and the effects of disbond in the sandwich structure was numerically investigated. The numerical results show that there is a strong correlation between the debonding and reduction in impact force and the contents of high-frequency sound pressure. From the impact response analysis on composite rotor blade, it is observed that this numerical model can be applied to real structure. The results presented above show that it is possible to detect damage in a composite structure using an impact force or sound pressure by comparing either the time history or the corresponding spectrum with signals from a structure.

ACKNOWLEDGEMENTS

This study was supported by the ‘Study on the light structure and structural integrity improvement technology’ program funded by the National Research Council of Science & Technology, Republic of Korea.

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Figure 24. Comparison of spectra of sound between undamaged and debonded sandwich plate.

Figure 25. Finite element model of composite rotor blade.


FFT-ApEn Analysis for the Vibration Signal of a Rotating Motor

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(Received 6 July 2017; accepted 26 January 2018)

This paper presents the FFT-ApEn analysis method for the fault detection of an electric motor under different rotating speeds. Motor vibration signals are analyzed using the Fast Fourier Transform (FFT) and Approximate Entropy (ApEn) to obtain the fault factor of a motor under different rotating speeds. The effectiveness of the proposed FFT and FFT-ApEn analyses for predicting the fault is verified through the experimental data. It is found that the FFT-ApEn analysis for the vibration signal can more precisely identify the fault as compared to the conventional FFT analysis method. In addition, the magnitudes of the frequency components are extracted for the recognition of the fault modes. The frequency spectrum analysis is used for distinguishing four operating statuses, i.e., normal, carbon brush failure, abnormal noise and bearing failure. Moreover, the FFT-ApEn method can successfully discriminate four different operating statuses of a motor without removing any motor parts. Hence, the FFT-ApEn analysis method is of great significance for a motor to have a real-time monitoring ability.

1. INTRODUCTION

A motor is an indispensable component in the modern industry. The temperature, noise, voltage, current and vibration signals are the most common parameters for detecting whether the status of a motor is normal or faulty. Among all state-monitoring techniques, the vibration signal analysis is the most classical technique to evaluate the status of a rotating machine. The analysis of a vibration signal is very critical to effectively monitor and control if a rotating motor possessed high productivity and reliability. Generally, for a fault diagnosis technique, the characteristic features of vibration signals were analyzed in frequency domain or chaotic phase space. For examples, Arslan et al. disclosed the relationship between statistical vibration parameters, tool wear, and surface roughness of a work piece during high speed turning operation. Javed et al. presented a method for feature extraction or selection, and the proposition was applied to time-frequency analysis of non-stationary signals using discrete wavelet transform. Zhang et al. used variational mode decomposition to detect the defect signals of different locations in a multistage centrifugal pump. They studied the failure mechanism of rolling bearings, and established the defect signal models of different locations, and simulated the fault signals of outer race defect, inner race defect, as well as the rolling element defect. Also, empirical mode decomposition (EMD) has been widely used for analyzing non-stationary signals due to its ability to self-adaptive decomposition of non-stationary signals. The EMD method can accurately identify and diagnose the running state and bearing fault type at early stages of their development. Generally speaking, for a motion system, the complexity and chaos degree can be described by the Approximate Entropy (ApEn). The higher the complexity and chaos degree of a motion system are, the bigger the ApEn value of the system becomes. The ApEn analysis method has a strong ability of resisting noise interference in random signals, deterministic signals or these two mixed signals. Since this method was proposed, it has been successfully applied to the analyses of heart rate variability and endocrine hormone release pulsatility. Sparacino et al. studied a distorted portrait of the secretion rate at the gland level by ApEn analysis. They reported whether and how this distortion can influence the regularity of hormone pulsatility. On the other hand, in conventional condition monitoring, the commonly used method is the vibration analysis in frequency domain through Fast Fourier Transform (FFT). FFT is an algorithm to realize discrete Fourier transform and able to convert the vibration signal from its time-domain representation to its equivalent frequency-domain representation. Gao et al. presented an algorithm, FFT-AFD (Adaptive Fourier Decomposition), reducing the computational complexity of the AFD. AFD is originated with the purpose of positive frequency decomposition of signals. They have proven the effectiveness, accuracy, and reliability of the FFT-AFD algorithm, as well as laid a foundation for its practical applications. Nevertheless, frequency analysis is only one aspect of interpreting the information contained in a vibration signal. In view of this, this paper presents a FFT-ApEn method for the fault detection of a motor on the basis of both the chaotic space and frequency-domain analysis of vibration signals at different ro-
tating speeds. It is found that the FFT-ApEn can more precisely identify the faulty type of a rotating motor as compared to the only FFT or ApEn method.

2. RESEARCH METHOD

2.1. The Definition of the ApEn

In order to define \( ApEn(r, m, N) \) for the N-dimensional time series \( \{u(1), u(2), ..., u(N)\} \) with given parameters \( m \) and \( r \), the \( m \)-dimensional sequence vector \( x(i) \) should be embedded.\(^{1,11}\) Then, the ApEn is defined as:

\[
ApEn(m, r, N) = \lim_{N \to \infty} \frac{1}{N} \ln \left( \frac{\varphi^m(r) - \varphi^{m+1}(r)}{\varphi^m(r)} \right);
\]

where:

\[
\varphi^m(r) = \left( \frac{1}{N-m+1} \right) \sum_{i=1}^{N-m+1} \ln C_i^m(r);
\]

\[
C_i^m(r) = \left( \frac{1}{N-m+1} \right) \sum_{j=1}^{N-m+1} \theta(d(x(i), x(j)) - r), \quad (1 \leq i \leq N-m+1, \quad i \neq j)
\]

\[
\theta(x) = \begin{cases} 
1 & \text{if } x > 0 \\
0 & \text{otherwise}
\end{cases} \quad (4)
\]

\[
d(x(i), x(j)) = \max_{0 \leq k \leq m-1} |x(i+k) - x(j+k)|, \quad (1 \leq i \leq N-m+1, 1 \leq j \leq N-m+1, i \neq j)
\]

Obviously, the estimate value of the ApEn depends on \( m \) and \( r \). As suggested by Pincus et al.,\(^{11}\) \( m \) can be taken as 2 and \( r \) be taken as \((0.1 - 0.25)\) SD, where SD is the Standard Deviation from the original data sequence. As a rule in engineering practice, more than 100 data are needed to meet the requirements for estimating a robust value of the ApEn. Consequently in this paper, the ApEn is calculated under the following conditions: \( N \geq 100, \quad r = 0.15\)SD, \( m = 2 \).

2.2. The FFT-ApEn Analysis Method

Seven motors that have been judged as four operating statuses were tested to obtain four sets of vibration signals, i.e., “normal” for two motors, “bearing failure” for two motors, “carbon brush failure” for two motors, and “abnormal sound” for a motor. The motors are series motors of the drum washing machines, and the DC voltage of 0–72 V is applied to them to get various rotating speeds.

Step 1: Acquiring the vibration signal for every typical operating status of a motor, and the frequency range of the vibration signal is divided into five equal regions.

Step 2: Through the FFT algorithm,\(^{15,17}\) computing the main frequency regions of the vibration signal for every operating status of the motor at low and high rotating speeds, respectively.

Step 3: Constructing the time series required for the calculation of the ApEn.

Step 4: Confirming the SD threshold \( \delta_0 \) and average-value threshold \( \Psi_0 \) of the ApEn for a normal-status rotating motor at various rotating speeds.

Step 5: The SD and average value of the ApEn of the vibration signal for every operating motor are calculated and compared with \( \delta_0 \) and \( \Psi_0 \), at various rotating speeds. The flowcharts for the description of steps are revealed in Fig. 1 and 2.

3. RESULTS AND DISCUSSION

3.1. Vibration Signals in Time and Frequency Domains

As mentioned above, seven motors that have been judged as four operating statuses were tested to obtain four sets of vibration signals. The hardware components of the data acquisition system are Lens LC0105 accelerometer and NI9235 data acquisition card, as revealed in Fig. 3. Vibration signals of the motors in time or frequency domain at the rotating speeds of 915 and 2250 rpm are shown in Figs. 4, 5, 6, and 7, respectively. The rotating speed of a motor is obtained from the location feedback by a photoelectric code-disc located at the motor revolution axis. From the vibration signals in time domain as seen in the Figs. 4 and 5, the four operating statuses have no obvious distortion phenomenon, so that it is difficult to evaluate the running states of the motors. Through the FFT algorithm, the vibration signals in frequency domain present obvious and different main frequency regions for the four operating-status motors at the same rotating speed of 915 rpm, as seen in the Fig. 6.

3.2. Analysis of FFT Algorithm

According to the vibration signal in frequency domain as shown in Fig. 6 and 7, the frequency range of the vibration signal for a motor is divided into five regions: \((0, 500), (500, 1000), (1000, 1500), (1500, 2000), (2000, 2500)\). By use of the FFT algorithm, main frequency regions of vi-
vibration signals are partially overlapped for the four operating-status motors at a low rotating speed of 915 rpm, as shown in Fig. 6 and Table 1. Besides, the main frequency regions tend to be consistent for the four motors at a high rotating speed of 2250 rpm, as revealed in Fig. 7.

3.3. The Average Value and Standard Deviation of the ApEn

Table 2 shows the average value (AV) and standard deviation of the ApEn of the vibration signal for every operating-status motor at different rotating speeds. From Table 2, the AV and SD of the ApEn of the normal motor are larger than those of the other faulty motors under the same rotating speed. When the motor rotating speed gets faster, the AV and SD differences between a normal motor and a faulty motor become larger. Therefore, comparing the AV and SD of the ApEn of a tested motor with those of a normal motor can judge if the tested motor was running normally. Unfortunately, when the AV and SD of the ApEn of a tested motor have been calculated out, it is difficult to discern the specific fault type by referring to Table 2 only.

3.4. Judging the Operating Status of a Motor by FFT-ApEn Analysis

To illustrate the FFT-ApEn method clearly, one normal motor (labeled as A) and three faulty motors (labeled as B, C, D) are tested as follows. Firstly, vibration signals are analyzed by ApEn algorithm for the four motors at a high rotating speed of 2250 rpm. Then, the operating status of every motor is evaluated as normal or not, as revealed in Table 3. Next, vibration signals are analyzed through the FFT algorithm for the four

Table 1. Main frequency features of vibration signals for four operating-status motors at different rotating speeds.

<table>
<thead>
<tr>
<th>Motor speed</th>
<th>Main frequency regions (Hz)</th>
<th>Operating status</th>
</tr>
</thead>
<tbody>
<tr>
<td>915 rpm</td>
<td>(500, 1000), (2000, 2500)</td>
<td>normal</td>
</tr>
<tr>
<td>915 rpm</td>
<td>(500, 1000)</td>
<td>bearing failure</td>
</tr>
<tr>
<td>915 rpm</td>
<td>(1500, 2000), (2000, 2500)</td>
<td>abnormal sound</td>
</tr>
<tr>
<td>915 rpm</td>
<td>(500, 1000), (2000, 2500)</td>
<td>carbon brush failure</td>
</tr>
<tr>
<td>2250 rpm</td>
<td>(1000, 1500)</td>
<td>normal</td>
</tr>
<tr>
<td>2250 rpm</td>
<td>(1000, 1500)</td>
<td>bearing failure</td>
</tr>
<tr>
<td>2250 rpm</td>
<td>(1000, 1500)</td>
<td>abnormal sound</td>
</tr>
<tr>
<td>2250 rpm</td>
<td>(1000, 1500)</td>
<td>carbon brush failure</td>
</tr>
</tbody>
</table>
motors at a low rotating speed of 915 rpm, and the main frequency regions are extracted out. Finally, referring to Table 1 and Table 2, the operating status of every motor can be judged clearly, as shown in Table 4. For example, the AV (ApEn) of tested motor B is 0.0621087 at a rotating speed of 2250 rpm, as shown in Table 3. Refering to Table 2, the operating status of motor B could be either bearing or carbon brush failure. The main frequency of tested motor B is only (500, 1000) at a rotating speed of 915 rpm, as shown in Table 4. Then, referring to Table 1, the operating status of motor B is judged as bearing failure.

3.5. Discussion About Nonlinearity and Chaos

By using the FFT algorithm, main frequency regions of the vibration signal for a motor are found out. According to the principle of nonlinear dynamics, the time series of variables already contain information about the system variables. In order to quantify the complexity of nonlinear sequence data, Pincus\(^{18}\) proposed a concept of the ApEn, which solved the puzzle of getting the entropy of a chaotic system. Approximate entropy can be used to represent the complexity of a system, and it is widely used in the fields of the weather forecast and detection of the mechanical vibration.\(^{19}\)

Figure 6. Vibration signals in frequency domain for four operating-status motors at the same rotating speed of 915 rpm. (a) normal (b) bearing failure (c) abnormal (d) carbon brush failure.

Figure 7. Vibration signals in frequency domain for four operating-status motors at the same rotating speed of 2250 rpm. (a) normal (b) bearing failure (c) abnormal (d) carbon brush failure.

4. CONCLUSIONS

Using FFT-ApEn analysis for a vibration signal, we investigated the relationship between the ApEn and stability of a rotating motor through the viewpoint of nonlinearity and chaos. The average value and standard deviation of the ApEn are significantly correlated with the operating status of a motor. For a normal motor, the value of the ApEn is increasing with the increasing of the rotating speed. To sum up, the FFT-ApEn method can successfully distinguish the operating status and is of great significance for a motor to have a real-time monitoring ability.

ACKNOWLEDGEMENTS

This work was supported by Pdjh2016b0752 and Jiangmen Science and Technology Project.

REFERENCES


International Journal of Acoustics and Vibration, Vol. 23, No. 2, 2018
Table 2. The average value and standard deviation of the ApEn for four operating-status motors at each rotating speed.

<table>
<thead>
<tr>
<th>Motor speed</th>
<th>Operating status</th>
<th>Av (ApEn)</th>
<th>SD (ApEn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>915 rpm</td>
<td>normal</td>
<td>0.0445834</td>
<td>0.0163492</td>
</tr>
<tr>
<td>915 rpm</td>
<td>bearing failure</td>
<td>0.0296041</td>
<td>0.0110936</td>
</tr>
<tr>
<td>915 rpm</td>
<td>abnormal sound</td>
<td>0.0410724</td>
<td>0.0120309</td>
</tr>
<tr>
<td>915 rpm</td>
<td>carbon brush fault</td>
<td>0.0175405</td>
<td>0.00861866</td>
</tr>
<tr>
<td>1200 rpm</td>
<td>normal</td>
<td>0.154326</td>
<td>0.0225397</td>
</tr>
<tr>
<td>1200 rpm</td>
<td>bearing failure</td>
<td>0.0348034</td>
<td>0.0120916</td>
</tr>
<tr>
<td>1200 rpm</td>
<td>abnormal sound</td>
<td>0.0283015</td>
<td>0.0163492</td>
</tr>
<tr>
<td>1200 rpm</td>
<td>carbon brush fault</td>
<td>0.0100699</td>
<td>0.0169607</td>
</tr>
<tr>
<td>1700 rpm</td>
<td>normal</td>
<td>0.3082297</td>
<td>0.0465132</td>
</tr>
<tr>
<td>1700 rpm</td>
<td>bearing failure</td>
<td>0.0081955</td>
<td>0.0182974</td>
</tr>
<tr>
<td>1700 rpm</td>
<td>abnormal sound</td>
<td>0.0284481</td>
<td>0.0181129</td>
</tr>
<tr>
<td>2250 rpm</td>
<td>normal</td>
<td>0.320216</td>
<td>0.04602</td>
</tr>
<tr>
<td>2250 rpm</td>
<td>bearing failure</td>
<td>0.0362530</td>
<td>0.0328796</td>
</tr>
<tr>
<td>2250 rpm</td>
<td>abnormal sound</td>
<td>0.154306</td>
<td>0.013487</td>
</tr>
<tr>
<td>2250 rpm</td>
<td>carbon brush fault</td>
<td>0.0743915</td>
<td>0.0186085</td>
</tr>
</tbody>
</table>

Table 3. The Av and SD of the ApEn for four tested motors at a high rotating speed of 2250 rpm.

<table>
<thead>
<tr>
<th>Tested motor (2250 rpm)</th>
<th>Av (ApEn)</th>
<th>SD (ApEn)</th>
<th>Operating status</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.320727</td>
<td>0.04588</td>
<td>normal</td>
</tr>
<tr>
<td>B</td>
<td>0.062108</td>
<td>0.0296779</td>
<td>abnormal</td>
</tr>
<tr>
<td>C</td>
<td>0.011106</td>
<td>0.010192</td>
<td>abnormal</td>
</tr>
<tr>
<td>D</td>
<td>0.0732915</td>
<td>0.0185985</td>
<td>abnormal</td>
</tr>
</tbody>
</table>

Table 4. Main frequency features for four tested motors at a low rotating speed of 915 rpm.

<table>
<thead>
<tr>
<th>Tested motor (915 rpm)</th>
<th>Main frequency (Hz)</th>
<th>Operating status</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(500, 1000), (2000, 2500)</td>
<td>normal</td>
</tr>
<tr>
<td>B</td>
<td>(500, 1000)</td>
<td>bearing failure</td>
</tr>
<tr>
<td>C</td>
<td>(1300, 2000)</td>
<td>abnormal sound</td>
</tr>
<tr>
<td>D</td>
<td>(500, 1000), (2001, 2500)</td>
<td>carbon brush fault</td>
</tr>
</tbody>
</table>

Attributes of a Vibration Isolator Design with Stiffness Nonlinearities

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(Received 4 August 2017; accepted 26 January 2018)

Inclusion of stiffness nonlinearity in a vibration isolator has been shown to exhibit some advantages such as an increase in the frequency range of isolation. In some engineering applications, it is common to design the vibration isolator such that the stiffness in one direction is significantly different from the stiffness in other directions. Such a design is commonly used for vibration isolators in applications where the packaging and performance requirements along different axes are drastically different. One such example is the vibration isolator for a motorcycle powertrain. This study proposes a design that incorporates stiffness nonlinearities into the vibration isolator along two axes to complement the spring-damper system along the axis of displacement of the single degree-of-freedom system. These stiffness nonlinearities are incorporated into the Maxwell-Voigt (MV) and Maxwell-Maxwell-Voigt (MMV) models for elastomeric isolators. The proposed design is expected to increase the range of vibration isolation and allow some design flexibility in placing the natural frequency of the system while satisfying the specific requirements of a range of products. Results from all the models investigated in this study indicate that adding stiffness nonlinearity, in the form of spring elements along the non-isolating axes, can provide a designer with additional flexibility in placing the natural frequency of the isolation system while enhancing the overall isolation range.

NOMENCLATURE

- $F$: Amplitude of external excitation force.
- $X$: Amplitude of base displacement.
- $Y, Y_1, Y_2$: Amplitude of displacement of the isolated mass and the nodes of the Maxwell elements.
- $\varphi, \varphi_1, \varphi_2$: Phase angles for the displacement of the mass and the nodes.
- $k_h, k_{hx}, k_{h2}$: Stiffness of horizontal spring element.
- $l_0, l$: Free length and compressed length of spring element.
- $x(t)$: Base displacement.
- $f_y$: External excitation force.
- $y, y_1, y_2$: Displacement of the isolated mass and the nodes of the Maxwell elements.
- $c_0, c_1, c_2$: Damping constants.
- $k_{10}, k_{11}, k_{2}$: Stiffness constants.
- $m$: Mass.

1. INTRODUCTION

The use of passive vibration isolators is widespread in multiple engineering applications. Designers have been increasingly investigating the use of stiffness and damping nonlinearities to overcome some of the constraints posed by linear vibration isolators. These nonlinearities are specifically incorporated as per design intent; however, there are multiple aspects of a passive elastomeric isolator such as cyclical softening, temperature dependent behavior, etc. that make the behavior inherently nonlinear. Nonlinearities that have been investigated in the literature include attributes resulting from smart material elements, X-shape structured lever-type design, scissor-like structured platform, etc. Passive isolators exhibit a complex behavior that needs to be modeled in order to accurately predict system response before undertaking detailed design of the isolation system. There are multiple models with varying characteristics in the existing literature that have been used to represent specific features of a vibration isolation system, one such example is the use of Maxwell-Voigt and Maxwell Ladder models for multi-degree-of-freedom isolation systems.

Some of the recent literature on nonlinear vibration isolation includes discussion on negative stiffness mechanisms, quasi-zero-stiffness, high-static-low-dynamic stiffness, among other possible designs that take advantage of nonlinear behavior. The negative frequency mechanism is reported to increase the frequency range of vibration isolation, and such a mechanism is also found to significantly mitigate the response at resonance. The quasi-zero-stiffness design is found to be particularly beneficial for low frequency response, while a multi-direction quasi-zero-stiffness isolator is found to improve the isolation effect in multiple directions simultaneously in addition to providing design flexibility.

Nonlinear designs and nonlinear design attributes have been found to be particularly appealing since they can offer design flexibility in a passive vibration isolator and allow the isolation system to overcome some of the trade-offs associated with the use of a passive isolator. Nonlinearities have been specifically used for designing high-static-low-dynamic stiffness isolators, for modeling hysteretic behavior, etc. Many quasi-zero-stiffness isolators have been observed to possess high-static-low-dynamic stiffness characteristics, the existing literature provides examples of isolator designs with both characteristics. Alternative models have also been proposed in the literature to identify the viscoelastic behavior of an elastomeric isolator by developing a generalized Maxwell model, or by developing a constitutive model that repre-
The governing equations of motion (EOM) for the models presented in this section are nonlinear; as a result, two methods have been used for analysis. The first method is the Harmonic Balance Method (HBM), with the underlying assumption that a harmonic input yields a harmonic output that may contain one or more harmonics. HBM is used as the primary method for analysis in this study. HBM has been used in multiple studies in the literature for the analysis of nonlinear vibration isolation, one such example is the analysis of cubic damping in a vibration isolator. For all the derivations from the HBM in this section, only the first harmonic is used and the higher order harmonics have been ignored. The system of equations has been derived by substituting for the higher powers of trigonometric ratios and also by making use of the Binomial theorem. The second method used for computing the system response is numerical, using a variable-step, variable-order solver, and has been used to check the validity of the solution. The numerical method has been primarily used for computing the time response.

The first model presented in this section for a single degree-of-freedom (DOF) system incorporates a horizontal spring element into the Maxwell-Voigt (MV) model. A generalized model with multiple Maxwell elements and multiple spring elements is shown in Fig. 2. Similar models such as the Generalized-2 Maxwell model have been used in the literature to represent the viscoelastic behavior of elastomeric isolators. For the MV model with horizontal spring elements, \( k_{h1} = c_{h1} = 0 \) (also \( y_2 = 0 \)), \( k_{h2} = 0 \), and \( k_{h3} = k_{h} \), while using the generalized model in Fig. 2. In Fig. 2, the vertical \((y)\) axis represents the direction of motion and the governing EOM for this model are derived as follows:

\[
\begin{align*}
 m \ddot{y} + k_0 y + c_0 \dot{y} + k_1 (y - y_1) + 2k_h \left(1 - \frac{l_0}{\sqrt{l^2 + y^2}}\right) y &= f_y; \\
k_1 (y - y_1) &= c_1 \dot{y}_1.
\end{align*}
\]

In Eqs. (1) and (2), \( k_0 \) and \( c_0 \) are the spring and damping constants in the vertical \((y)\) direction of motion, \( y_1 \) is the displacement at the node of the Maxwell element. In Eq. (1), \( k_h \) is the stiffness of the horizontal spring element, \( l \) is the compressed length of the spring, and \( l_0 \) is the free-length of the horizontal spring before assembly. Furthermore, \( m \) is the mass of the single DOF system and \( f_y \) is the external excitation force. In Eq. (2), \( k_1 \) and \( c_1 \) are the spring and damping constants of the Maxwell element in the MV model.
Using the Binomial theorem, the first equation-of-motion from Eq. (1) can be expressed as:

\[ m\ddot{y} + k_0y + c_0\dot{y} + k_l(y - y_1) + 2k_h \left[ 1 - \frac{l_0}{l} \left( 1 - \frac{1}{2} \frac{y^2}{l^2} + \frac{3}{8} \frac{y^4}{l^4} - \frac{5}{16} \frac{y^6}{l^6} + \cdots \right) \right] y = f_y. \]  

(3)

In Eq. (3), higher order terms have been ignored. Furthermore, it may be noted that Eq. (3) holds for \(-l^2 < y^2 < l^2\), this is a reasonable assumption since the response of the system is expected to be small and can be verified from the simulation results in Section 3. Using the HBM, for a sinusoidal input of \(f_y = F \sin(\omega t - \varphi)\) with an amplitude \(F\) and phase \(\varphi\), the output displacements are \(y = Y \sin \omega t\) with an amplitude \(Y\) and \(y_1 = Y_1 \sin(\omega t - \varphi_1)\) with an amplitude \(Y_1\) and a phase of \(\varphi_1\). Substituting these expressions in Eq. (3) and equating the sine and cosine coefficients between the two sides from Eq. (2) and Eq. (3) yields the following:

\[ -m\omega^2 Y + k_0 Y + k_1 Y - k_l Y_1 \cos \varphi_1 + 2k_h \left[ 1 - \frac{l_0}{l} \left( 1 - \frac{1}{2} \frac{Y^2}{l^2} + \frac{3}{8} \frac{Y^4}{l^4} - \frac{5}{16} \frac{Y^6}{l^6} + \cdots \right) \right] Y = F \cos \varphi; \]

(4a)

\[ c_0\omega Y + k_1 Y_1 \sin \varphi_1 = -F \sin \varphi; \]  

(4b)

\[ c_1\omega Y_1 \sin \varphi_1 = k_1 Y - k_l Y_1 \cos \varphi_1; \]  

(4c)

\[ c_1\omega Y_1 \cos \varphi_1 = k_l Y_1 \sin \varphi_1. \]  

(4d)

The system of equations in Eq. (4) results from the comparison of first harmonic coefficients from Eq. (2) and Eq. (3). It may be noted that all higher order harmonics have been ignored.

For the Maxwell-Maxwell-Voigt (MMV) model in conjunction with a horizontal spring element, Fig. 2 can be used for reference with \(k_{hx} = 0\) and \(k_{hx} = k_h\). It may be noted that the use of two Maxwell elements has been found to enhance the capability of the model by allowing the model to predict dynamic stiffness and loss angle through the entire frequency range. The governing EOM for this model are as follows:

\[ m\ddot{y} + k_0y + c_0\dot{y} + k_l(y - y_1) + 2k_h \left( 1 - \frac{l_0}{\sqrt{l^2 + y^2}} \right) y = f_y; \]  

(5)

\[ k_1(y - y_1) = c_1\dot{y}_1; \]  

(6)

\[ k_2(y - y_2) = c_2\dot{y}_2. \]  

(7)

In Eq. (5), \(y_1\) and \(y_2\) are the displacements at the two nodes of the Maxwell elements shown in Fig. 2. Using similar steps to the ones used for the MV model along with the use of the Binomial theorem and the HBM, the following system of equations can be derived for the MMV model with horizontal spring elements:

\[ -m\omega^2 Y + k_0 Y + k_1 Y - k_l Y_1 \cos \varphi_1 + 2k_h \left[ 1 - \frac{l_0}{l} \left( 1 - \frac{1}{2} \frac{Y^2}{l^2} + \frac{3}{8} \frac{Y^4}{l^4} - \frac{5}{16} \frac{Y^6}{l^6} + \cdots \right) \right] Y = F \cos \varphi; \]

(8a)

\[ c_0\omega Y + k_1 Y_1 \sin \varphi_1 + 2k_2 Y_2 \sin \varphi_2 = -F \sin \varphi; \]  

(8b)

\[ c_1\omega Y_1 \sin \varphi_1 = k_1 Y - k_l Y_1 \cos \varphi_1; \]  

(8c)

\[ c_1\omega Y_1 \cos \varphi_1 = k_l Y_1 \sin \varphi_1; \]  

(8d)

\[ c_2\omega Y_2 \sin \varphi_2 = k_2 Y - k_2 Y_2 \cos \varphi_2; \]  

(8e)

\[ c_2\omega Y_2 \cos \varphi_2 = k_2 Y_2 \sin \varphi_2. \]  

(8f)

It may be noted that the following responses have been assumed for the rigid body and the two nodes to derive the system in Eq. (8): \(y = Y \sin \omega t\), \(y_1 = Y_1 \sin(\omega t - \varphi_1)\), \(y_2 = Y_2 \sin(\omega t - \varphi_2)\), for an input of \(f_y = F \sin(\omega t - \varphi)\). In Eq. (8), \(Y_1\) and \(Y_2\) are the displacement amplitudes of the two nodes respectively, and \(\varphi_1\) and \(\varphi_2\) are the phase angles associated with the motion of the two nodes respectively.

The MMV model with two horizontal stiffness elements along \(x\) and \(z\) axes is specifically used to incorporate the need for different stiffness properties along the non-isolating axes. Figure 2 can be used for reference with \(k_{hx} = k_h\) (also \(y_2 = 0\)). The governing EOM for this model are derived to be as follows:

\[ m\ddot{y} + k_0y + c_0\dot{y} + k_l(y - y_1) + 2k_{hx} \left( 1 - \frac{l_{0x}}{\sqrt{l_x^2 + y^2}} \right) y + 2k_{hz} \left( 1 - \frac{l_{0z}}{\sqrt{l_z^2 + y^2}} \right) y = f_y; \]  

(9)

\[ k_1(y - y_1) = c_1\dot{y}_1. \]  

(10)

In Eq. (9), \(k_{hx}\) and \(k_{hz}\) represent the stiffness elements along \(x\) and \(z\) axes, respectively. The free-length of the two springs is \(l_{0x}\) and \(l_{0z}\), and \(l_x\) and \(l_z\) are the compressed lengths of the two springs respectively at assembly. The rest of the variables in Eq. (9) and Eq. (10) are identical to the other MMV model in Fig. 2.
The following system of equations is derived for the MV model with two horizontal stiffness elements by using the HBM:

\[
-\omega^2 m\ddot{y} + k_0 \dot{y} + k_1 (y - y_1) + k_2 (y - y_2) = 0
\]

For this model, using the HBM for the EOM in Eqs. (12), (13) and (14) yields the following system of equations:

\[
-\omega^2 m\ddot{y} + k_0 \dot{y} + k_1 \dot{y} - k_1 Y_1 \cos \varphi_1 + k_2 \dot{y} - k_2 Y_2 \cos \varphi_2 = 0
\]

The system of equations in Eq. (15) needs to be solved for the displacement amplitude of the rigid body, \(Y\), and the displacement amplitudes of the nodes, \(Y_1\) and \(Y_2\). The corresponding phase angles, \(\varphi\), \(\varphi_1\), and \(\varphi_2\), associated with the input force and the displacement of the two nodes are also calculated from the system of equations in Eq. (15). A non-linear least-squares based method is used to solve the system of equations derived in this section for all the models. This method is primarily based on Newton’s method in conjunction with the Powell Dogleg procedure. This method is reported to be robust and capable of overcoming problems related to singularities and convergence.

For the MV model with horizontal spring elements along \(x\) and \(z\) axes, a base excitation of \(x = \dot{X} \sin(\omega t - \varphi)\) is used to compute the displacement transmissibility and the effect of the parameters associated with the horizontal stiffness elements. Substitution of base excitation and the use of the HBM results in the following system of equations for base excitation for this model:

\[
-\omega^2 m\ddot{y} + k_0 \dot{y} + k_1 \dot{y} - k_1 Y_1 \cos \varphi_1 + k_2 \dot{y} - k_2 Y_2 \cos \varphi_2 = 0
\]
Eq. (17) is solved for six unknowns — $Y$, $Y_1$, $Y_2$, $\phi$, $\phi_1$, and $\phi_2$ for a unit displacement amplitude of base excitation.

The models presented in this section have been used for multiple simulations in Section 3. All the variables associated with the model are identified by using load-deflection characteristics from a commercially available vibration isolator, shown in Figure 1a, that has been tested for this study.

3. RESULTS

The simulation results for the models of the proposed design are presented in this section. Test results from an elastomeric isolator are used to characterize the MV and MMV models along the isolating axis. The vibration isolator shown in Fig. 1a has been used for characterization. The load-deflection data is collected from a single-axis test for the elastomeric isolator at multiple frequencies, and an optimization program is used to identify the variables associated with each model. It may be noted that the variables associated with the horizontal spring elements have not been determined from model characterization. The variables for the MV model are as follows: $k_0 = 251.26$ N/mm, $c_0 = 3.23$ Ns/mm, $k_1 = 237.38$ N/mm, $c_1 = 121.92$ Ns/mm. The variables for the MMV model are found to be as follows: $k_0 = 251.26$ N/mm, $c_0 = 3.23$ Ns/mm, $k_1 = 237.38$ N/mm, $c_1 = 121.92$ Ns/mm, $k_2 = 180.59$ N/mm, $c_2 = 1.89$ Ns/mm. A mass of 125 kg is used for the single DOF system and a free length of 50 mm is used for the horizontal spring elements in all the simulations in this section. The three variables associated with the horizontal spring element — stiffness, free length and pre-compression — have been varied in order to understand the influence of these variables on the frequency response as well as the time response.

Figure 3 shows the frequency response of the MV model with one horizontal spring element at varying levels of horizontal stiffness ($k_h$) for 10% pre-compression. It may be noted that the output is derived for a sinusoidal input with unit amplitude. Also, $k_h = 0$ corresponds to a conventional Maxwell-Voigt model without any horizontal spring elements.

Increasing horizontal stiffness is seen to result in a slight reduction in the natural frequency, but the response is seen to increase at lower frequencies without significantly influencing the response at higher frequencies. Also, the peak response shows an increasing trend with an increase in stiffness. However, the response at lower frequencies does not show a trend. Adding a horizontal spring element along another axis to the MV model exhibits similar results for 10% pre-compression, leading to a reduction in the natural frequency. However, the shift in the natural frequency is seen to be accentuated and the frequency response is seen to significantly reduce at lower frequencies for a substantial increase in stiffness. This can be seen from Fig. 4 for the response of the MV model with two horizontal spring elements along the two non-isolating axes, it can be seen that the response is reduced through the entire frequency range for $k_h = 8k_0$. It may be noted that $k_h$ indicates the stiffness of the springs in both directions ($x$ and $z$), even though the model is capable of accommodating different stiffness along the two non-isolating axes.

Pre-compression of the horizontal spring elements is seen to significantly influence the frequency response, this can be seen from the response in Fig. 5 and Fig. 6. In Fig. 5, increase in stiffness is seen to significantly reduce the response through the entire frequency range at 40% pre-compression of the two
horizontal spring elements, this can be directly compared to the results in Fig. 4. This phenomenon can be further observed in Fig. 6 as the pre-compression is changed from 10% to 40% at a constant level of horizontal stiffness for the MV model. Overall, the results for the MV model indicate that an incorporation of horizontal stiffness and pre-compression of the horizontal spring elements can be successfully used to adjust the natural frequency of the system and control the response amplitude with limited trade-offs. Particularly, significant benefits are observed at high levels of stiffness in conjunction with high levels of pre-compression.

The MMV model exhibits characteristics that are similar to the observations from the response of the MV model. The MMV model exhibits a significant reduction in response through the entire frequency range with the increasing stiffness of the horizontal elements in conjunction with a relatively higher pre-compression. One such result for the MMV model can be seen in Fig. 7 and Fig. 8 with the amplitude and phase response at 40% pre-compression. The amplitude response is seen to reduce through the entire frequency range for $k_h > 2k_0$, as seen in Fig. 7.

Another result for the MMV model can be seen in Fig. 9 and Fig. 10 for a pre-compression of 40% with two horizontal spring elements, the response is seen to decrease with increasing stiffness for all levels of horizontal stiffness with limited trade-offs at lower frequencies. The results for the MV and MMV models are similar to the results reported for the Voigt model in the existing literature. Figure 10 shows the phase angle associated with the response shown in Fig. 9. The phase angle for $k_h = 0$ is similar to a damped system with a very low damping ratio, but the phase angle remains lower than 20 deg. through the entire frequency range for all non-zero values of horizontal stiffness. This indicates that the change in the frequency response resulting from increasing horizontal stiffness or pre-compression is not analogous to an increase in the damping ratio. Instead, the parameters of the horizontal spring elements significantly reduce the phase lag between excitation input and the system response, as seen in Fig. 10.

Time response of the models has been investigated by using a quasi-constant step implementation of the backward difference method. In the algorithm, the time step is reduced only if convergence is not achieved initially and the Jacobian is updated if the problem is found to be significantly stiff. The time response has been used to compare the results of the
HBM with numerical integration and to determine the step response for the models discussed in this paper. The step response of the MV model with one horizontal spring element at 10% pre-compression is shown in Fig. 11. An increase in horizontal stiffness is seen to result in an increase in the amplitude of the step response. A similar step response is exhibited by the MMV model with one or two horizontal spring elements. Figure 12 shows the transient force transmitted by the isolator due to a step input acting on the rigid body supported by the isolator. The transmitted force in the transient response is seen to reduce with an increase in horizontal stiffness. All MV and MMV models investigated in this study are seen to exhibit similar trends for transmitted force due to a step input. The trends of the results from the time response correspond to the results seen from the frequency response. This can be observed by comparing the results from Fig. 3 and Fig. 11. For instance, the increase in rise time, as defined by the time taken for the response to reach 90% of the steady state value, corresponds to a decrease in the natural frequency, as seen from Fig. 3 for increasing horizontal stiffness at the same level of pre-compression. The numerical solution was not able to converge to a solution for higher levels of stiffness, this needs to be investigated further.

The models proposed in this study have also been investigated for base excitation. It can be seen from the results in Fig. 13 and Fig. 14 that an increasing stiffness of the horizontal spring elements results in shifting the peak transmissibility to a lower frequency, and this shift is accompanied by an increase in displacement transmissibility at lower frequencies. However, a substantial increase in stiffness \( (k_h = 8k_0) \) results in a reduction in displacement transmissibility through the entire frequency range. This is consistent with the results derived from the frequency response.

In order to compare the capability of the models discussed in this study with a commonly used model, the displacement transmissibility of the MV model with two horizontal spring elements at 10% pre-compression is compared with the results from a Voigt model (one spring and one damper element in parallel along the isolating axis). The results from this comparison are shown in Fig. 15. The advantages of the model proposed in this study are expected to be similar to the quasi-zero-stiffness
(QZS) mechanism that has been investigated in the literature. Direct numerical comparisons with the QZS mechanism have not been performed in this study since the results for both the models depend on a large number of variables. However, the trends exhibited by the models discussed in this section are similar to the QZS models, this includes the ability of the isolation system to mitigate the response over a larger frequency range and the ability of the isolation system to reduce the response at relatively lower frequencies.

As can be seen from the results in Fig. 15, the MV model with horizontal spring elements along $x$ and $z$ axes is able to significantly mitigate the transmissibility at resonance and at lower frequencies, as compared to the Voigt model. However, it is important to note that these results may vary with the choice of variables associated with the MV model. In general, the simulation results have pinpointed the use of pre-compression and stiffness of the horizontal spring elements as important variables that can be used in the design of the isolation system to reduce the response at relatively lower frequencies.

As a follow up to this study, the coupled influence of stiffness and damping nonlinearities will be investigated in the future for the models analyzed in this paper. The numerical solution will be investigated further to compute the time response at higher levels of stiffness and to comprehend whether the assumptions associated with the HBM are appropriate for all the configurations investigated in this study. The output frequency response function (OFRF) approach will be used to account for the influence of higher harmonics in future work. The models discussed in this paper will also be numerically compared to other similar models in the literature such as the QZS model.

4. CONCLUSIONS

In this paper, the effect of stiffness nonlinearity has been investigated by incorporating stiffness elements along non-isolating axes into the MV and MMV models for a vibration isolator. An alternative design of a vibration isolator is investigated in this study with significantly different stiffness properties along multiple axes of the isolator. The main advantage of this design is an ability to control the frequency response over a relatively larger frequency range. Such a design could mitigate some of the trade-offs typically associated with the design of a passive isolator. This design can also be used to accommodate multiple performance constraints posed on an isolation system while requiring the system to effectively mitigate vibration response. These performance constraints are important in applications such as motorcycles where the stiffness requirements of the isolation system are significantly different along the non-isolating axes due to handling and packaging requirements. A drawback of the proposed design is a significant enhancement of design complexity and related challenges associated with manufacturing the vibration isolator.

Results indicate that the incorporation of stiffness nonlinearity, as investigated in this study, can be useful in enhancing vibration isolation characteristics of a passive isolator while allowing the design to meet other performance criteria that the isolation system may be required to satisfy. Specifically, the stiffness nonlinearity is seen to significantly reduce the response amplitude at lower frequencies with limited trade-offs for relatively higher frequencies. The stiffness of the horizontal spring elements in conjunction with the level of pre-compression are found to be critical in controlling the frequency response, time response as well as displacement transmissibility. The simulation results do not vary much between the MV and the MMV models, and the MMV model is not seen to exhibit any specific advantages for the models investigated in this study.

As a follow up to this study, the coupled influence of stiffness and damping nonlinearities will be investigated in the future for the models analyzed in this paper. The numerical solution will be investigated further to compute the time response at higher levels of stiffness and to comprehend whether the assumptions associated with the HBM are appropriate for all the configurations investigated in this study. The output frequency response function (OFRF) approach will be used to account for the influence of higher harmonics in future work. The models discussed in this paper will also be numerically compared to other similar models in the literature such as the QZS model.

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Rolling Bearing Fault Trend Prediction Based on Composite Weighted KELM

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(Received 28 August 2017; accepted 11 December 2017)

The abilities of different degradation feature types to characterize rolling bearing fault trend are distinctive. And even the characteristic ability of the same degradation feature can change at various times. Thus these feature samples possess heteroscedasticity. However, traditional kernel extreme learning machine (KELM) model assumes that different input samples’ effects on the predicted value are equal, which results in low prediction precision and low computing efficiency. To solve this problem, a novel composite weighted KELM (CWKELM) prediction model, which is fused with explicit weighting and implicit weighting, is proposed. In both feature type scale and sample time scale, the feature samples and the prediction model are weighted according to the prediction error. An adaptive mutation particle swarm optimization (AMPSO) algorithm is applied in optimizing the penalty factor and the kernel parameter in the model. Taking various entropy features as the input samples, the proposed model is adopted to conduct one-step and multi-step prediction for rolling bearing fault trend. Experimental results show that this prediction model has higher prediction accuracy and computing efficiency compared with the traditional KELM model.

1. INTRODUCTION

Rolling bearing is one of the key components that are widely used and affect the health status of rotating machinery. In order to prevent bearings and equipment from failure or damage, it’s of great safety significance and economic value to carry out rolling bearing fault trend prediction. The key of rolling bearing fault trend prediction is to extract accurate degradation characteristics and establish a good predictive model.

On the one hand, as the input sample of the prediction model, the degradation characteristics need to be sensitive and robust to the degradation state of the rolling bearings in the whole life cycle. The nonlinear complexity characteristics based on the information entropy theory can measure the probability distribution difference of variable bearing vibration signals in different degradation stages from multiple perspectives. And these degradation characteristics can reveal the developing trend of the ball bearing degradation state in essence. Therefore, entropy characteristics, including multi-scale entropy, energy spectrum entropy, singular spectrum entropy and spatial information entropy etc. have been widely utilized in rotating machinery fault diagnosis and prediction. These characteristics are combined with time-frequency analysis methods and perform well in fault diagnosis and degradation state identification. Zhang et al proposed a novel hybrid bearing fault classification method based on permutation entropy (PE) and ensemble empirical mode decomposition (EEMD) which can calculate the multi-scale intrinsic characteristics as the fault classification features. In order to extract accurate fault features from vibration signals, Li et al employed multi-scale permutation entropy (MPE) to characterize the complexity of the product function (PF) components which are computed by local mean decomposition (LMD). Zheng et al put forward a new and effective bearing fault diagnosis methodology based on fuzzy entropy (FuzzyEn) and a self-adaptive time-frequency analysis method named local characteristic-scale decomposition (LCD) which deals with rolling bearing vibration signals. In summary, the information entropy features are mostly applied in rolling bearing fault classification and diagnosis, and the effect is significant. However, there are few applications of entropy features in the characterization and prediction of the whole degradation life of rolling bearings. This is mainly due to the degradation state of rolling bearings changing at every moment in their full life cycles. Bearing vibration signals have a large amount of non-stationary and unbalanced data that are difficult to process. When dealing with degradation data during the bearings’ whole life cycles, there are wide performance variations of different single entropy features on both type scale and time scale and it can very likely result in a large prediction error. Therefore, it’s better to carry out an effective weighted fusion pro-
cessing of different entropy features.

On the other hand, the current fault prediction theories are mainly divided into three categories including physical prediction model, data-driven prediction methodology and fusion prediction methodology.\textsuperscript{10} Among them, physical prediction models utilize mathematical models such as damage rules to describe failure mechanism and failure modes. However, these damage rules are either linear or multi-linear and that implies physical models fail to meet the mechanical failure prediction requirements. Moreover, physical prediction models rely heavily on the analysis of physical mathematical model which is difficult to establish for rotating machinery with complex structure and multiple fault modes. Therefore, the physics-based prediction approach makes the prognosis a risky task. With the rapid development of computer science and artificial intelligence (AI), the data-driven prediction methodologies have covered a great number of new technologies and AI algorithms such as time series prediction model, particle filtering, regression analysis, hidden Markov model (HMM), artificial neural network (ANN), support vector machine (SVM) and extreme learning machine (ELM) etc.\textsuperscript{11} The main idea of the date-driven prediction methodology is to employ data from past operations and current bearing conditions in order to predict the bearing fault trend or even forecast the remaining useful life (RUL).\textsuperscript{12} It is interesting to find out that there are both connections and differences among ANN, SVM and ELM. And these three kinds of methodologies have been widely studied and utilized in the area of mechanical prognostics.\textsuperscript{13–15} ELM model is a learning machine based on single hidden layer feedforward neural networks which is proposed by Huang G.B. When using this model for predicting, the only thing you should do is to set the node number of hidden layer neurons, and the input layer weight and the implicit layer deviations are randomly generated. Furthermore, the weight of the output layer can be obtained, and the whole process is completed through one training without iteration. Therefore, this model has a faster computing speed compared with ANN model. However, due to the random generation of some parameters of the model, the generalization ability is insufficient. Meanwhile, the prediction performance is susceptible to parameters such as the number of implicit nodes, and the stability is relatively poor as a result. Huang et al in 2012 borrowed the kernel mapping from SVM model to replace the random mapping in ELM model and put forward the concept of kernel extreme learning machine (KELM) to improve the deficiency of the original ELM model.\textsuperscript{16} And the proposed new model has been widely studied and utilized in time series prediction in the past five years, and its prediction ability is stronger than SVM.\textsuperscript{17,18} However, there are few application cases of KELM model in terms of rolling bearing condition monitoring or fault prognostics. The main reason is that the rolling bearing monitoring data has strong nonlinear and non-stationary characteristics and is imbalanced in both date type scale and time changing scale. Han and Wang\textsuperscript{19} indicate that the KELM model ignores that different prediction window sample points have a different influence upon the prediction performance and these sample points exhibit heteroscedasticity which can be harmful to the fault prognostics. Furthermore, Han and Wang\textsuperscript{19} point out that it’s necessary to assign different weights to the sample points in time scale to improve the prediction accuracy. This significant standpoint shows us a new road to the higher prediction accuracy.

Based on the above analysis, this paper proposes a novel bearing fault trend prediction model named composite weighted KELM (CWKELM). On the basis of the KELM structure, the training input samples come from three kinds of entropy features and these features are explicit weighted in the feature fusion layer. And the model structure is implicit weighted in the model optimization layer at the same time. In the proposed model, the prediction error of CWKELM is reduced by balancing the data structure in both feature type scale and time scale. Further more, explicit weighting can reduce the input samples’ dimension and the calculation efficiency of the prediction model can be improved. An adaptive mutation particle swarm optimization (AMPSO) algorithm is employed to optimize the key parameters such as penalty factor and kernel parameter in the model in order to improve the prediction accuracy further. The optimized CEKELM model is then applied in bearing fault trend prediction based on bearing vibration monitoring signals. Several kinds of information entropy features extracted from vibration signals are taken as the input samples and the root mean square (RMS) which monitors the bearing’s whole life from normal state to failure is taken as the predicted target value. After these steps, single step and multi-step prediction of bearing fault trend is conducted at last. The advantages of this model are verified in both predicting accuracy and computational efficiency compared with traditional models such as ELM, KELM, SVM and BP neural networks.

The rest of this paper is organized as follows. The basic principles of CWKELM model are explained in Section 2. In detail, Section 2.1. presents the process of the entropy feature extraction and the explicit weighting. Later, the KELM methodology and the implicit weighting are exhibited in Section 2.2. Furthermore, Section 2.3. gives the specific steps of CWKELM for rolling bearing fault trend prediction. Section 3 describes the bearing fatigue life test and explains the experimental data used in this paper. Section 4 shows how the model parameters are optimized with AMPSO algorithm. The predicted results with the novel model and other methodologies are discussed and analysed in Section 5. And the conclusion is reached in Section 6.

2. PRINCIPLES OF CWKELM

2.1. Entropy Feature Extraction and Explicit Weighted Fusion

The first step of forecasting the rolling bearing degradation state is to extract different kinds of information entropy characteristics as the input samples of the model from vibration signals during bearings’ whole life. Of these entropy characteristics, multi-scale entropy (MSE), energy entropy, and singular entropy have a better performance and have been applied in many cases. The calculation process of MSE is detailed by Li et al.\textsuperscript{20} The fault developing trend of rolling bearings can be better characterized by describing how far the rolling bearing monitoring state deviates from the normal state with relative entropy theory. Therefore, relative energy entropy (REE) and relative singular entropy (RSE) are defined on the basis of energy spectrum entropy and singular spectrum entropy as follows:

One single set of vibration data at rolling bearing normal
state $x_0(t)(t = 0, 1, 2, \ldots, N - 1)$ and $M$ sets of vibration data at rolling bearing degradation state $x_i(t)(i = 1, 2, \ldots, M, t = 0, 1, 2, \ldots, N - 1)$ are selected and then time-frequency spectrum $L_0(t, f)$ and $L_i(t, f)$ are computed by time-frequency analysis. And these two spectrums’ energies at characteristic frequency $f_g$ are respectively $E^0_g$ and $E^i_g$ calculated as follows:

$$E^0_g = \sum_{t=0}^{N-1} L_0(t, f_g)^2; \quad (1)$$

$$E^i_g = \sum_{t=0}^{N-1} L_i(t, f_g)^2; \quad (2)$$

where $g = 1, 2, \ldots, G$ represents the frequency characteristics of fault. Based on this, the energy spectrum of samples at normal state and samples at fault state can be represented as $E_0 = [E^0_1, E^0_2, \ldots, E^0_G]$ and $E_i = [E^i_1, E^i_2, \ldots, E^i_G]$.

The total energy at characteristic frequency of all the samples can be computed by the following formula:

$$E^g = \sum_{i=1}^{M} E^i_g + E^0_g; \quad (3)$$

The proportions of the energy of $x_0(t)$ and $x_i(t)$ at characteristic frequency $f_g$ accounting for the total energy are respectively as below:

$$p^0_g = \frac{E^0_g}{E^g}; \quad (4)$$

$$p^i_g = \frac{E^i_g}{E^g}; \quad (5)$$

where $p^0_g + \sum_{i=1}^{M} p^i_g = 1$.

According to the theory of relative entropy, REE between the fault sample set and the normal sample set is calculated as follows:

$$REE^g = \sum_{g=1}^{G} p^g_0 \log \frac{p^g_0}{p^g}; \quad (6)$$

The time-frequency spectrum $L_0(t, f)$ of normal state and $L_i(t, f)$ of fault state are processed by singular value decomposition (SVD). The singular value spectrum of normal samples is $\sigma_0 = [\sigma_0^1, \sigma_0^2, \ldots, \sigma_0^J]$ and the single value spectrum of fault samples is $\sigma_i = [\sigma_i^1, \sigma_i^2, \ldots, \sigma_i^J]$ where $J$ is the order of the diagonal matrix in the decomposition of singular values.

Combined with singular spectrum entropy theory, two corresponding probabilities are defined as below:

$$q^i_j = \frac{\sigma^i_j}{\sigma^0_L}; \quad (7)$$

$$q^0_j = \frac{\sigma^0_j}{\sigma^0_L}; \quad (8)$$

where $1 \leq j \leq J, \sigma^0_L = \sum_{i=1}^{M} \sigma^i_1 + \sigma^0_1$.

And RSE between the fault sample set and the normal sample set is calculated as follows:

$$RSE^g = \sum_{j=1}^{J} q^j_0 \log \frac{q^j_0}{q^j}; \quad (9)$$

The above information entropy features can form a three-dimensional eigenvector: $X_1 = [\text{MSE REE RSE}]$. Three kinds of information entropy features measure the complexity changing of vibration signals from different angles and there is much difference in abilities of characterizing the bearing fault trend. It will result in large prediction error that $\lambda_1$ is taken as the input samples of KELM directly. Therefore, in the stage of model training, each entropy feature’s prediction error for bearing operating states is evaluated respectively and conduct explicit weighting on every entropy feature. The weighted input samples are more suitable for prediction model training and fault prognostics. Relative Root Mean Square Error (RRMSE) and Correlation Coefficient of Trend Change $\alpha$ are two different kinds of indicators commonly used in evaluating the forecasting results. The actual feature sequence is set as $Y = \{y_j \mid j = 1, 2, \ldots, n\}$ and the predicted feature sequence is written as $Y_1 = \{\hat{y}_j \mid j = 1, 2, \ldots, n\}$. The formulas of RRMSE and $\alpha$ are as follows:

$$RRMSE = \sqrt{\frac{1}{n} \sum_{j=1}^{n} \left( \frac{y_j - \hat{y}_j}{y_j} \right)^2} \times 100\%; \quad (10)$$

$$\alpha = \frac{\sum_{j=1}^{n-1} [(y_{j+1} - y_j)(\hat{y}_{j+1} - \hat{y}_j)]}{\sqrt{\sum_{j=1}^{n-1} [(y_{j+1} - y_j)^2(\hat{y}_{j+1} - \hat{y}_j)^2]}}. \quad (11)$$

RRMSE indicates the deviation and $\alpha$ reflects the consistency between the predicted values and the actual values respectively. Based on this, the weight evaluation factor is defined as follows:

$$F = \alpha / RRMSE. \quad (12)$$

The merits of single entropy feature prediction are evaluated by calculating the weight evaluation factors of single feature prediction results. And finally the input predicting features of the prediction model is expressed as the following formula.

$$X_2 = [\text{MSE REE RSE}] [F_{\text{mseg}} F_{\text{ree}} F_{\text{rse}}]^T. \quad (13)$$

### 2.2. KELM and Implicit Weighting

The ELM model is a kind of single hidden layer feedforward neural networks and its output function can be represented as

$$f(x) = \sum_{i=1}^{L} \beta_i h_i(x) = h(x)\beta. \quad (14)$$

In order to guarantee the minimum prediction error of output values, the corresponding objective function equation is set as follows:

$$\min \| f(x) - f_0(x) \|$$

$$= \min \| \sum_{i=1}^{L} \beta_i h_i(x) - f_0(x) \| = 0. \quad (15)$$

In the above equations, $L$ is the number of neurons in the hidden layer. $h(x)$ is the random mapping in the hidden layer and $f_0(x)$ is the predictive function value. When there are $N$ inputs named $x_i(i = 1, 2, L, \ldots, N)$, the matrix form of Eq. 14 can be represented as follows:

$$f(x) = H\beta; \quad (16)$$

where $H$ is the random mapping matrix in the hidden layer. And matrix $H$ can be denoted by $h(x)$ as follows:

$$H = \begin{bmatrix} h(x_1) \\ h(x_2) \\ \vdots \\ h(x_N) \end{bmatrix}. \quad (17)$$
The output weight vector $\beta$ can be calculated by the least square method as the following equation:

$$\beta = H^+ O = H^T (H H^T)^{-1} O$$

$$= H^T \left( \frac{I}{C} + H H^T \right)^{-1} O; \quad (18)$$

where $H^+$ represents the Moore-Penrose generalized inverse matrix of $H$ and $O$ represents the predictive expectation matrix. $C$ is the penalty factor that is generally positive and this parameter can improve the generalization ability of the model. And the output function of the model can be indicated as follows:

$$f(x) = H \beta = H H^T \left( \frac{I}{C} + H H^T \right)^{-1} O. \quad (19)$$

However, unlike ELM, KELM model uses the kernel mapping in the hidden layer instead of random mapping in ELM, which can be shown as the following formula 17.

$$\Omega_{ELM} = H H^T : \Omega_{ELM, i,j} = h(x_i) \cdot h(x_j) = K(x_i, x_j); \quad (20)$$

where $K(x_i, x_j)$ is a kind of kernel function, which is always selected as RBF kernel function. The general form is given as follows:

$$K(x_i, x_j) = \exp \left( - \frac{\| x_i - x_j \|^2}{2 \sigma^2} \right); \quad (21)$$

where $\sigma$ is the kernel parameter. And the output function of KELM model can be represented by the following equation.

$$f(x) = \left[ \begin{array}{c} K(x, x_1) \\ M \\ K(x, x_N) \end{array} \right]^T \left( I/C + \Omega_{ELM} \right)^{-1} O. \quad (22)$$

On the time scale, the predictive ability of the models input samples is different. That is to say, the closer the prediction points are, the greater the impact of training samples on prediction results will be. Therefore, there is necessity conducting implicit weighting on the input samples of KELM model on the time scale. Concretely, weighting coefficients are used to weigh the prediction error variables of the model, and then the objective function 15 can be rewritten as follows:

$$\min P(\beta, \xi) = \frac{1}{2} \beta^T \beta + \frac{1}{2} \sum_{i=1}^{N} \nu_i \xi_i^2; \quad (23)$$

s.t. $h(x_i) \beta = a_i - \xi_i, i = 1, L, \ldots, N;$

where $P(\beta, \xi)$ is the cost function of the prediction model. $x_i$ is the value of the input samples. $a_i$ is the prediction expectation value. $\xi_i$ is the error variance. $N$ is the number of training samples. Combined with Eq. 17 and Eq. 18, the output function of the implicit weighted KELM model is shown as follows.

$$f(x) = \left[ \begin{array}{c} K(x, x_1) \\ M \\ K(x, x_N) \end{array} \right]^T \left( V_C + \Omega_{ELM} \right)^{-1} O; \quad (24)$$

where $V_C = \text{diag} \left\{ \frac{1}{C_{11}}, \ldots, L, \frac{1}{C_{NN}} \right\}$ and the weighting coefficient is $\nu_k = \sqrt{\frac{\sum_{i=1}^{N} \xi_i^2}{\xi_k}}.$

### 3. FATIGUE LIFE TEST FOR ROLLING BEARINGS

In this paper, the fatigue test data of rolling bearings that are analysed by the methodology mentioned above are gathered in Bearing Test Research Center, Hangzhou, China. As shown in Fig. 2, the experimental platform is mainly composed of three parts, including ABLT-1A bearing test machine, signal acquisition module and status monitoring module. Figure 3 is the system diagram. Four CA-YD-139 acceleration sensors are arranged in the four bearing test stations, and connected with the DH-5920 dynamic signal acquisition instrument. In this way, four sets of rolling bearings can be tested at one time and multiple sets of full life vibration data can be stored in the end. Meanwhile, four thermal resistances on stations and one YD-1 acceleration sensor is connected with one signal amplifier to monitor the operation indexes such as temperature, kurtosis and RMS. When these indexes exceed the alarm threshold, the test machine will stop working.

The single row deep groove ball bearing 6204, which is commonly used in mechanical equipment, is taken as the test object of the life test, and it is shown in Fig. 4(a). During the test, the motor speed is 1500 r/min, and the vibration signals are sampled every 10 minutes. The sampling time is 1 s, and the sampling frequency is 25.6 kHz. When the test runs to 9600 minutes, the kurtosis index is over eight while it is about three at normal state. The rolling bearing has a serious failure and the test machine stops. After the shutdown and examination, the bearing of No.4 Station fails due to the inner ring erosion as is shown in Fig. 4(b) and 960 groups of rolling bearing vibration data is collected.

The change curve of Root Mean Square (RMS) over time is shown in Fig. 5. This monitoring index monitors and records the whole process of 6204 bearing from normal state to failure.
state and has a good ability to follow and reflect the fault developing trend of the bearing. Therefore, predicting the fault trend of the rolling bearing can be equivalent to predicting the RMS monitoring curve. According to the change of curve of time, the fault development of rolling bearing over time can be divided into four stages: (1) normal state: 0–6820 min; (2) slight degradation state: 6830–8450 min; (3) serious fault state: 8460–9330 min; (4) failure state: 9340–9600 min. Liu et al. 22 point out that the second and the third state which are referred as “soft failure” account for most of mechanical failures. Considering that there is some regularity in time sequence of the period, we take 6840–9330 min as the analysis interval of the prediction model where the training interval is 6840–8830 min, 200 sets of data in total; the test range is 8840–9330 min, 50 sets of data in total.

![Figure 1. Flow chart of the methodology.](image1)

Figure 1. Flow chart of the methodology.

![Figure 2. Bearing test platform.](image2)

Figure 2. Bearing test platform.

![Figure 3. Structure of the test system.](image3)

Figure 3. Structure of the test system.

4. OPTIMIZATION OF THE MODEL PARAMETERS

Similar to the support vector machine (SVM) model, the penalty factor \( C \) in KELM model is mainly used to balance the complexity of the model and the empirical risk value to improve the generalization performance of the model. In order to control the model complexity, the factor \( C \) is always relatively small but cannot be too small, avoiding experience error. 22 When the kernel parameter is too large, the response velocity of the model can be slow and the ability to adjust is poor. While the kernel parameter \( \sigma \) is too small, the model can be too sensitive and this can cause a great error too. Therefore, it’s necessary to optimize the penalty factor \( C \) and the kernel parameter \( \sigma \). At present, commonly used optimization algorithms include Grid Search Algorithm, Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) algorithm.

Compared with the Grid Search Algorithm and GA, PSO algorithm converges fast and has strong universality but its search precision is poor and it is easy to get into local optimal. However, GA has relatively high precision despite relatively complex calculation and low iteration efficiency. Therefore,
the adaptive mutation particle swarm optimization (AMPSO) algorithm is proposed to be applied to \( C \) and \( \sigma \) optimization. The main idea of this algorithm is to add the variation operation to the particle variable and initialize the population to the optimal solution. The training period data is used for the single step prediction analysis. The number of particle group dimension is set to 2, and the number of particles is set to 20, and the number of iterations is limited to 100. \( C \in [0.1, 1000] \), \( \sigma \in [0.01, 100] \). The fitness objective function is the weight evaluation factor \( F \). Parameter setting and specific steps of the algorithm can be seen in Qu et al.\textsuperscript{23} and Zhang.\textsuperscript{24} At the same time, GA and PSO algorithm are used for comparison and further analysis. The detail specification of the calculating device is given as below: CPU is Intel\textsuperscript{®} Core\textsuperscript{TM} i5-4590 CPU@3.30GHz, RAM is 4.00GB, and the operating system (OS) is 32-bit Windows 7. With this calculating device, the converge curves are shown in Fig. 6. The optimization results are shown in Table 1.

The AMPSO algorithm achieves the optimal value of 108.5, which is basically consistent with the optimal solution of GA, only through eighteen iterations. The computational efficiency of AMPSO is higher than GA. The PSO algorithm, which doesn’t add the variation operation, is in the local optimum after the 11\textsuperscript{th} iteration, and the optimization result is not ideal. Finally the model parameters are selected as \( C = 9.8 \), \( \sigma = 1.02 \).

5. THE ANALYSIS AND COMPARISON OF FAULT TREND PREDICTION

There are 200 sets of data in the training period that are used to train the prediction model. The information entropy features are weighted by the one-step prediction weighting evaluation factor. Firstly, the information entropy characteristics including MSE, REE and RSE are extracted from the vibration signals, and then three groups of feature samples are respectively used as the model input to predict the changing curve of RMS over time. The prediction results are shown in Figure 7, 8, 9. The predicted values which are calculated from MSE, REE and RSE deviate from the real values more or less in the second half of the training period. There are relatively more error points in the predicted curve which is calculated from MSE. However, the error points are fewer in the predicted curve which is calculated from RSE. The three information entropy features’ abilities to predict the RMS curve differ from each other and the training results need further quantitative analysis.

The RRMSE and Correlation Coefficient of Trend Change \( \alpha \) of the above mentioned one-step prediction training results are respectively calculated and the weight evaluation factor \( F \) is computed according to this. The results are shown in Table 2.

Among the three features, the correlation coefficient of RSE is the largest, and the RRMSE of RSE is similar to that of MSE. The weight evaluation factor of RSE is the largest, and that indicates the predictive ability of RSE is the best. This result is consistent with the prior qualitative analysis. Meanwhile, the indexes of RSE and REE are close to each other and all of them are within reason. That means it’s reasonable and effective to utilize information entropy features for rolling bearing fault prognostics. On the other hand, in order to utilize the comprehensive entropy information in the samples, explicit weighting is conducted on the input samples of KELM model according to Eq. 13. The fusion feature \( X_2 \) is used as a new input characteristic to train KELM model. The training result is shown in Fig. 10.

After the explicit weighting processing, the predicted residuals in the middle and posterior segment of the single information feature are effectively cut down and the predicted error is further reduced. In order to illustrate the effectiveness of the
Table 1. Comparison of the optimization results.

<table>
<thead>
<tr>
<th>Optimization Algorithm</th>
<th>Parameter C</th>
<th>Parameter σ</th>
<th>Target Value F</th>
<th>Iteration Number</th>
<th>Computation Time/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMPSO</td>
<td>9.8</td>
<td>1.02</td>
<td>108.5</td>
<td>18</td>
<td>12.235</td>
</tr>
<tr>
<td>GA</td>
<td>10.5</td>
<td>1.12</td>
<td>109</td>
<td>23</td>
<td>15.363</td>
</tr>
<tr>
<td>PSO</td>
<td>5.7</td>
<td>0.84</td>
<td>85.3</td>
<td>11</td>
<td>9.275</td>
</tr>
</tbody>
</table>

Table 2. The evaluation indexes of single entropy feature.

<table>
<thead>
<tr>
<th>Information Entropy Features</th>
<th>Correlation Coefficient</th>
<th>RRMSE</th>
<th>Weight Evaluation Factor (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE</td>
<td>6.4430</td>
<td>7.54</td>
<td>85.4152</td>
</tr>
<tr>
<td>REE</td>
<td>6.7806</td>
<td>7.77</td>
<td>87.3206</td>
</tr>
<tr>
<td>RSE</td>
<td>7.4317</td>
<td>7.58</td>
<td>97.9829</td>
</tr>
</tbody>
</table>

Table 3. The comparison of fusion processings.

<table>
<thead>
<tr>
<th>Fusion Processing</th>
<th>Correlation Coefficient</th>
<th>RRMSE</th>
<th>Weight Evaluation Factor (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multi inputs (without processing)</td>
<td>7.0093</td>
<td>6.40</td>
<td>109.5788</td>
</tr>
<tr>
<td>Explicit weighting</td>
<td>7.6351</td>
<td>3.15</td>
<td>242.6733</td>
</tr>
<tr>
<td>Composite weighting</td>
<td>8.3976</td>
<td>1.68</td>
<td>501.3071</td>
</tr>
</tbody>
</table>

Figure 10. The explicit weighted training result of one-step forecast.

Figure 11. The multi-input training result of one-step forecast.

Figure 12. The composite weighted training result of one-step forecast.

Figure 13. CWKELM test result of one-step forecast.

explicit weighting, the three single entropy features which are not weighted are used as the KELM input samples for training and the training time is 0.2027s. However, the weighting fusion features training time is only 0.0913s. The one-step forecast result of the multi input features which are not weighted is shown in Fig. 11. Compared with this result, the predicted error of the explicit weighted training result in Fig. 10 is relatively smaller, and that demonstrates the effectiveness of the explicit training.

On the basis of explicit training, implicit weighting is conducted on the characteristic sequence $X_2$ on the time scale in KELM model and the result is shown in Fig. 12 below. Within the training interval, the predicted values are consistent with the true values and the error is relatively small. The further comparative analysis can be seen in Table 3.

By the comparative analysis of Table 2, Table 3 and Figure 12, we can conclude that the prediction accuracy is significantly improved after the composite weighting in KELM and training samples. The proposed model’s prediction result has a high consistency with real value in the range of one-step forecast and the prediction curve fits the monitoring curve well in the training stage. The CWKELM prediction model is used to predict 50 groups of data in the test period, as is shown in Fig.13. The KELM multi-input prediction model is adopted for comparison and the result is shown in Fig. 14. By comparing Fig. 13 and Fig. 14, we find that the prediction effect of CWKELM for rolling bearing fault trend is better than KELM. In the middle and posterior section of the test interval, the predicted curve is basically consistent with the true curve and there is only a small margin of deviation in the initial phase of the test interval. In comparison, the prediction error of KELM model is relatively large and the KELM model is sensitive to the change of the data samples. And this results that small amplitude of data fluctuation can cause a large prediction error. Therefore, the CWKELM prediction model has some engineering significance in balancing the data structure and reducing the prediction error.

In order to further illustrate the robustness and practical value of CWKELM model in rolling bearing fault trend prediction, a certain length of data is randomly selected as the training sample and ELM, KELM and CWKELM model are adopted to the one-step, five-step, ten-step, fifteen-step and
twenty-step prediction of the normalized RMS. The result is shown in Fig. 15.

From Fig. 15, the relative errors of the three models have increased in different degrees with the improvement of the number of prediction step. Compared with the ELM and KELM models, the relative error of CWKELM model is relatively small, and it is always below 10% and the performance of CWKELM model is better. While the relative error of ELM model and KELM model is over 15% when the number of prediction step reaches 20. The reliability of the predicted results is significantly reduced.

As a special kind of single hidden layer feedforward neural network, KELM combines the design concept of ANN and the kernel learning methodology of SVM, and has been widely applied in the field of fault diagnosis and prognostics. CWKELM is a kind of advanced KELM model. In order to illustrate the advantages of CWKELM in rolling bearing fault trend prediction, SVM regression prediction model and the typical back propagation (BP) neural network are utilized for comparison and the results are shown in Table 4.

Table 4 shows that the RRMSE of CWKELM model is relatively smaller than that of SVM and BP model and the fault trend changing correlation coefficient between the predicted results and the real values is larger. This result can demonstrate that the CWKELM model can better achieve the goal of rolling bearing fault trend prediction. In terms of computational efficiency, CWKELM prediction model has a shorter calculation time. The main reason is that the composite weighting changes multi inputs into single fusion inputs and the structure is simple, so the computation speed is faster. However, there are always multiclass sample inputs in the SVM model and BP neural network and the computation process is complex and needs more time. In the control experiment, BP neural network takes empirical risk minimization as its principal and need a large number of training samples to train. However, due to the limited number of samples in the study, there will be a large error. CWKELM model can balance the sample data structure and modify the input samples in both sample data scale and time scale by explicit weighting and implicit weighting. That ensures the CWKELM model also has certain adaptability for small sample data.

6. CONCLUSIONS

In order to improve the rolling bearing fault trend prediction accuracy and computational efficiency, this paper proposes a composite weighted KELM (CWKELM) model and applies it to forecast the condition monitoring index RMS. Through comparison and analysis, the following conclusions can be reached.

(1) CWKELM model can overcome the differences of the input samples in both type scale and time scale and balance the data structure by explicit weighting and implicit weighting. In this way, the input samples’ ability to characterize different degradation states can be enhanced. In the one-step prediction and multi-step prediction of rolling bearing fault trend, the CWKELM model is more accurate than traditional KELM model.

(2) Composite weighting can improve the prediction accuracy of the KELM model. At the same time, explicit weighting can convert multi inputs into single inputs and thus improve the computational efficiency to some degree. Compared with KELM, SVM and BP neural network, CWKELM model has faster computing speed.

(3) AMPSO algorithm is applied in the parameter optimization of CWKELM model and this algorithm can effectively avoid the local optimal problem which exists in PSO algorithm. Meanwhile, compared with GA, AMPSO algorithm has advantages such as fewer iterations, faster convergence speed and is more suitable for real-time online parameter optimization, and then the prediction accuracy of CWKELM model is improved rapidly.

7. ACKNOWLEDGEMENTS

This project is supported by National Natural Science Foundation of China (Grant no. 51541506).

REFERENCES


Vibration Analysis of an Axially Moving Plate Based on Sound Time-Frequency Analysis

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(Received 12 September 2017; accepted 11 December 2017)

In some types of non-contact measuring systems, vibrating motion has negative effects on accuracy. Therefore, there is a need to analyze and monitor the motion state of a moving part using a low-cost scheme. This study focused on the vibration analysis and monitoring of a poly-crystalline silicon solar wafer carried by a pair of parallel moving strings, separated by a distance. Based on the sound data series picked up by a low-cost microphone and using a time-frequency analysis method, namely, the Hilbert-Huang transform (HHT) method, the low frequency features of the moving wafer could be determined quantitatively. The results showed that the motion of the moving wafer was sensitive to speed and string tension. By comparison, the average marginal spectrum should be treated as the basis for quantitative vibration monitoring, especially for a system with a strict requirement in terms of motion smoothness.

1. INTRODUCTION

The dynamics of axially moving systems has been studied over many years. Recently, a review paper on the characteristics of axially moving continua has been published.1 Because of the complexity of the mathematical model of an axially moving plate, many types of numerical methods, such as the mixed Finite Element Method (FEM), modal spectral element method, and finite strip method have been used.2–8 In addition, Ghayesh and Amabili reported the geometrical nonlinear dynamics of an axially moving plate based on the direct time integration method.9 Using the pseudo-arclength continuation technique, Ghayesh et al. investigated the nonlinear dynamics of the forced motion of an axially moving plate and the effect of system parameters, such as the axial speed and pretension on resonant responses.10 The finite difference method, perturbation techniques, direct time integration, and pseudo-arclength continuation technique are also used to explore the nonlinear dynamic behaviors of moving continua.11–19 Considering viscoelastic characteristics, some researchers have taken an interest in viscoelastic moving panels, strings, plates, and webs.20–24

The characteristics of axially moving continua under special constraints have been reported. Based on the Von Kármán large deflection equations of thin plates, considering the influences of the axial movement of the plate, axial tension, fluid-structure interaction, and foundation displacement, and by adopting a numerical method and the approximate analysis method, Li studied the characteristics of 1:3 internal resonances and their bifurcations for an axially moving unidirectional plate, partially immersed in a fluid under foundation displacement excitation.25

Tang presented a vibration characteristic analysis and numerical confirmation of an axially moving plate with viscous damping.26 The complex frequencies, complex modes, and critical speeds of an axially moving plate, with viscous damping, were investigated using the complex mode approach. The effects of some parameters, such as viscous damping coefficients, axial speeds, aspect ratios, stiffness ratios, and support stiffness parameters, on complex frequencies and critical speeds were also examined. The natural frequencies, complex modes, and critical speeds of an axially moving rectangular plate, which is partially immersed in a fluid and subjected to a pre-tension, were investigated by using the classical thin plate theory.27 The nonlinear vibrations of an axially accelerating multi-supported string were discussed by using the Hamilton principle and multiple timescale methods.28

Experimental studies of an axially moving system have also attracted attention.29–35 The authors presented experimental results on the vibration characteristics of axially moving strings.32, 33 The vibrating characteristics of a moving plate were demonstrated by using the Hilbert-Huang transform (HHT) method.34 To quantitatively express the motion (including pitch and roll) of a moving plate, three capacitive sensors were used and a motion indicator was introduced.

In engineering applications, such as some types of non-contact measuring systems, vibrating motion has negative effects on accuracy; therefore, there is a need to monitor the motion state of the moving part by using a low-cost scheme. In the present study, through sound signal analysis, we focus on...
the vibration analysis and monitoring of a poly-crystalline silicon solar wafer, carried by a pair of parallel moving strings separated by a distance. The experimental system is similar to that presented by Wu et al.\(^4\) Based on a sound data series, the time-frequency features of the moving wafer were analyzed using HHT, which is considered a powerful time-frequency analysis method.\(^35\)–\(^39\) In Section 2, the experimental system and its parameters are briefly introduced. As an example, a data series was investigated using HHT to examine the features of vibration. The Hilbert spectra and marginal spectra of the moving part (solar wafer in this case) are detailed; the effects of the system parameters on motion are presented in Section 3; finally, the conclusions are presented in Section 4.

## 2. EXPERIMENTAL SYSTEM

A system schematic is shown in Fig. 1, which includes two seamless parallel polyester strings, with a diameter of 6 mm and separated by 90 mm. The parameters of the test system and the sample solar wafers are listed in Table 1.\(^34\) To drive the system, a step motor is used with different velocities. Two polytetrafluoroethylene guides, that almost cover the span, are arranged to reduce string vibrations to an acceptable level. The clearance \(C\) (between the string and the guide), and the initial tension \(P\) in the strings, can be adjusted separately. A capacitive-type microphone (D-78), with a frequency response range of 70 Hz–20 kHz, and sensitivity of \(−3 \text{ dB} \pm 3 \text{ dB}\), was used to pick up the sound when the wafer passed by; a 16 kHz sampling rate was used. Capacitance sensors S2, S3, and S4 were used to monitor the transverse vibrations; however, the details are not covered in this paper. In this experiment, clearance \(C\) remains at 1.4 mm and uniform axial velocities of 152.5, 305, 457.5, 610, 762.5, and 915 mm/s are used. Two initial tensions in the strings, of 20.5 and 30 N, which correspond to different velocities, were set by the tensioners.

### 3. TIME-FREQUENCY ANALYSIS

In this section, first, a short introduction of the HHT method is presented. As an example, based on a sound series, the features were extracted through the Ensemble Empirical Mode Decomposition (EEMD), Hilbert Spectrum (HSP) and Marginal Spectrum (MSP). Even though it requires more computation time, EEMD, in which finite-amplitude white noises are added to the series to be processed and ensemble means as decomposition results, was chosen for this analysis to avoid possible inter-wave modulations and to suppress possible intermittence.\(^32\),\(^33\),\(^36\)

#### 3.1. HHT

The HHT method consists of Empirical Mode Decomposition (EMD), EEMD, and Hilbert spectral analysis.\(^35\),\(^36\)

##### 3.1.1. EMD

Given a time series \(x(t)\), through a sifting process, one can write

\[
x(t) = \sum_{k=1}^{n} C_k(t) + r_n;
\]

where \(r_n\) is the residue of \(x(t)\), and \(C_k\) represents the intrinsic mode functions (IMFs), which are simple oscillatory functions with varying amplitude and frequency that satisfy the following two requirements: (1) over the entire dataset, the number of extrema and zero crossings must either be equal or differ mostly by one; (2) at any data point, the mean value of the envelope defined by local maxima and the envelope defined by local minima is zero.\(^35\) The envelopes are generally produced using a natural cubic spline, to fit the local maxima or local minima, respectively. The IMFs carry the characteristic time scales, which are directly derived from, and based on, the data, without any assumptions. If the residue is excluded, \(x(t)\) can be expressed as follows:

\[
x(t) = \sum_{k=1}^{n} \alpha_k(t) \cos \theta_k(t) = \sum_{k=1}^{n} \alpha_k(t) \cos \left[ \int_{0}^{t} \omega_k(\tau) \, d\tau \right];
\]

where \(\alpha_k(t)\) is the amplitude of the IMF, \(\theta_k(t)\) is the phase, and \(\omega_k\) is the instantaneous frequency. After using EMD to obtain the IMFs, the instantaneous frequency (IF) of each IMF can be calculated using many possible methods such as Direct Quadrature, Normalized Hilbert Transform, or the generalized Zero-crossing Method. If necessary, the orthogonality of IMFs can be verified, and a significance test against white noise can

---

**Table 1. Parameters for test system and sample solar wafers.**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of driving/driven pulley</td>
<td>86 mm</td>
</tr>
<tr>
<td>Length of guide</td>
<td>610 mm</td>
</tr>
<tr>
<td>String mass per unit length</td>
<td>0.0399 kg/m</td>
</tr>
<tr>
<td>Eccentricities of driving pulleys</td>
<td>25, 36 µm</td>
</tr>
<tr>
<td>Eccentricities of driven pulleys</td>
<td>8, 25 µm</td>
</tr>
<tr>
<td>Solar wafer size</td>
<td>156×156 mm</td>
</tr>
<tr>
<td>Weight of wafer #1</td>
<td>11.09 g</td>
</tr>
<tr>
<td>Weight of wafer #2</td>
<td>11.08 g</td>
</tr>
<tr>
<td>Weight of wafer #3</td>
<td>11.44 g</td>
</tr>
</tbody>
</table>

\(^*\) The driving and driven pulleys were paired up as follows: a driving pulley with an eccentricity of 25 µm matches a driven pulley with an eccentricity of 8 µm; in the other pair, 36 µm corresponds to 25 µm.

![Figure 1. The schematic of test stand.](image-url)
be performed. Finally, the HSP, designated as $H(\omega, t)$, and defined as the energy density distribution in the time-frequency space, was divided into equally sized bins of $\Delta t \times \Delta \omega$, each with the value $\alpha_{i,j}^2/(\Delta t \times \Delta \omega)$, is defined as follows:

$$H(\omega, t_i) = \left( \sum_j \sum_i \alpha_{i,j}^2 \right) / (\Delta t \times \Delta \omega); \quad (3)$$

where $t_i = t_0 + i \times \Delta t$, $\omega_j = \omega_0 + j \times \Delta \omega$. From Eq. (3), MSP is defined as follows:

$$\omega(\omega) = \frac{1}{T} \int_0^T H(\omega, t)dt = \sum_{i=1}^N H(\omega, t_i); \quad (4)$$

where $T$ is the total data span and $N$ is equal to $T/\Delta t$.

### 3.1.2. EEMD

Based on the original EMD method, Wu and Huang proposed EEMD by adding finite-amplitude white noise to the data and using a sifting process similar to that of EMD. The noise-impregnated data could be decomposed into IMFs and a residue. After a number of trials, in which the added noise series were different and the amplitudes were finite (expressed as the ratio $\sigma$ of the root mean square of the added noise and the standard deviation of the targeted data), the ensemble means of the corresponding IMFs of the decomposition were treated as the final results.

Wang and Yeh et al. proved that the time complexity of the EMD/EEMD is, in fact, a computationally efficient method. Finally, a fast EEMD program is presented.

### 3.2. Sound Analysis

In this section, by using the above mentioned HHT method, the energized component extraction effects of system parameters on the vibration of the plate and marginal spectrum-based monitoring, are discussed. By comparing the MSP corresponding to the loading wafer to that of the background noise, distinctions between them could be observed in most cases. This allowed for the possibility of extracting useful features.

#### 3.2.1. Main component extraction

In Fig. 2a, an original data series (length of 57,373 points), corresponding to load wafer #2 passing by the Mic, is shown. Fig. 2b shows the original background noise, or environmental noise (length of 21,436 points), picked up without the load wafer, when the transporting system was running. As shown in Fig. 1, two guides constrain the vibration of each string unilaterally. In this experiment, only one solar wafer was carried by the strings at a time. The thin plate is the load on the moving strings. The interactions between the strings, guide and plate, and driving motor, were the main sound sources. Through the EEMD in which $\sigma$, the ratio of the root-mean-square value of the added white noise to the standard deviation of the data series, was set to 0.2, and the ensemble number $n$ (defined in the program 37, usually set between 100 and 500) was set to 400. In this study, the same values were used in all EEMD calculations. The detailed calculation procedures can be found in. Time-frequency features such as HSP, MSP, and energized IMFs and IFs were extracted.

By decomposing, orthogonal checking, and combining, and using the significance test of the IMFs, 36 HSPs could be obtained and are presented in Fig. 3. As shown in Fig. 3a, within 0–1 s, there was an area of energy concentration, which corresponded to the period of the wafer passing over the Mic. However, the energy distribution of the background noise in Fig. 3b is dispersive. Note that, here, the significance test was based on the assumption that the noise was white. To check this assumption, the Hurst exponents were estimated using the Matlab function wbemstic and the highest frequency components, which were obtained by decomposing the original sound series, shown in Fig. 2a, by EMD. The Hurst exponents were 0.0146, 0.4609, and 0.4816, respectively corresponding to the highest, sum of two highest, and sum of three highest components used to estimate the exponents. In this case, there were obvious differences between the estimated Hurst and 0.5. The background noise in Fig. 2b should not have been white. The significance test was only used to filter out the highest component after EEMD.

Figure 4a shows the marginal spectra corresponding to the data series in Fig. 2a and 2b. Compared to the MSP of background noise, there is a resonance peak at approximately 40 Hz in the MSP with the load wafer. Considering that the central frequencies of the HSPs, of the two cases in Fig. 3, are
the same (approximately 200 Hz), there exists strong energy mixing when the wafer is passing. It is difficult to separate the characteristic component of the moving wafer at this frequency. Fig. 4b shows the comparison of the two MSPs, when the initial tension is at 30 N. Corresponding to the moving wafer, 3 peaks can be seen. The main components, under the two initial tensions and their instantaneous frequencies, are presented in Fig. 5. In Fig. 5b, the average IF is approximately 41.142 Hz. In Fig. 5d, the average IFs are 102.6225, 46.1130, and 25.5520 Hz, for IMF3, IMF4, and IMF5, respectively.

Figure 6 shows the comparison of MSPs in six cases. In Fig. 6a, b, c, and d, the differences in marginal spectra between the background noise and sound with the load wafer are obvious; therefore, the main vibrating component(s) of the wafer can be extracted with higher credibility. In Fig. 6e, when the speed is 915 mm/s, the two curves have high similarity, and the signal corresponding to the moving wafer is swamped out by background noise. In Fig. 6f, the low frequency component, in the range of 20–50 Hz, can be extracted.

3.2.2. Effects of system parameters

System parameters such as speed, string initial tension, and varying loading, exert effects on wafer vibration. In this section, the results related to a few cases involving five speeds and two initial tensions are presented. The data series lengths were 22,000, 14,000, 8,600, 7,000, and 6,500 points, corresponding to 152.5, 305, 457.5, 610, and 762.5 mm/s, respectively.

Figure 4. a) Marginal spectra, speed 152.5 mm/s, initial tension 20.5 N, with and without load wafer; b) MSPs, 152.5 mm/s, 30 N, with and without load wafer.

EEMD ($\sigma = 0.2, nt = 400$) was used and the averaged MSP was obtained by averaging 10 results for each case.

Figure 7 shows the average marginal spectra under five speeds, with an initial tension of 30 N. The differences between the spectra are obvious. At speeds of 152.5 and 305 mm/s, besides a main energy peak, there were more than two sub-resonance peaks. From these curves, one can see that the motion is very sensitive to the moving speed.

Figure 8 presents the effects of initial tension in the MSP strings. In these cases, low tension induces high spectral density.

In Fig. 9a, when the speed is 762.5 mm/s, there are relatively small differences caused by the wafers. At 610 mm/s, the MSPs in Fig. 9b tend to converge. This shows that under certain conditions, the average MSP can be stable. As shown in Fig. 9c and 9d, MSP curves of wafer 1# and 2# under two speeds are plotted; their weight difference is very small, the vibration details are not the same, but the overall shapes are similar.

3.2.3. Comparisons of the results using capacitance sensor S2 and Mic

In Fig. 10, using a capacitance sensor S2 and the Mic, the MSPs of wafer 1# under two speeds are shown. From the plots, the main energetic peaks coincide; the spectra at the relative high frequencies obtained by capacitance sensor fall rapidly. The reason for this should be the averaging effect of the capacitance sensor.
3.2.4. Monitoring method

From the above analysis, it follows that the vibrating motion of the plate/wafer is sensitive to system parameters. In a certain system configuration, the average marginal spectrum demonstrates a certain stability. In particular, for a certain measuring system with a strict requirement of motion smoothness, the average marginal spectrum, or a certain threshold based on the maximum energy density, can be used to monitor the abnormal variations in string tension, speed, and motion of the plate during long-term running.

4. CONCLUSIONS

Based on a sound data series and by using HHT, which is a time-frequency analysis method, the low frequency features of the moving wafer could be determined quantitatively. The motion of the moving wafer sensitive to speed and string tension is verified.

In comparison to the expensive capacitive sensor, a low-cost microphone could be used to pick up the sound caused by the interactions in the system. Through data analysis, the average marginal spectrum should be treated as a basis of comparison for qualitative vibration monitoring, particularly for a certain measuring system with a strict requirement of motion smoothness.

REFERENCES


Figure 6. Comparison of MSPs. a) 305 mm/s, 20.5 N; b) 457.5 mm/s, 20.5 N; c) 610 mm/s, 20.5 N; d) 762.5 mm/s, 20.5 N; e) 915 mm/s, 20.5 N; f) 915 mm/s, 30 N.

Figure 7. Comparison of MSPs, five speeds, same tension of 30 N.


Figure 8. Comparison of MSP. a) Average marginal spectra, 610 mm/s; b) Average marginal spectra, 762.5 mm/s.


Figure 10. Comparison of MSP using S2 and MIC. a) MSP using capacitance sensor S2 and Mic, 762.5 mm/s, 20.5 N; b) MSP, 610 mm/s, 20.5 N.


Acoustic Emission Signal Analysis and Event Extraction through Tuned Wavelet Packet Transform and Continuous Wavelet Transform While Tensile Testing the AA 2219 Coupon

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(Received 21 October Month 2017; accepted 20 December 2017)

Aluminium Alloy AA 2219 is the principal metal for the production of propellant reservoir used in launch vehicles. The propellant tanks are often proof tested with acoustic emission technique (AET) to ensure its health. Acoustic Emission Testing during the structural health monitoring and proof testing is complex and unrealistic occasionally as it is performed in the noisy environment. Identification of signature corresponds to crack and its extraction from noise signatures is a major challenge in AET. Wavelet Packet Transform is an efficient mathematical tool for the analysis of AE signals. This paper recommends a novel combination of normalized cross correlation, Wavelet Packet Transform and Continuous wavelet transform to detect and extract the event related to failure. Experiments were carried out on AA 2219 tensile coupons at different threshold conditions. The recorded AE hits contain signals related to different events such as atmospheric noise, rubbing noise and other noise signals along with the signals from cracks. By applying the fine tuned wavelet packet transform technique in combination with CWT, the extraction of denoised single event related to crack was executed. Based on the frequency and the wavelet coefficient the crack related hits and the noisy hits are categorized.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<td>AE</td>
<td>Acoustic Emission</td>
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<td>AET</td>
<td>Acoustic Emission Testing</td>
</tr>
<tr>
<td>STFT</td>
<td>Short Term Fourier Transform</td>
</tr>
<tr>
<td>WT</td>
<td>Wavelet Transform</td>
</tr>
<tr>
<td>WPT</td>
<td>Wavelet Packet Transform</td>
</tr>
<tr>
<td>CWT</td>
<td>Continuous Wavelet Transform</td>
</tr>
<tr>
<td>MWT</td>
<td>Mother Wavelet</td>
</tr>
<tr>
<td>RA</td>
<td>Rise Angle</td>
</tr>
<tr>
<td>PLB</td>
<td>Pencil Lead Break</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
</tbody>
</table>

1. INTRODUCTION

Fabrication of low weight hardware of huge dimensions has been a challenge always in the field of aerospace. The material selection and its further processing also plays prominent role in the fabrication of such hardware. AA 2219 is an alloy of aluminium that contains copper as the major sub-system and becomes a prominent material in the aerospace industry. The propellant tank of satellite launch vehicles is often fabricated from Aluminium Alloy (AA 2219) metal sheets.1–3 The material has superior mechanical strength, corrosion resistance properties and fracture toughness. Its exceptional weldability and applicability up to 260°C from cryogenic temperature makes the material an unavoidable one in aerospace industry.1

Acoustic Emission is a well known and exceptional non-destructive testing (NDT) method for local damage identification of materials and structural members. Acoustic emissions are the elastic stress waves originated by rapid discharge of strain energy.4–6 While performing the AE test, it is often found that the background noise (extraneous & intermittent acoustic signals carrying no relevant data) is high. The noise includes mechanical noise, hydraulic noise, electromagnetic noise and miscellaneous noise. Presence of these noise signals lead to unreliable analysis of the AE signals. Hence it is necessary to eliminate such noisy signals to perform the correct analysis. While using the threshold technique, the genuine AE signatures with low amplitude may also be eliminated along with noise.4

Knowledge about various acoustic signatures related to distinct damage mechanism is indispensable for signal investigation and to make decision. Acoustic emission signal interpretation methods are basically separated into two major groups parameter related approach and waveform based approach. First method stores only the AE parameters such as Amplitude, AE count, RA value, Energy, etc. Recently waveform based signal examination methods have received developing consideration in the detection of the crack sources more accurately.5 Acquired raw AE signals are basically in time domain format. As the time as well as the frequency domain signal analysis is capable of giving important information, a time frequency investigation will be fairly efficient. Short Term Fourier Trans-
form [STFT] can be appropriate for such analysis. But recently STFT is not popularly used for the analysis of AE signals since all the frequencies are interpreted with identical resolution.\(^6\)

Wavelet Transform (WT) is an additional time frequency analysis method. The wavelet transformation technique for the analysis of AE signals has growing attention in recent decades. The frequency/Wavelet transformation based analysis can be effectively incorporated to discriminate different failure mechanisms.

Failure mode discrimination on glass/polypropylene specimen was evaluated by C.R. Ramirez Jimenez, et al. on the basis of fast Fourier transform of the waveform collected from tensile test. Based on the results the relationship between micro mechanical event and the specific frequency was proven.\(^7\)

The peak frequency value was efficiently utilized for the discrimination of five distinct failure modes of thermal barrier coatings through unsupervised k-mean clustering algorithm.\(^8\)

Md Yeasin Bhuian, et al. focussed on the failure mode analysis while fatigue testing the Al 2024 tensile coupon. They proposed waveform based analysis despite of conventional AE parameters. The phenomena related to crack resonance was identified through peak frequency based analysis.\(^9\)

Wavelet transform based frequency approach was efficiently utilized by Yang et al. for the classification of different failure mechanisms of thermal barrier coatings.\(^10\) Investigation on the damage behaviour of the thermal barrier coatings attacked by molten calcium-magnesium-alumino-silicate was executed using AE system by L.Yang, et al. The discrimination of failure modes is obtained using wavelet packet transform technique.\(^11\)

Hence it is a proven truth that the frequency is a prominent parameter in the discrimination of distinct failure modes.

In addition, Wavelet transform technique was efficiently utilized by various researchers\(^6,12-17\) to detect single event, denoising, and leak detection. Except Majid ahadi, et al.\(^6\) no one else has proposed a proper method for the selection of optimum mother wavelet. Majid ahadi, et al.\(^6\) used visual inspection method to find the resemblance between the most occurring pattern and mother wavelet. However, it is quite hard to match the profile of two signals visually. Hence in this work, normalized cross correlation method is proposed to find the appropriate mother wavelet. After the determination of the optimum mother wavelet, wavelet packet transform will be effectively performed to decompose the signal into sub bands. Then Shannon entropy based denoising and CWT will also be performed to detect and extract the event efficiently.

The proposed approach has three stages

**AE signal acquisition & classification:**

Reference signatures such as atmospheric noise, rubbing noise, pencil lead break signature and crack related signature are acquired and classified to have a good data base.

**Fine tuning of Mother wavelet & Wavelet based decomposition:**

Selection of optimized mother wavelet is performed on the basis of normalized cross correlation coefficient. Wavelet packet decomposition is then carried out with the selected optimum mother wavelet.

**Denoising and event extraction:**

Shannon entropy based denoising operation is performed and the crack related event is extracted with the use wavelet diagram (CWT).

The following section discusses the basics of WPT, cross correlation and Shannon entropy based denoising and the successive sections cover the experimental set up, discussion of results and conclusion.

## 2. OVERVIEW OF PROPOSED SIGNAL ANALYSIS TECHNIQUES

### 2.1. Wavelet Packet Transform

Wavelet packet transform is a general form of wavelet transform and it grants multiresolution investigation.\(^15\) Through the WPT technique, decomposition of signal can be executed on both the scaling and wavelet coefficients. This technique provides complete decomposition hierarchy which makes the decomposition extremely adaptable by giving uniform frequency secondary groups.\(^18\)

A fixed energy signal \(\psi(t)\) represented as mother wavelet (MWT), is a continuous fluctuating function of extremely small duration and is given in Eq. 1.

\[
\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-\tau}{s}\right), \text{s > 0; } -\infty < \tau < \infty.
\]

(1)

Where \(\psi_{s,\tau}(t)\) family contains all normalized expressions (dilations) in time \(t\) assigned by \(s > 0\) (scale factor) and translation in time \(t\) is assigned by \(-\infty < \tau < \infty\). Wavelet transformation of a signal \(x(t)\) is described by cross correlation of \(x(t)\) with \(\psi_{s,\tau}(t)\)\(^6,14,17,18\) and is expressed in Eq. 2.

\[
WT_x(s, \tau) \equiv \int x(t)\psi_{s,\tau}(t)dt.
\]

(2)

The execution of WPT with filter banks make the determination efficient by recursive schemes. Details (High frequency components) and approximations (Low frequency components) at each resolution level are achieved by passing the signal \(x(t)\) through a two-channel filter. WPT technique decomposes both details and approximations at every resolution level where the wavelet transform technique decomposes only the approximations not the details.

### 2.2. Cross Correlation

Cross correlation analysis is a mathematical approach to identify the similarity between two signals. Consider two sets of signals \(x_i\) and \(y_i\), where \(i = 0, 1, 2, \ldots N - 1\). The function of normalized cross correlation with zero time lag is described in Eq. 3.

\[
R = \frac{\sum x_i y_i}{(\sum x_i^2)(\sum y_i^2)^{1/2}}.
\]

(3)

The normalized cross correlation estimates the resemblance in profile among two signals as a numerical quantity between 0 and 1. Two signals of identical profile yield a normalized cross correlation coefficient (NCC) of 1.0.\(^19\)

### 2.3. Shannon Entropy Based Denoising

In order to detect the unnecessary signatures in an acoustic emission signal acquired during a test, an informative entropy dependent algorithm is utilized. In this method the informative entropy is weighed as a cost function. The method is intended...
to pick only the sub bands which focus the major information carried by the signal.

Normally, if \( X_j = (x_j, k) \) be a cluster of coefficients of a specified sub band of the wavelet packet transform (WPT) tree at stage of resolution \( j \), the Shannon entropy \( H(X_j) \) is given in Eqs. 4 and 5.

\[
H(X_j) = - \sum_k P_k \ln(P_k). 
\]  
(4)

\[
P_k = \frac{|x_{j,k}|^2}{\|X_j\|^2}.
\]  
(5)

With \( \|X_j\|^2 = \sum_k x_{j,k}^2 \) denoting a norm of \( X_j \). If the value of \( H(X_j) \) is large, the signal is in higher disorder and carries less information. If this occurs, the equivalent sub band and its subordinates are omitted. It means the entropy executes an energy correlation among the sub bands. At this moment the aim is to select stream through the WPT tree transporting the minor disorders to be specific having minimum conceivable energy. If the informative entropy is smaller at a sub band in comparison with subsequent resolved sub band the whole data is saved, or else a lesser energy level of resolution is necessary. The Acoustic Emission signal is then reconstructed with the preferred sub bands and subsequently saves the major significant data and the complementary component is known to be noise.

3. EXPERIMENTAL PROCEDURE

3.1. Tensile Test

Aluminium Alloy AA 2219 under T0 condition sheet type specimens are fabricated as per ASTM standard (Fig. 1). The specimens have 200 mm length, 12.5 mm width and 2 mm thick. 20 specimens are tested with different AE threshold conditions. Tensile tests are performed in servo-hydraulic driven DAK universal testing machine. The specimens are loaded till their failure and the corresponding stress strain values are continuously recorded.

3.2. AE Acquisition System

Acoustic Emission (AE) during the tensile tests is recorded by AEWin software powered by Mistras group, Physical Acoustic Corporation (PAC). A pair of nano 30 resonant type piezo electric sensors is positioned on the face of the test specimen for AE data acquisition (Fig. 2). Silicon vacuum grease is applied as a couplant in between the specimen and sensor interface. Hsu Neilson pencil lead break calibration is executed to ensure the efficient acquisition of AE signals. The 20 number of specimens are grouped into four sets (A, B, C and D). Specimens of set A are tensile tested with an acoustic emission threshold of 25 dB subsequently the sets B, C and D are tested with AE threshold of 30 dB, 35 dB and 40 dB respectively.

4. RESULTS AND DISCUSSION

Average number of hits from typical acoustic emission tensile tests is tabulated in Table 1. It is observed from the results that, the AE data of first two sets are strongly influenced by atmospheric noise signals.

4.1. Construction of Reference Signal

In order to perform the signal analysis efficiently, good knowledge about the AE signals related to mechanical fracture of material is essential. Signature from a typical pencil lead break test is shown in Fig. 3. The events from cracks are in some respect comparable with the events of pencil lead break (PLB) signatures. The waveforms are often portrayed by a solid rise subsequently a relatively calm period and afterward the reflected signals are observed.

To demonstrate that the suggested methodology is efficient in the recognition of crack related event, a reference signal with 1 k samples is constructed in MATLAB 2016 b workspace. The reference signal is constituted with pencil lead break (PLB) signal, atmospheric noise, rubbing noise and random gaussian white noise. The constructed reference signal and its signal to noise ratio (SNR) are shown in Figs. 4 and 5. SNR is a measure to compare the level of useful signal and the background noise. By definition, SNR is the ratio between the power of useful signal and the power of background noise. A typical power ratio of 1:1 yields the SNR value of 0 dB. Negative magnitude in SNR is the indication of heavy background noise. Result (Fig. 5) shows the magnitude of SNR as -10.99 which indicate the strong influence of background noise. The subsequent section deals with the interpretation of...
the constructed reference signal by applying the proposed algorithm.

4.2. Fine Tuning of Mother Wavelets

One of the real challenges in the wavelet analysis method is the determination of best suited mother wavelet for the given application. The choice of mother wavelet (MWT) mainly depends on the resemblance between the signal acquired and the mother wavelet. Noor Kamal Al Qazzaz, et al. found normalized cross correlation coefficient as an efficient parameter to select the optimum MWT for EEG signal. In this AE signal analysis work, the same method is suggested for the selection of best MWT. Totally 45 mother wavelets from three orthogonal families including symlets (sym1 – sym 20), daubechies (db1 – db20) and coiflets (coif1 – coif5) are selected for this work. The normalized cross correlation coefficient between the most occurred pattern and 45 MWTs are given in Fig. 6. Among the 45 MWTs, db 4 shows maximum cross correlation coefficient of 0.8. Hence db 4 from daubechies family is selected as the optimum wavelet for the current work.

4.3. WP Decomposition, Denoising and Event Extraction

WP decomposition of the reference signal is performed by db 4 mother wavelet to a level of 4 to obtain 16 sub bands. The denoising operation is executed by Shannon entropy based algorithm. In order to extract the AE event related to failure/crack, CWT is performed on the denoised signal version of constructed signal. The denoised signal and its corresponding wavelet diagram are shown in Figs. 7 and 8. From the diagram, it is observed that a strong event occurs between 142 µs and 248 µs with very good wavelet coefficients. This strong crack related event is then extracted from the denoised version of the signal. To prove the pre-eminence of the proposed method in denoising, normalized cross correlation is performed again between the extracted signal and PLB signature. Correlation yields a strong coefficient of 0.97 which shows that the noise signatures in the reference signal are almost eliminated.

In addition to the reference signal, two more signals (Test 1 signal and Test 2 signal) from a typical tensile test also analysed with the proposed methodology and the results are shown in Figs. 9 and 10. The categorization of crack signals and noise signals shall be made on the basis of three valuable parameters of CWT. The crack related events do have high frequency, high wavelet coefficient and poor duration. Meanwhile the noise events have low frequency, low wavelet coefficient and longer duration. Test 1 signal shows a typical waveform which contains a strong crack related event with good frequency in between 68 µs and 139 µs. Meanwhile Test 2 signal indicates...
a very long event with poor frequency and poor wavelet coefficient. Hence test 2 shall be considered as pure noise signal carries no useful information in it.

The methodology is then efficiently utilized for the interpretation of hits acquired from AE tensile testing. Entire hits acquired during the tensile test of the first specimen from each set are considered for the further analysis and the results are categorized in Table 2. Since the number of signals are too large for set A, initially few of the atmospheric noise signals are analysed with the methodology and then based on the signal profile and parameter (Amplitude and Frequency) similarity, rest of the atmospheric signals are categorized as noise signal. The other signals in set A are completely analysed and then categorized. Signals have poor frequency, poor wavelet coefficients and long duration are taken under category I and hence the signals under category I are purely noise signal. In category II all the signals show good frequency and wavelet coefficients.

But still few of the signals have different signal pattern which may be an indication of noise signals. In order to ensure it, other AE parameters can be utilized. The results are found to be satisfactory and reliable. Hence the frequency, wavelet coefficient and the duration of events are excellent sources for the clustering the AE crack related events and noise events.

5. CONCLUSION

Wavelet packet transform technique is an efficient mathematical tool for the analysis and denoising of acoustic emission signals. In this work, the compatibility of 45 mother wavelet functions from symlets, coiflets and daubechies families were elected. The normalized cross correlation between the MWT and most occurring pattern were performed to optimize the MWT. From the results db4 wavelet is selected as most appropriate MWT for current work. WP decomposition with db4 mother wavelet and Shannon entropy based denoising were performed on the constructed reference signal. With the help of wavelet diagram (CWT) of the denoised signal, the event related to crack was detected and extracted. The extracted event shows a good correlation coefficient of 0.97 with PLB signature which ensures the effectiveness of the suggested methodology. Successively, tensile test signals are also analysed with the same approach and yield reliable results. These results reveal that the frequency and the wavelet coefficient are reliable resources for the clustering of AE hits which carry useful crack related signature.

6. AKNOWLEDGEMENTS

This research work is supported by Indian Space Research Organization (ISRO) under ISRO RESPOND program. The authors wish to thank Shri.Purusothaman from VSSC, Trivandrum for his valuable support and guidance in doing the experiment.

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Analysis on a Kinetic Theoretical Model of the Straight-Curved Pipe Conveying Fluid

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(Received 26 October 2017; accepted 12 February 2018)

A kinetic theoretical model for the straight-curved pipe conveying fluid was proposed and studied in this paper. Firstly, the new equations of static equilibrium and motion about equilibrium position were derived by applying perturbation method to the motion equations of the curved pipe conveying fluid. Then, the kinetic equations of the straight pipe were derived by ignoring the terms containing the curvature of the curved pipe in these equations. Subsequently, considering different factors, four segmental kinetic theoretical models of the straight-curved pipe were built. Lastly, based on the present theoretical models and finite element method, the static deformations and natural frequencies of the pipes with three typical boundary conditions were simulated. The simulation results show that the applicability of these kinetic theoretical models is closely related to the boundary conditions: 1) For the pipes with any one of three boundary conditions, the geometrical non-linearity and the nonlinear force caused by the deformation of straight pipe segment, have a great effect on the static deformation of the pipes, while they have little effect on the natural frequencies of the pipes. 2) When the natural frequencies of the pinned-pinned pipe or the pinned-sliding bearing-pinned pipe are solved, the static deformation of the pipes must be considered. 3) When the natural frequencies of the pinned-pinned-pinned pipe are solved, the static deformation of the pipe can be ignored.

1. INTRODUCTION

Flow-induced vibration of the pipe conveying fluid is a difficult problem in many industrial projects. It contains so many dynamic phenomena and has attracted the attention of many scholars. For the past decades, scholars at home and abroad have carried out a large number of related researches and obtained so many fruitful results. From the perspective of pipe geometries, the previous literatures mainly focused on the vibration research of single-configuration pipes, such as straight pipes or curved pipes separately. However, flow-induced vibration of multi-conformational pipe conveying fluid has been studied little. In actual project, the pipe conveying fluid is always composed of straight pipe segments and curved pipe segments. Therefore, the geometric model of multi-configuration pipe is closer to engineering practice, and its research results have more practical significance.

According to the different boundary conditions, straight pipe can be divided into two kinds: pipes supported at both ends and cantilever pipes. The two forms of pipe have differ-
ent mechanical characteristics, and the reasons of instability caused by fluid in the pipes are also different. Based on the linear theory, Paidoussis, Dai, Wang, Qian, et al., Wen, Yang, Li, et al. studied the stability of straight pipe conveying fluid supported at both ends. The results show that with increasing the flow velocity of fluid, the pipe buckles firstly, and then loses stability with coupled-mode flutter. Rahmati, Mirdamadi, et al. dealt with the investigation of probabilistic stability of pipes conveying fluid with stochastic flow velocity. The results delineate that the critical mean flow velocity of fluid is closely related to the power spectral density of the random velocity, the boundary condition, viscoelastic damping and mass ratio. Holmes studied the stability of straight pipes supported at both ends by using the nonlinear theory and found that the pipe supported at both ends will not flutter if the nonlinear force caused by pipe deformation was taken into account. Luezzo and Czerwiński and Czerwiński and Luezzo analyzed the non-planar vibration of a statically deformed pipe conveying fluid with experiments and numerical simulations. The results show that the pipe’s curvature and geometrical nonlinearities result in coupling of in-plane and out-plane vibration, which in certain conditions give rise to non-planar vibrations. As for cantilever pipes conveying fluid, fluid in the pipe could cause the pipe flutter. The motion of cantilever pipe conveying fluid is very complex when some nonlinear factors are considered, and the change of any parameter of the system could result in a great change of the motion of pipe.

The history of the study on the curved pipe conveying fluid has not been long before. Chen has derived the in-plane vibration equation of the curved conveying pipe by using Hamilton’s principle and Newton’s method, and analyzed the stability of the curved pipe. The results show that: 1) with increasing the flow velocity of the fluid, the curved pipe supported at both ends has buckling instability firstly; 2) the buckling form of the cantilever curved pipe is closely related to the subtended angle of the curved pipe: under the influence of the internal flow fluid, if the ratio between the subtended angle and \(2\pi\) is less than 0.25, the pipe only has flutter instability, and if the ratio is larger than 0.25 the pipe has buckling instability first. Misra, Paidoussis and Van derived the motion equations of the curved pipe conveying fluid with complex shape by using Newton method. In order to simplify the calculation, these equations were linearized to establish the equations of static equilibrium of the curved pipe conveying fluid and the equations governing the motion of the curved pipe conveying fluid about the equilibrium position. It should be noted that the motion equations about equilibrium position can be degenerated into the equations in literatures by ignoring the parameters about the equilibrium state. The results show that if the static deformation of pipe was considered, the curved pipe supported at both ends could not lose the stability; as for the buckling form of the semi-circular cantilever curved pipe conveying fluid, the predicted results of Misra’s and Chen’s theoretical models are qualitatively consistent. So Chen’s theoretical model is still used by other scholars to study the kinetic behavior of the cantilever curved pipe conveying fluid, such as Ni, Wang, Qian, Wang, Ni, Huang, Wang, Ni and Wang. Jung and Chung added the terms about the geometrical non-linearity of the pipe in the equations of static equilibrium derived by Misra, Paidoussis and Van, and studied the stability of the curved pipe supported at both ends. The result shows that when the flow velocity of the fluid is large, the terms about geometrical non-linearity of the pipe should be considered to avoid discontinuities in the pipe static deformation.

There is little reported literature on the vibration problem of the three-dimensional curved pipe conveying fluid, and the existing literature is limited to the analysis on linear vibration characteristics of the system. Koo and Park studied the vibration performance and stability of the three-dimensional pipe conveying fluid, and obtained the amplitude-frequency characteristic of the pipe with the motion equations of the straight pipe, based on the idea of “replacing curves with straight”’. Dai, Wang, et al. thought that the equations in literatures didn’t consider the influence of the static axial force of pipe, so the conclusions of the related researches could be doubted. Therefore, the static axial force was introduced into the equation of motion of the curved pipe segment, and the vibration performance of the three-dimensional pipe conveying fluid in the literature was recalculated. Besides, there is little literature on the vibration problem of the three-dimensional curved pipe conveying fluid.

In summary, as for the pipe conveying fluid containing the configuration characteristics of straight pipe segment and the curved pipe segment, its vibration performance may be closely related to the nonlinear force caused by the deformation of the straight pipe segment, static deformation of the pipe and geometrical non-linearity of the pipe. However, the theoretical models in the literatures didn’t consider these factors, and the theoretical model in the literature just considered the static axial force generated by the static deformation of the pipe. Therefore, the current kinetic theoretical model of the multi-configuration pipe conveying fluid should be optimized. It is necessary to further clarify the internal connections between the vibration characteristics of pipe and the factors such as the nonlinear force caused by the deformation of the straight pipe segment, static deformation of the pipe and geometrical non-linearity of the pipe.

In this paper, a kinetic theoretical model of the straight-curved pipe conveying fluid is studied. The geometrical configuration of the straight-curved pipe contains straight pipe configuration and curved pipe configuration. Firstly, the new equilibrium equations, the motion equations of the curved pipe conveying fluid about the equilibrium position were derived. Then, the equilibrium equations of the straight pipe conveying fluid and the motion equations of the straight pipe conveying fluid about the equilibrium position were derived by ignoring the terms that contained the curvature of the curved pipe in these equations. Subsequently, the four segmental kinetic theoretical models of the straight-curved pipe conveying fluid were built according to different factors. Following, the equations of the present theoretical models are analyzed by FEM, and the finite element forms of the equations are derived. Lastly, the static deformation and generalized eigenvalues of straight-curved pipe conveying fluid with several typical boundary conditions are solved. With the comparative analysis of the calculated results, the applicability of the four theoretical models is studied and the stability of the straight-curved pipe conveying fluid...
fluid under various typical constraints is predicted.

2. THEORETICAL MODEL OF THE STRAIGHT-CURVED PIPE

The geometry configuration of the straight-curved pipe conveying fluid is shown in Fig. 1. The total length of the pipe is $L$, and the length of the straight pipe segment is $L_1$, so as the length of the curved pipe segment is $L_2$, and the radius of the curved pipe is $R$. Point $A$, $B$ and $C$ are the left end of the straight pipe segment, the right end of the curved pipe segment, and the connection point between the straight pipe segment and the curved pipe segment respectively.

In this paper, the segmental kinetic theoretical model of straight-curved pipe conveying fluid was derived. The theoretical model of straight pipe conveying fluid and the curved pipe conveying fluid, which can embody the geometry features of straight pipe and curved pipe well.

2.1. The Theoretical Model of the Curved Pipe Segment

Misra, Paidoussis and Van\textsuperscript{15} derived the equations of motion for a curved pipe conveying fluid with complex shape by using Newton’s method. Ignoring the equation of motion about out-of-plane motion, and ignoring some other factors such as outpipe fluid, gravity and structural damping, the kinetic equation could be obtained:

\[ - (A_f P_t - Q_x z) \dot{v} + (EI/R)(v'''' + u''''/R) + (1/R)[(A_f P_t - Q_x z)(v' + u/R)] + (M_f U^2/R)(v' + u/R) - M_f U(\ddot{v}' - \ddot{u}/R) - (M_t + M_f) \ddot{u} = 0; \]

(1)

\[ EI(v'''' + u''''/R) + [(A_f P_t - Q_x z)(v' + u/R)]' + (1/R)(A_f P_t - Q_x z) + M_f U^2(v'' + u'' + R + 1/R) + 2M_f U(\dot{v}' + \dot{u}/R) + (M_t + M_f) \ddot{u} = 0; \]

(2)

where the prime represents a derivative with respect to $s$, and the superposed dot represents a derivative with respect to time.

The following dimensionless variables and parameters are introduced:

\[ \xi = \frac{s}{L}; \]

\[ \eta_1 = \frac{u}{v}; \]

\[ \eta_2 = \frac{v}{L}; \]

\[ \tau = \frac{t}{L^2} \left( \frac{EI}{M_f + M_t} \right)^{1/2}; \]

\[ \pi = \frac{LU}{M_f }; \]

\[ \theta = \frac{L}{R}; \]

\[ A = \frac{A_f L^2}{T}; \]

\[ \beta = \frac{M_f}{M_t + M_f}; \]

\[ \Pi_p = \frac{A_f P_t L^2}{EI}; \]

\[ \Pi = \frac{(A_f P_t - Q_x z) L^2}{EI}. \]

(3)

Substituting Eq. (3) into Eqs. (1) and (2), the dimensionless equations of motion for the in-plane motion of the curved pipe are:

\[ - \Pi' + \theta (\eta_2'' + \eta_1''') + \Pi \theta (\eta_2' + \eta_1) + \pi^2 \eta (\eta_2' + \theta \eta_1) - \beta^{1/2} \pi (\eta_2' - \theta \eta_2) - \ddot{\eta}_1 = 0; \]

(4)

\[ (\eta_2'' + \theta \eta_1'') + [\Pi (\eta_2 + \theta \eta_1)]' + \Pi + \pi^2 (\eta_2' + \theta \eta_1') + \Pi \theta (\eta_2' + \theta \eta_1) = 0; \]

(5)

It is assumed that the displacement consists of static term and disturbance term, namely as follows,

\[ \eta_1 = \eta_{1s} + \eta_{1d}; \]

(6a)

\[ \eta_2 = \eta_{2s} + \eta_{2d}; \]

(6b)

where the subscript $s$ indicates the static quantity, and the subscript $d$ indicates the disturbance quantity.

According to the extensible theory in literature,\textsuperscript{16} the axial force of the curved pipe segment is:

\[ Q_x = E A_f \left( \frac{\dot{\theta}_u}{\delta s} - \frac{v}{R} \right). \]

(7)

By substituting Eq. (7) into Eqs. (4) and (5), and ignoring the time-related terms, the static equilibrium equation of the curved pipe conveying fluid can be derived:

\[ - A (\eta_{1s} - \eta_{1ds})' - (\eta_{1ds} + \eta_{1s}) - \Pi A (\eta_{2s} + \theta \eta_{1s}) - \pi^2 \eta_2 (\eta_{2s} + \theta \eta_{1s}) + \Pi \theta (\eta_{2s} + \theta \eta_{1s}) = 0; \]

(8)

\[ (\eta_{2s}'' + \theta \eta_{1s}''') + (\Pi_p + \pi^2) (\eta_2'' + \theta \eta_1'') - \Pi (\eta_{2s} - \theta \eta_{2s}) + \theta (\Pi_p + \pi^2) - A[(\eta_{1s} - \theta \eta_{2s})(\eta_{1s}'' + \theta \eta_{1s})] = 0. \]

(9)

Equations (8) and (9) contain nonlinear terms about geometric nonlinear deformations. If the terms about geometrical non-linearity are ignored, Eqs. (8) and (9) can be degenerated into the linear static equilibrium equations derived in the literature,\textsuperscript{16}

Substituting Eq. (6) into Eqs. (4) and (5), the equations of motion of the curved pipe conveying fluid about the equilibrium position can be obtained as following:

\[ - \theta (\eta_{2ds} + \theta \eta_{1ds}) - \theta \Pi_s (\eta_{2ds} + \theta \eta_{1ds}) + \Pi A (\eta_{2ds} + \theta \eta_{1ds}) - A (\eta_{2ds} - \theta \eta_{2ds}) - \pi^2 \eta_2 (\eta_{2ds} + \theta \eta_{1ds}) + \beta^{1/2} \pi (\eta_{2ds} - \theta \eta_{2ds}) - \Pi (\eta_{2ds} + \theta \eta_{1ds})' + \Pi_s (\eta_{2ds} + \theta \eta_{1ds})' - \Pi \theta (\eta_{2ds} - \theta \eta_{2ds}) + \pi^2 (\eta_{2ds} + \theta \eta_{1ds}) + 2 \beta^{1/2} \pi (\eta_{2ds} + \theta \eta_{1ds}) + \eta_{2ds} = 0; \]

(10)

where $\Pi_s$ is the static value of the axial force, and its expression is

\[ \Pi_s = \Pi_p - \Pi A (\eta_{1s} - \theta \eta_{2s}). \]

(12)

2.2. The Theoretical Model of the Straight Pipe Segment

The straight pipe can be regarded as the curved pipe with the curvature being 0, that is, $\theta = 0$. When the axial displacement
of the two ends of the straight pipe segment are constrained, the axis of the pipe will extend due to the bending deformation, and then produce the added axial force, so the axial force of the pipe is

\[ Q_x = EA_t \left( \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial v}{\partial x} \right)^2 \right). \]  

(13)

Then the following static equilibrium equations of the straight pipe conveying fluid and the equations of motion of the straight pipe conveying fluid about the equilibrium position can be derived.

\[ -A(\eta'_s + \eta''_s) = 0; \]  

(14)

\[ \eta'''_{2s} + (\Pi_p + \pi^2)\eta''_{2s} - A(\eta'_s, \eta''_s) - \frac{3A}{2} \eta''_s \eta'''_s = 0; \]  

(15)

\[ -A\eta'_{1d} + \bar{\eta}_{1d} - A(\eta'_d, \eta''_d)' = 0; \]  

(16)

\[ \eta'''_{2d} + \pi^2 \eta''_{2d} + (\Pi_s, \Pi'_d)\eta'_d + 2\beta^{1/2} \pi \eta''_{2d} + \bar{\eta}_{2d} - A(\eta'_d, \eta''_d)' - A(\eta'_d, \eta''_d)' = 0. \]  

(17)

Equations (14) and (15) are the static equilibrium equations of the straight pipe conveying fluid, containing the nonlinear terms about the geometrical non-linearity of the pipe and the additional axial force. Equations (16) and (17) are the equations of motion of the straight pipe conveying fluid about the equilibrium position. If the terms about the steady parameters are ignored, Eqs. (16) and (17) can be degenerated into the equation of motion of the straight pipe in the axial and transverse direction derived in literatures.\(^{23,24}\) Compared with the theoretical models in the literature,\(^{2}\) the steady parameters considered in Eqs. (16) and (17) are more comprehensive, and the calculation results should be more accurate.

### 2.3. The Theoretical Model of the Straight-Curved Pipe

The geometry of the straight-curved pipe conveying fluid is complex, often containing the straight pipe segments and curved pipe segments. Based on the idea of "replacing curves with straight", the vibration problem of the curved pipe can still be solved by using the theoretical model of straight pipe. However, according to the literatures,\(^{15,16}\) the initial deformation of the pipe must be considered to study the vibration characteristics and stability of the curved pipe. Otherwise, the calculation result is not reliable. From Eq. (9), it can be seen that the reason for the static deformation of the curved pipe is that there exists an external force term in the equation, that is, \(\theta(\Pi_p + \pi^2)\). When a straight pipe element is used to simulate a curved pipe element, the external force term will be ignored as in literatures.\(^{23,24}\) Literature\(^{2}\) added the static axial force in the equations of motion of the straight pipe conveying fluid to consider the effect of the static deformation of the curved pipe, which is the revision of the theoretical model in literatures.\(^{23,24}\) However, it is not comprehensive enough to consider the initial deformation of the curved segment. Therefore, in this paper, for the vibration problem of straight-curved pipe conveying fluid, a segmental kinetic theoretical model is established by combining the theoretical models of straight pipe and curved pipe.

According to Eqs. (8)–(11), (14)–(17), the following segmental theoretical model of the planar pipe conveying fluid can be obtained.

### Table 1. Four theoretical models for calculating the natural frequencies of straight-curved pipe conveying fluid.

<table>
<thead>
<tr>
<th>Theoretical model</th>
<th>(\alpha_1, \alpha_2, \alpha_3)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model I</td>
<td>(\alpha_1 = 0)</td>
<td>The static deformation of pipe is ignored.</td>
</tr>
<tr>
<td>Model II</td>
<td>(\alpha_1 = 1), (\alpha_2 = 0)</td>
<td>The linear static deformation of pipe is considered.</td>
</tr>
<tr>
<td>Model III</td>
<td>(\alpha_1 = 1), (\alpha_2 = 1), (\alpha_3 = 0)</td>
<td>The geometrical non-linearity of the pipe is considered.</td>
</tr>
<tr>
<td>Model IV</td>
<td>(\alpha_1 = 1), (\alpha_2 = 1), (\alpha_3 = 1)</td>
<td>The geometrical non-linearity of the pipe and the additional axial force of the straight pipe segment are considered.</td>
</tr>
</tbody>
</table>

(1) **Straight pipe segment:**

Equations of static equilibrium of the straight pipe are:

\[ -A(\eta'_s + \alpha_2 \alpha_3 \eta''_s) = 0; \]  

(18)

\[ \eta'''_{2s} + (\Pi_p + \pi^2)\eta''_{2s} - \alpha_2 A(\eta'_s, \eta''_s) - \frac{3\alpha_2 \alpha_3}{2} \eta''_s \eta'''_s = 0. \]  

(19)

Equations governing the motion about the static equilibrium position are given by:

\[ -A\eta'_{1d} + \bar{\eta}_{1d} - \alpha_1 \alpha_2 \alpha_3 A(\eta'_d, \eta''_d)' = 0; \]  

(20)

\[ \eta'''_{2d} + \pi^2 \eta''_{2d} + (\Pi_s, \Pi'_d)\eta'_d + 2\beta^{1/2} \pi \eta''_{2d} + \bar{\eta}_{2d} - \alpha_1 A(\eta'_d, \eta''_d)' - \alpha_2 \alpha_3 A(\eta''_{2d})' = 0. \]  

(21)

(2) **Curved pipe segment:**

Equations of static equilibrium of the curve pipe are:

\[ -A(\eta'_s - \theta_{2s})' - \theta(\eta''_{2s} + \eta'''_s) - \Pi_p (\eta'_{2s} + \theta_{1s}) - \pi^2 \eta'_s - \eta''_{2s} + \alpha_2 A(\eta'_s - \theta_{2s} - \Pi_s, \Pi'_d) A(\eta'_s - \theta_{1s})' = 0; \]  

(22)

\[ (\eta'''_{2s} + \theta''_{2s}) + (\Pi_p + \pi^2) (\eta'_{2s} + \theta_{1s}) - A(\eta'_s - \theta_{2s}) + \theta(\Pi_p + \pi^2) - \alpha_2 A(\eta'_s - \theta_{2s}) (\eta'_{2s} + \theta_{1s})' = 0. \]  

(23)

Equations governing the motion of the curve pipe about the equilibrium position are given by:

\[ -\theta(\eta''_{2d} + \theta'''_d) - \alpha_1 \Pi_s (\eta''_{2d} + \theta_{1d}) + \alpha_2 A(\eta''_{2d} + \theta_{1s}) (\eta'_{1d} - \theta_{2d}) - A(\eta''_{2d} + \theta_{1d}) - \eta''_{2d} + \alpha_2 A(\eta''_{2d} + \theta_{1s}) (\eta'_{1d} - \theta_{2d}) + \Pi_s (\eta''_{2d} + \theta_{1s}) (\eta'_{1d} - \theta_{2d}) + \Pi_s (\eta''_{2d} + \theta_{1s}) (\eta'_{1d} - \theta_{2d}) = 0. \]  

(24)

\[ \eta'''_{2d} + \theta''_{2d} + \alpha_1 \Pi_s (\eta''_{2d} + \theta_{1s}) - \alpha_1 A(\eta''_{2d} + \theta_{1s}) (\eta'_{1d} - \theta_{2d}) + \Pi_s (\eta''_{2d} + \theta_{1s}) + 2\beta^{1/2} \pi \eta''_{2d} + \bar{\eta}_{2d} = 0. \]  

(25)

In Eqs. (18)–(25), \(\Pi_{ss}\) and \(\Pi_{cs}\) are the static value of the axial force of straight pipe segment and the curved pipe segment respectively. Their corresponding expressions can be described as follows respectively:

\[ \Pi_{ss} = \Pi_p - A \left( \theta_{1s} + \alpha_3 \frac{1}{2} \eta''_{2s} \right); \]  

(26)

\[ \Pi_{cs} = \Pi_p - A (\eta''_{2s} - \theta_{2s}). \]  

(27)

The coefficients \(\alpha_1, \alpha_2, \alpha_3\) are introduced in Eqs. (18)–(25) with a value of 0 or 1. Items that contain \(\alpha_1\) are terms related to static deformation of the pipe, items that contain \(\alpha_2\) are
3.1. Displacement Model of the Pipe Element

Considering only the in-plane deformation and vibration of the straight-curved pipe conveying fluid, both the straight pipe and the curved pipe can be separated by a beam element with two nodes and six degrees of freedom. Each node has such three degrees of freedom as \( \eta_1, \eta_2 \) and \( \eta_3 \), and the displacements in any part of the element can be expressed by the interpolation functions as

\[
\eta_1 = [N_1] \{\eta\}_i^c; \\
\eta_2 = [N_2] \{\eta\}_i^c; \\
\eta_3 = [N_3] \{\eta\}_i^c;
\]

where \([N_1] \) and \([N_2] \) are shape functions, \( \{\eta\}_i^c \) is the displacement vector quantity of the node.

In order to ensure that the solution of the equation has good calculation accuracy, the displacement function is assumed to be a polynomial as follows:

\[
\eta_1 = a_1 + a_2 \zeta; \\
\eta_2 = a_3 + a_4 \zeta + a_5 \zeta^2 + a_6 \zeta^3;
\]

where \( a_i \) is the generalized coordinates, \( \zeta \) is the local coordinate of the element.

Then, Eqs. (30) and (31) can be converted to

\[
\eta_1 = [\Phi_1] \{a_i\}; \\
\eta_2 = [\Phi_2] \{a_i\};
\]

where

\[
[\Phi_1] = \begin{bmatrix} 1, \zeta, 0, 0, 0, 0 \end{bmatrix}; \\
[\Phi_2] = \begin{bmatrix} 0, 0, 1, \zeta, \zeta^2, \zeta^3 \end{bmatrix}; \\
\{a_i\}^T = \begin{bmatrix} a_1, a_2, a_3, a_4, a_5, a_6 \end{bmatrix}.
\]

Utilizing Eqs. (28), (29), (32) and (33), it can be obtained that:

\[
\eta_1 = [\Phi_1] [A]^{-1} \{\eta\}_i^c; \\
\eta_2 = [\Phi_2] [A]^{-1} \{\eta\}_i^c.
\]

where

\[
[A] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \zeta_e \\ 0 & 0 & 1 & \zeta_e & \zeta_e^2 & \zeta_e^3 \\ 0 & 0 & 0 & 1 & 2\zeta_e & 3\zeta_e^2 \end{bmatrix}.
\]

Here, \( \zeta_e \) is the length of the pipe element.

3.2. Analysis of the Static Equilibrium

Substituting Eqs. (37) and (38) into Eqs. (18), (19), (22) and (23), and considering the two ends of the pipe as hinge constraint or clamped constraint, and with the calculus of variation, the finite element equations calculating the static deformation of the planar pipe conveying fluid can be derived.

The finite element equation of the \( i \)th element of the straight pipe segment is

\[
([K]_{si}^c + [K]_{si}}^c_{\text{stiff}}) \{\eta\}_i^c = 0; \tag{40}
\]

where \([K]_{si}^c\) and \([K]_{si}}^c_{\text{stiff}}\) are the linear part and nonlinear part of the stiffness matrix respectively. Their corresponding expressions could be described as follows respectively:

\[
[K]_{si}^c = [A]^{-1T} \left( [A] J_1 + [J_2] - (\Pi_p + \pi^2) [J_3] \right) [A]^{-1}; \tag{41}
\]

\[
[K]_{si}}^c_{\text{stiff}} = [A]^{-1T} \int_0^{c_i} \left\{ \alpha_2 [A] [\Phi_2]^T [\Phi_1]' [A]^{-1} \{\eta\}_i^c [\Phi_2]' - \alpha_2 \alpha_3 [A] [\Phi_2]^T [\Phi_2]' [A]^{-1} \{\eta\}_i^c [\Phi_2]' - \frac{3}{2} \alpha_2 \alpha_3 A [\Phi_2]^T [\Phi_2]' [A]^{-1} \{\eta\}_i^c [\Phi_2]' \right\} d\zeta [A]^{-1}. \tag{42}
\]

The finite element equation of the \( i \)th element of the curved pipe segment is

\[
([K]_{sci}^c + [K]_{sci}}^c_{\text{stiff}}) \{\eta\}_i^c = \{F\}_i^c; \tag{43}
\]

where \([K]_{sci}^c\) and \([K]_{sci}}^c_{\text{stiff}}\) are the linear part and nonlinear part of the stiffness matrix respectively, while \([F]_i^c\) is the external load. Their corresponding expressions can be described as follows respectively:

\[
[K]_{sci}^c = [A]^{-1T} \left\{ \alpha_2 [A] (\theta [\Phi_1] + [\Phi_2])^T - \theta (\Pi_p + \pi^2) [J_6] + \theta [J_7] \right\} [A]^{-1}; \tag{44}
\]

\[
[K]_{sci}}^c_{\text{stiff}} = [A]^{-1T} \left\{ \int_0^{c_i} \alpha_2 A [\theta [\Phi_1] + [\Phi_2]]^T \left( [\Phi_1] - \theta [\Phi_2] \right) [A]^{-1} \{\eta\}_i^c [\Phi_2]' d\zeta \right\} [A]^{-1}; \tag{45}
\]

\[
\{F\}_i^c = -\theta (\Pi_p + \pi^2) [A]^{-1T} \int_0^{c_i} [\Phi_2] T d\zeta. \tag{46}
\]

3.3. Analysis of Motion About Static Equilibrium Position

3.3.1. Analysis of Motion of the Straight Pipe Segment About Static Equilibrium Position

The motion equations, Eqs. (20) and (21), of the straight pipe segment contain the static parameters \( \Pi_{ss} \) and \( \eta_{2s} \). For convenience, it is assumed that the static parameters in each element are linear functions, that is,

\[
\Pi_{ss} = a_1 + a_2 \xi; \tag{47}
\]

\[
\eta_{2s} = b_1 + b_2 \xi. \tag{48}
\]
where
\[ a_1 = \Pi_{s \mid j} = \Pi_p - \mathcal{A} \left( \eta_{1s} + \frac{1}{2} \alpha_3 \eta_{2s}^2 \right)_j; \] (49a)
\[ a_2 = (\Pi_{s \mid j+1} - \Pi_{s \mid j}) / \xi_c; \] (49b)
\[ b_1 = \eta_{2s}'; \] (49c)
\[ b_2 = (\eta_{2s}'' + \eta_{2s}) / \xi_c. \] (49d)

Substituting Eqs. (37)–(38), (47)–(48) into Eqs. (20)–(21), and considering the two ends of the pipe as hinge constraint or clamped constraint, and with the calculus of variation, the finite element equation of the \( i \)th element of the straight pipe segment to calculate the perturbation displacements can be derived, as follows:
\[ [M]_s^{e di} \{ \ddot{\eta} \}_s^{d i} + [D]_s^{e di} \{ \dot{\eta} \}_s^{d i} + [K]_s^{e di} \{ \eta \}_s^{d i} = 0; \] (50)
where \([M]_s^{e di}, [D]_s^{e di}\) and \([K]_s^{e di}\) are the mass matrix, damping matrix and stiffness matrix of the \( i \)th element, respectively, and their corresponding expressions are
\[ [M]_s^{e di} = [A]^{-1T} \left\{ [J]_s + [J]_{s1} \right\} [A]^{-1}; \] (51)
\[ [D]_s^{e di} = 2\beta^{1/2} \pi [A]^{-1T} \left\{ [J]_{s1} \right\} [A]^{-1}; \] (52)
\[ [K]_s^{e di} = [A]^{-1T} \left\{ [A][J]_s + [J]_{s2} + \alpha_1 \alpha_2 \alpha_3 \mathcal{A} \left( b_1[J_{s0}] + b_2[J_{s1}] \right) - \alpha_1 a_2 (J_{s1}) + \alpha_2 \mathcal{A} \left( b_1[J_{s0}] + b_2[J_{s1}] \right) + \alpha_1 \alpha_2 \alpha_3 \mathcal{A} \left( b_2^2[J]_s + 2b_1b_2[J_{s1}] + b_2^2[J_{s2}] \right) \right\} [A]^{-1}. \] (53)

### 3.3.2. Analysis of Motion of the Curved Pipe Segment About Static Equilibrium Position

The motion equations, Eqs. (24) and (25), of the curved pipe segment contain such static parameters as \( \Pi_{c \mid s}, \Pi_{c \mid s}' \), \( \eta_{1s} \), and \( \eta_{2s} \). For convenience, it is assumed that the static parameters in each element are the linear functions. Among them, the parameter \( \Pi_{c \mid s} \) is assumed according to Eq. (47), and the corresponding coefficients in the equation at this moment are
\[ a_1 = \Pi_{c \mid s} = \Pi_p - \mathcal{A} (\eta_{1s} - \theta_{2s})_j; \] (54a)
\[ a_2 = (\Pi_{c \mid s+j} - \Pi_{c \mid s}) / \xi_c; \] (54b)

The parameter \( \eta_{1s} \) is assumed according to Eq. (48), and the parameters \( \Pi_{c \mid s}', \Pi_{c \mid s}' \) are assumed as Eqs. (55) and (56) respectively,
\[ \eta_{1s} = c_1 + c_2 \xi; \] (55)
\[ \Pi_{c \mid s}' = d_1 + d_2 \xi; \] (56)
where
\[ c_1 = \eta_{1s}; \] (57a)
\[ c_2 = (\eta_{1s} + \theta_{2s})_j / \xi_c; \] (57b)
\[ d_1 = \Pi_{c \mid s}'; \] (58a)
\[ d_2 = (\Pi_{c \mid s+j} - \Pi_{c \mid s}) / \xi_c; \] (58b)

Substituting Eqs. (37)–(38), (47)–(48), (55)–(56) into Eqs. (24)–(25), and considering the two ends of the pipe as hinge constraint or clamped constraint, and with the calculus of variation, the finite element equation of the \( i \)th element of the curved pipe segment to calculate the perturbation displacements can be derived as follows:
\[ [M]_c^{e di} \{ \ddot{\eta} \}_c^{d i} + [D]_c^{e di} \{ \dot{\eta} \}_c^{d i} + [K]_c^{e di} \{ \eta \}_c^{d i} = 0; \] (59)
where \([M]_c^{e di}, [D]_c^{e di}\) and \([K]_c^{e di}\) are the mass matrix, damping matrix and stiffness matrix of the \( i \)th element, respectively, and their corresponding expressions are
\[ [M]_c^{e di} = [A]^{-1T} \left\{ [J]_c + [J]_{c1} \right\} [A]^{-1}; \] (60)
\[ [D]_c^{e di} = \beta^{1/2} \pi [A]^{-1T} \left\{ [J]_{c1} \right\} [A]^{-1}; \] (61)
\[ [K]_c^{e di} = [A]^{-1T} \left\{ [A][J]_c + [J]_{c2} + \alpha_1 \alpha_2 \alpha_3 \mathcal{A} \left( b_1[J_{c0}] + b_2[J_{c1}] \right) - \alpha_1 a_2 (J_{c1}) + \alpha_2 \mathcal{A} \left( b_1[J_{c0}] + b_2[J_{c1}] \right) + \alpha_1 \alpha_2 \alpha_3 \mathcal{A} \left( b_2^2[J]_c + 2b_1b_2[J_{c1}] + b_2^2[J_{c2}] \right) \right\} [A]^{-1}. \] (62)

The integral \([J]_i\) in the equations above are referred to the Appendix A.

### 4. THE STATIC DEFORMATION AND STABILITY OF STRAIGHT-CURVED PIPE CONVEYING FLUID

It is assumed that point \( A \) in Fig. 1 is the origin of coordinates, then \( \xi_A = 0, \xi_c = L_1/L, \xi_B = 1 \). The other calculation parameters are: \( \xi_c = 1/2, \beta = 0.2, \mathcal{A} = 10000, \Pi_p = 0, \pi \in [0, 3\pi] \).

Considering the following three boundary conditions:

![Figure 1. The straight-curved pipe conveying fluid.](image-url)
4.1. The Simulation Results of the Static Deformation

Among the four theoretical models in the Table 1, models II–IV contain the equations of static equilibrium, and can be used to calculate the static deformation of the pipe conveying fluid. The finite element equations of the straight pipe element and the curved pipe element, which are corresponding to Eq. (40) and (43) respectively, are derived from the equations of static equilibrium. With the element integration method, the following finite element equation to calculate the static deformation of the straight-curved pipe conveying fluid can be obtained:

\[
\begin{bmatrix} [K]_{st} + [K]_{sn} \end{bmatrix} \{\eta\} = \{F\};
\]

where \([K]_{st}\) and \([K]_{sn}\) are the linear part and nonlinear part of the total stiffness matrix respectively, and \(\{F\}\) is the nodal load vector.

The equations of static equilibrium of the theoretical model III and IV are nonlinear, and they were solved by Newton-Raphson iterative technique in this paper.

4.1.1. The Deformation of the Pipe

The static deformations of the pinned-pinned-pinned pipe are shown in Fig. 2. The difference of the results of the static deformation calculated by theoretical model II, III and IV increases with the increase of the flow velocity. When the flow velocity is increased, the deformation of the pipe is magnified by a factor of 10 for clarity.

Figure 2. The static deformation of the pinned-pinned-pinned pipe.
velocity increases from \( \pi = 2.4\pi \) to \( 2.5\pi \), a mutation occurs in the static deformation calculated by the theoretical model II, and the direction of bending deformation of the straight pipe segment changes from down to upward, as shown in Fig. 2(a) and 2(b). When the flow velocity increases from \( \pi = 2.5\pi \) to \( 2.6\pi \), similar mutation occurs in the static deformation calculated by the theoretical model III, as shown in Fig. 2(b) and 2(c). The mutation of the deformation does not appear in the static deformations calculated by the theoretical model IV, and it cannot be explained from the physical perspective. So, when the flow velocity is large, the theoretical model IV is more suitable for the calculation of the static deformation of the pinned-pinned-pinned straight-curved pipe conveying fluid.

The static deformations of the pinned-pinned pipe are shown in Fig. 4. The difference of the static deformation calculated by the theoretical model II and III increases with the increase of the flow velocity. The computing results of the theoretical model II show that when the flow velocity is large, the straight pipe segment will have large lateral displacement. However, the computing results of the theoretical model III show that the lateral deformation of the straight pipe segment is always small. So, when the flow velocity is large, the theoretical

Figure 3. The static deformation of the pinned-sliding bearing-pinned pipe.

(a) \( \pi = 2.4\pi \) (Deformation is magnified by a factor of 5)
(b) \( \pi = 2.5\pi \) (Deformation is magnified by a factor of 5)
(c) \( \pi = 2.8\pi \) (Deformation is magnified by a factor of 5)
(d) \( \pi = 3.0\pi \) (Deformation is magnified by a factor of 5)

Figure 4. The static deformation of the pinned-pinned pipe.

(a) \( \pi = 1.7\pi \) (Deformation is magnified by a factor of 3)
(b) \( \pi = 2.0\pi \) (Deformation is magnified by a factor of 10)
(c) \( \pi = 2.5\pi \) (Deformation is magnified by a factor of 2)
(d) \( \pi = 3.0\pi \) (Deformation is magnified by a factor of 3)
model III is more suitable for the calculation of the static deformation of the pinned-pinned straight-curved pipe conveying fluid.

### 4.1.2. The Static Axial Force of the Pipe

The static axial force of the pipe can be calculated according to the static deformation of the pipe and Eqs. (26) and (27).

With specified flow velocity, the variation curves of the static axial force of the pinned-pinned-pinned pipe varying with the location of the cross section of the pipe are shown in Fig. 5. The additional axial force caused by the bending deformation of the straight pipe segment were not considered in the theoretical model II and III, so only the static axial force of curved pipe segment could be calculated. The theoretical model IV has considered the added additional axial force, so the static axial force of the straight pipe segment and the curved pipe segment could be calculated completely. However, compared with the static axial force of the curved pipe segment, that of the straight pipe segment is very small. For example, if \( \pi = 3.0 \pi \), the static axial force of the straight pipe segment is about 3.5 (shown in Fig. 5(d)), while that of the curved pipe segment is about 89 (shown in Fig. 5(c)). With specified flow velocity, the static axial force of the curved pipe segment calculated by the theoretical models II–IV always undulate around \(- (\pi)^2\). When the flow velocity is small, the magnitude of the difference between the value of the static axial force of the curved pipe segment calculated by the theoretical models II–IV and \(- (\pi)^2\) is \(10^{-1}\). When the flow velocity is large, the magnitude of the difference between the static axial force of the curved pipe segment calculated by the theoretical model II–IV and \(- (\pi)^2\) is \(10^{0}\), while the magnitude of the difference between the value of the combined force of the curved pipe section calculated by the theoretical model IV and \(- (\pi)^2\) remains \(10^{-1}\).

With specified flow velocity, the variation curves of the static axial force of the pinned-sliding bearing-pinned pipe and the pinned-pinned pipe varying with the location of the cross section of the pipe are shown in Fig. 6 and 7 respectively.

From Figs. 6 and 7, the static axial forces of the pinned-sliding bearing-pinned pipe or the pinned-pinned pipe calculated by theoretical models II and III always undulate around \(- (\pi)^2\). Even when the flow velocity is large, the magnitude of the difference between \(- (\pi)^2\) and the calculated forces remains within \(10^{0}\). In addition, the axial force of the pipe is not continuous at the connection position. The reasons for this are that: 1) Equation (26) neglected the angular displacement of the straight pipe element. If the angular displacement was taken into account, the equation for calculating static axial force of the straight pipe element should be

\[
\Pi_{ss} = \Pi_p - A \left( \frac{1 - \cos \varphi}{\cos \varphi} + \frac{1}{\cos \varphi} \eta_1 \right) ;
\]  

where \( \varphi \) is the angular displacement of the straight pipe element. The derivation of this equation refers to Appendix B. When \( \varphi \) is small, \( \cos \varphi \approx 1 \), therefore Eq. (67) could be degenerated into Eq. (26). Obviously, if the angular displacement \( \varphi \) of the straight pipe element was taken into account, the calculation results of the static axial force would be decreased. 2) In

![Figure 5](image-url)
Figure 6. The variation curves of the static axial force of the pinned-sliding bearing-pinned pipe with the location of the cross section of the pipe.

Figure 7. The variation curves of the static axial force of the pinned-pinned pipe with the location of the cross section of the pipe.
the derivation of Eq. (27), it is assumed that the axial elongation caused by the radial displacement $v$ is $-vd\theta$, which is distinct with the actual situation. For the above two reasons, the phenomenon that the static deformation is continuous (as shown in Figs. 3 and 4) but the axial force is not continuous occurs.

### 4.2. The Stability of the Straight-Curved Pipe Conveying Fluid

From the motion equations of the four theoretical models in the Table 1, the finite element equations of the straight pipe and the curved pipe could be derived as Eqs. (50) and (59) respectively. Using the element integration method, the finite element equation to calculate the natural frequencies of the straight-curved pipe conveying fluid could be obtained:

$$[M]_d\{\ddot{\eta}\}_d + [D]_d\{\dot{\eta}\}_d + [K]_d\{\eta\}_d = 0;$$  \hspace{1cm} (68)

where $[M]_d$, $[D]_d$ and $[K]_d$ are the total mass matrix, total damping matrix and total stiffness matrix respectively.

The dimensionless eigenvalue $\omega$ of the straight-curved pipe conveying fluid is

$$\omega = \left(\frac{M_f + M_s}{EI}\right)^{1/2} \Omega L^2;$$  \hspace{1cm} (69)

where $\Omega$ is the circular frequency of the straight-curved pipe conveying fluid. Generally, the eigenvalues $\omega$ is a plural. According to the positive and negative of the real part of the eigenvalue $\omega$, the stability of the system can be analyzed.\(^3,9\)

The first-order eigenvalue of the pinned-pinned straight-curved pipe conveying fluid is shown in Fig. 8.

From the results of the static deformation of the pipe calculated by using the theoretical model II–III, a mutation occurs in the static deformation when the flow velocity reaches a certain value. However, the changing curves of the eigenvalues of the straight-curved pipe conveying fluid system are still continuous with increasing the flow velocity, which is different from the result of literature.\(^22\) It could be explained that when the static deformation of the pipe is abrupt, the static axial force of the curved segment does not change much, and the magnitude of the difference between the values of static axial force and $-(\pi)^2$ is still within $10^6$. The frequency of the straight-curved pipe conveying fluid system is not sensitive to the degree of the change of the static deformation.

The eigenvalues show that the buckling instability of the first-order mode occurs when the flow velocity reaches a certain value. The critical flow velocity in the theoretical model I and Koo and Yoo’s theoretical model\(^{24}\) is $2.5\pi$. The critical flow velocity in the theoretical model II–III is $2.6\pi$. Therefore, when the vibration characteristics of the pinned-pinned straight-curved pipe conveying fluid are qualitatively studied, the four theoretical models proposed in this paper and Koo and Yoo’s theoretical model\(^{24}\) are all right. By considering the static deformation and the geometric nonlinear deformation of the pipe, the additional axial force of straight pipe segment can only improve the accuracy of calculation.

The first-order eigenvalue of the pinned-pinned straight-curved pipe conveying fluid is shown in Fig. 9.

It can be seen that the results calculated by the theoretical model I and Koo and Yoo’s model\(^{24}\) are consistent. The results show that when $\pi = 2.5\pi$, the buckling instability of the first-order mode occurs. The results calculated by the theoretical models II–III are consistent. Their results show that the first-order frequency of the straight-curved pipe system changes little with the increase of the flow velocity, and the real part of the first-order eigenvalue of the system is 0, so the system is stable.

Therefore, when the vibration characteristics of the pinned-sliding bearing-pinned straight-curved pipe conveying fluid is studied, it is necessary to consider the static deformation of the pipe system, while it is not necessary to consider the geometric nonlinear deformation of the pipe.

The first-order eigenvalue of the pinned-pinned straight-curved pipe conveying fluid is shown in Fig. 10. The results calculated by the theoretical model I show that with the increase of flow velocity, the straight-curved pipe system experiences a flutter instability with the first-order modal, but the critical flow velocity cannot be determined. The results calculated by Koo and Yoo’s theoretical model\(^{24}\) show that the straight-curved pipe system occurs a buckling instability with the first-order modal when $\pi = 1.8\pi$. The results calculated by the theoretical models
II–III are consistent. Their results show that the first-order frequency of the straight-curved pipe system changes little with the increase of the flow velocity, and the real part of the first-order eigenvalue of the system is 0, so the system is stable.

Therefore, when the vibration characteristics of the pinned-pinned straight-curved pipe conveying fluid is studied, it is necessary to consider the static deformation of the system, while it is not necessary to consider the geometric nonlinear deformation of the pipe.

5. CONCLUSIONS

In this paper, the dynamic theoretical model for solving the vibration characteristics of the straight-curved pipe conveying fluid was studied and the following conclusions were drawn:

(1) The geometric nonlinear deformation of pipe and the nonlinear force caused by the bend deformation of the straight pipe segment had a great influence on the calculation of the static deformation of the pipe system, while they had little influence on the calculation of the natural frequency of the pipe system.

(2) When the natural frequency of the pipe system was calculated, the static deformation of the system was considered depending on the situation of the restraint of the system. When the natural frequencies of the pinned-pinned pipe or the pinned-sliding bearing-pinned pipe were solved, the static deformation of the pipes must be considered. When the natural frequencies of the pinned-pinned-pinned pipe were solved, the static deformation of the pipe could be ignored.

(3) When the fluid flows through the curved pipe, the pipe system would undergo static deformation. In this case, the value of the static axial force of the pipe was approximately the square of the flow velocity. In addition, the change of the static axial force of the pipe within the magnitude of 10^9 had no influence on the calculation of the frequency of the system. Therefore, the square of flow velocity could be used as the static axial force of the pipe and could be substituted into the equation of motion of the pipe about the static equilibrium position so as to simplify the theoretical models.

(4) The stability of the pipe system was related to the boundary conditions. When both ends of the pipe and the connecting point between the straight pipe segment and the curved pipe segment were pinned, the first-order modal buckling instability occurred. The system maintained stability when there was a pinned constraint on the two end points of the pipe and there was a sliding bearing or a non-constraint on the connection point between the straight pipe segment and the curved pipe segment.
ACKNOWLEDGEMENTS

This work is supported by the National Natural Science Foundation of China (Grant Nos: 11772273); the Opening Project of Sichuan Province University Key Laboratory of Bridge Non-destruction Detecting and Engineering Computing, China (Grant Nos: 2016QZJ03). Fund Project of Sichuan University of Science & Engineering in hit-haunting for tal-ling, China (Grant Nos: 2016QZJ03). Fund Project of Sichuan Bridge Non-destruction Detecting and Engineering Compu-

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APPENDIX A

Integrals \([J_i]\) are defined as follows:

\[
\begin{align*}
[J_1] &= \int_0^{\xi_1} [\Phi_1]^T[\Phi_1] d\zeta; \\
[J_2] &= \int_0^{\xi_2} [\Phi_2]^T[\Phi_2] d\zeta; \\
[J_3] &= \int_0^{\xi_3} [\Phi_3]^T[\Phi_3] d\zeta; \\
[J_4] &= \int_0^{\xi_4} [\Phi_4]^T[\Phi_4] d\zeta; \\
[J_5] &= \int_0^{\xi_5} [\Phi_5]^T[\Phi_5] d\zeta; \\
[J_6] &= \int_0^{\xi_6} [\Phi_6]^T[\Phi_6] d\zeta; \\
[J_7] &= \int_0^{\xi_7} [\Phi_7]^T[\Phi_7] d\zeta; \\
[J_8] &= \int_0^{\xi_8} [\Phi_8]^T[\Phi_8] d\zeta; \\
[J_9] &= \int_0^{\xi_9} [\Phi_9]^T[\Phi_9] d\zeta; \\
[J_{10}] &= \int_0^{\xi_{10}} [\Phi_{10}]^T[\Phi_{10}] d\zeta; \\
[J_{11}] &= \int_0^{\xi_{11}} [\Phi_{11}]^T[\Phi_{11}] d\zeta; \\
[J_{12}] &= \int_0^{\xi_{12}} [\Phi_{12}]^T[\Phi_{12}] d\zeta; \\
[J_{13}] &= \int_0^{\xi_{13}} [\Phi_{13}]^T[\Phi_{13}] d\zeta; \\
[J_{14}] &= \int_0^{\xi_{14}} [\Phi_{14}]^T[\Phi_{14}] d\zeta; \\
[J_{15}] &= \int_0^{\xi_{15}} [\Phi_{15}]^T[\Phi_{15}] d\zeta; \\
[J_{16}] &= \int_0^{\xi_{16}} [\Phi_{16}]^T[\Phi_{16}] d\zeta; \\
[J_{17}] &= \int_0^{\xi_{17}} [\Phi_{17}]^T[\Phi_{17}] d\zeta; \\
[J_{18}] &= \int_0^{\xi_{18}} [\Phi_{18}]^T[\Phi_{18}] d\zeta; \\
[J_{19}] &= \int_0^{\xi_{19}} [\Phi_{19}]^T[\Phi_{19}] d\zeta; \\
[J_{20}] &= \int_0^{\xi_{20}} [\Phi_{20}]^T[\Phi_{20}] d\zeta; \\
[J_{21}] &= \int_0^{\xi_{21}} [\Phi_{21}]^T[\Phi_{21}] d\zeta; \\
[J_{22}] &= \int_0^{\xi_{22}} [\Phi_{22}]^T[\Phi_{22}] d\zeta; \\
[J_{23}] &= \int_0^{\xi_{23}} [\Phi_{23}]^T[\Phi_{23}] d\zeta; \\
[J_{24}] &= \int_0^{\xi_{24}} [\Phi_{24}]^T[\Phi_{24}] d\zeta; \\
[J_{25}] &= \int_0^{\xi_{25}} [\Phi_{25}]^T[\Phi_{25}] d\zeta; \\
[J_{26}] &= \int_0^{\xi_{26}} [\Phi_{26}]^T[\Phi_{26}] d\zeta; \\
[J_{27}] &= \int_0^{\xi_{27}} [\Phi_{27}]^T[\Phi_{27}] d\zeta; \\
[J_{28}] &= \int_0^{\xi_{28}} [\Phi_{28}]^T[\Phi_{28}] d\zeta; \\
[J_{29}] &= \int_0^{\xi_{29}} [\Phi_{29}]^T[\Phi_{29}] d\zeta; \\
[J_{30}] &= \int_0^{\xi_{30}} [\Phi_{30}]^T[\Phi_{30}] d\zeta; \\
[J_{31}] &= \int_0^{\xi_{31}} [\Phi_{31}]^T[\Phi_{31}] d\zeta; \\
[J_{32}] &= \int_0^{\xi_{32}} [\Phi_{32}]^T[\Phi_{32}] d\zeta; \\
[J_{33}] &= \int_0^{\xi_{33}} [\Phi_{33}]^T[\Phi_{33}] d\zeta; \\
[J_{34}] &= \int_0^{\xi_{34}} [\Phi_{34}]^T[\Phi_{34}] d\zeta; \\
[J_{35}] &= \int_0^{\xi_{35}} [\Phi_{35}]^T[\Phi_{35}] d\zeta; \\
[J_{36}] &= \int_0^{\xi_{36}} [\Phi_{36}]^T[\Phi_{36}] d\zeta; \\
[J_{37}] &= \int_0^{\xi_{37}} [\Phi_{37}]^T[\Phi_{37}] d\zeta; \\
[J_{38}] &= \int_0^{\xi_{38}} [\Phi_{38}]^T[\Phi_{38}] d\zeta; \\
[J_{39}] &= \int_0^{\xi_{39}} [\Phi_{39}]^T[\Phi_{39}] d\zeta; \\
[J_{40}] &= \int_0^{\xi_{40}} [\Phi_{40}]^T[\Phi_{40}] d\zeta.
\end{align*}
\]

APPENDIX B

Take the infinitesimal segment of the straight pipe, with the axial displacement of the two ends not being constrained at the same time, as is shown in Fig. B.1. The elongation of the infinitesimal segment is

\[
ds = \frac{1 - \cos \varphi}{\cos \varphi} dx + \frac{1}{\cos \varphi} (u_2 - u_1). \tag{B.1}
\]

So, the axial force of the infinitesimal segment is

\[
Q_x = EA_\ell \left( \frac{1 - \cos \varphi}{\cos \varphi} + \frac{1}{\cos \varphi} \frac{\partial u}{\partial x} \right). \tag{B.2}
\]

According to Eq. (3), the following steady state value of the combined force of the infinitesimal segment can be achieved:

\[
\Pi_{ss} = \Pi_p - A \left( \frac{1 - \cos \varphi}{\cos \varphi} + \frac{1}{\cos \varphi} \eta_\ell \right). \tag{B.3}
\]
Location Optimization of Monopole Equivalent Sources in Wave Superposition Method

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(Received 10 December 2017; accepted 5 February 2018)

In wave superposition method, the prediction accuracy of acoustic pressure heavily depends on the locations of equivalent sources. In this paper, the prediction accuracy corresponding to monopole equivalent source is studied. According to analysis in this paper, when the velocities on some boundary nodes are inversely calculated using the predicted pressures, there is velocity reconstruction error, and the prediction error of the acoustic pressure and the velocity reconstruction error are closely related. The relationship between them is theoretically derived and indicates that the prediction error decreases with a decrease in velocity reconstruction error. Based on these findings, a method to determine the optimal locations by minimizing the normalized velocity reconstruction error is proposed. A frequency threshold criterion is devised to give the frequency range for certain number of equivalent sources within which good prediction accuracy of the acoustic pressure can be obtained. The proposed method is validated by simulation and experiment, respectively. The results show that the method significantly reduces prediction errors and is feasible.

1. INTRODUCTION

Boundary element method (BEM) is established as a well-known numerical tool for predicting acoustic pressure in infinite domain. In the BEM, boundary surface discretization is only required, and the Sommerfeld radiation condition at infinity is naturally satisfied. However, the system matrices of BEM are usually non-sparse and non-symmetric, which increases the processing time and storage requirements. Furthermore, the non-uniqueness problem occurs at characteristic wave numbers corresponding to interior problems. Different methods including the CHIEF and the Burton-Miller methods to overcome the non-uniqueness problem have been devised. In the CHIEF method, the integral formulation is modified by adding equations to enforce solutions to vanish at points in the interior. Thus, the fictitious solutions can be differentiated from the true ones. But there is a lack of rigorous criteria for selecting interior points and determining the limit of stability. The Burton-Miller method, in which the integral equation is combined with its normal derivative, theoretically precludes non-unique solutions. However, the various orders of singular integrals of Green’s function must be numerically described and can lead to inefficiencies in computation. During the past few decades, tremendous progresses in the development of BEM are moving the application of BEM in predicting acoustic radiation in infinite domain. For example, with the recent developments in fast multipole BEM, the computational efficiency is significantly improved. The relevant work of BEM is still on its way.

Koopmann et al. proposed the wave superposition method (WSM) based on the idea that the radiated acoustic field of a radiator can be constructed as a superposition of the fields generated by an array of equivalent sources located on an auxiliary surface in the radiator. The source strengths are determined by the specific normal velocity distribution on boundary surface. Thus, the singular integrals of Green’s function, which are involved in BEM when the acoustic pressure prediction points are located on the boundary surface, are eliminated. Only matrix operations are needed for the acoustic pressure prediction. Therefore, this method greatly simplifies the process of acoustic pressure prediction and is easily realized using computer programming. WSM has been applied to calculation of acoustic radiation. However, the prediction accuracy of acoustic pressure using this method strongly depends on the locations of the equivalent sources, particularly for the radiators with complex geometric shapes. Determining the locations becomes a problem. If the sources are far from the boundary surface, the system equations become ill-conditioned and the prediction accuracy of acoustic pressure is greatly affected. Alternatively, if equivalent sources are located near the boundary surface, the singularity occurs and erroneous predicted results will be obtained. The equivalent source locations are distinguishing for different structures. These lead to that until now there is no available commercial software of WSM to predict acoustic pressure.

To determine the equivalent source locations, some researches have been proceeded. Bai provided an effective methodology for finding the optimal distance for WSM ap-
lications, and indicated that the optimal distance was not a unique value and may well depend on many factors. Fahl-
line studied the prediction accuracy and stability of the wave
superposition method using singular-value decomposition, and
provided physical insight into the nature of an acoustic field
by the approximate singular-function expansion of the acous-
tic radiation. His research further showed that the nodes
should be equally spaced. The cause of the non-uniqueness
problem (due to too small singular value) was also discussed.
Hwang and Chang developed a regular integral equation for
the exterior acoustic radiation based on the surface source dis-
tribution. It has been indicated that the offset distance be-
tween the equivalent source and boundary surface must be
larger than one-quarter of element size for a certain meshing
pattern. Gounot and Musafir devised a genetic algorithm to
search for the optimal equivalent source sets. This ap-
proach permits a good reconstitution of the pressure field by
using very few monopoles. However, this search algorithm
fails for large number of sources. Zellers and Wu analyzed
the diagonal terms in related matrices. The diagonal terms
were substituted by the derived approximate analytical ex-
spression self-terms. As an example of the application of the tech-
nique, acoustic radiation from a uniformly pulsating sphere
was analyzed and compared with the analytic solution. The
results showed that the approximate analytical expressions of
the monopoles were only consistent with the analytical solu-
tions in the low-frequency range.

In this paper, the relationship between the prediction error
of the acoustic pressure and velocity reconstruction error on
the boundary is theoretically derived. An optimization method
to determine the locations of equivalent sources is proposed to
improve the prediction accuracy. A frequency threshold cri-
teron is devised to make the predicted pressure accuracy ac-
ceptable. The applicability of this proposed method is demon-
strated by numerically investigating the acoustic field of two
models including a uniformly pulsating sphere and designed
shell structure. The proposed method is further validated by
experiments of a cylindrical shell and a clamped plate.

2. THEORETICAL BACKGROUND

The wave superposition method (WSM) is based on the idea
that the radiated acoustic field of a radiator can be constructed
as a superposition of the fields generated by an array of sim-
ple sources, which are called equivalent sources and located
on an auxiliary surface enclosed in the radiator. The resulting
pressure at a receiver point \( r \) is defined as

\[
p(r) = i\rho \omega \int_V q(r_0) g(|r - r_0|) dV(r_0);
\]

where \( i^2 = -1 \); \( \rho \) is the mean density of the medium; \( \omega \) is
the angular frequency of the harmonic vibration of surface
\( S \), which encloses volume \( V \) of the radiator; \( q(r_0) \) is the
strength of the simple source distribution, which is evaluated
at \( r_0 \) inside \( V \), as shown in Fig. 1a; and \( g(|r - r_0|) =
-\frac{e^{ik|r-r_0|}}{4\pi |r-r_0|} \) is the free-space Green’s function,

\[
\hat{p}(r) = \sum_{i=1}^{N} s_i g(r, r_i);
\]

where \( \hat{p}(r) \) denotes an approximation of \( p(r) \); \( r_i \) is the location
of the \( i \)th equivalent source on the auxiliary surface, as shown
in Fig. 1b; and \( s_i \) is the source strength of the \( i \)th equivalent
source.

The boundary condition on the surface of the radiator is de-
defined by:

\[
 \nabla p(r_s) \cdot \mathbf{n}_s = -ik \rho c v(r_s) \text{ on } S;
\]

where \( \nabla \) represents the gradient operator with respect to the
receiver location \( r_s \) on the boundary surface; \( \mathbf{n}_s \) is the unit
normal vector at location \( r_s \) on the boundary surface; and \( v(r_s) \)
is the normal velocity at \( r_s \). Substituting Eq. (2) into Eq. (3)
and applying it to \( N \) nodes on the surface of the radiator, the

\[
 p(r) = i\rho \omega \int_V q(r_0) g(|r - r_0|) dV(r_0);
\]

\[
 \hat{p}(r) = \sum_{i=1}^{N} s_i g(r, r_i);
\]

\[
 \nabla p(r_s) \cdot \mathbf{n}_s = -ik \rho c v(r_s) \text{ on } S;
\]
source strengths can be determined. Thus, the radiated acoustic pressure at any field point is determined as:

$$\hat{p}(r) = \sum_{j=1}^{N} \left( \sum_{i=1}^{N} g(r, r_j)D_{ij}^{-1} \right) v(r_{sj});$$  (4)

where \( v(r_{sj}) \) is the normal velocity corresponding to the \( j \)th node on the boundary; and:

$$D_{ij} = \frac{1}{ik\rho c} \frac{\partial g(r_{xi}, r_j)}{\partial n_i}. $$  (5)

3. EQUIVALENT SOURCE LOCATION OPTIMIZATION

3.1. Velocity Reconstruction Error

Equation (3) indicates that the pressure and the vibration velocity on the boundary are not independent. In this section, the relationship between the prediction error of acoustic pressure and the surface velocity reconstruction error is analyzed. From Eqs. (3) and (4), the normal velocity at arbitrary location \( r_s \) on the boundary can be expressed as:

$$\hat{v}(r_s) = \sum_{j=1}^{N} \left( \frac{1}{ik\rho c} \sum_{i=1}^{N} \frac{\partial g(r_s, r_i)}{\partial n_s} D_{ij}^{-1} \right) v(r_{sj}) = \sum_{j=1}^{N} N_j(r_s) v(r_{sj});$$  (6)

where \( N_j(r_s) \) is the velocity interpolation function. When \( r_s = r_{sh} \) (\( r_{sh} \) is the location of the \( h \)th node on the boundary surface), \( N_j(r_{sh}) \) is written as:

$$N_j(r_{sh}) = \frac{1}{ik\rho c} \sum_{i=1}^{N} \frac{\partial g(r_{sh}, r_i)}{\partial n_h} D_{ij}^{-1} = \sum_{i=1}^{N} D_{hi} D_{ij}^{-1}. $$  (7)

Equation (7) indicates that \( N_j(r_{sh}) \) is the summation of the product of the \( h \)th row of matrix \( D \) and the \( j \)th column of \( D^{-1} \). Thus, \( N_j(r_{sh}) \) is exactly unity when \( h = j \) and zero when \( h \neq j \) due to \( DD^{-1} = E \), where \( E \) is the identity matrix. Therefore, \( N_j(r_{sh}) = \delta_{h,j} \), where \( \delta_{h,j} \) is the Kronecker delta. Because of the property of the Kronecker delta, the calculated surface normal velocities on the nodes are identical to the prescribed values, which are used to obtain the source strengths, i.e., \( \hat{v}(r_{sh}) = v(r_{sh}) \), \( h = 1, 2, \cdots, N \). Because the prescribed normal velocities on these surface nodes can be exactly reconstructed and the imaginary part of \( N_j(r_s) \) is zero at these nodes, the imaginary part of \( N_j(r_s) \) vanishes when \( N \) approaches infinity. However, it is not always zero for finite \( N \) values. This behavior results in velocity reconstruction errors on the surface, i.e., the reconstructed velocity \( \hat{v}(r_s) \) is not identical to the prescribed value \( v(r_s) \) when \( r_s \neq r_{sh} \).

In this paper, the errors of the predicted acoustic pressure and the reconstructed velocity on the boundary surface are represented by \( \varepsilon_p = p(r_s) - \hat{p}(r_s) \) and \( \varepsilon_v = v(r_s) - \hat{v}(r_s) \), respectively, where \( v(r_s) \cdot n_s = v(r_s) \) and \( \hat{v}(r_s) \cdot n_s = \hat{v}(r_s) \).

Because \( \hat{p}(r_s) \) and \( \hat{v}(r_s) \) satisfy Eq. (3), the following equation is obtained.

$$ik\rho c v(r_s) = \nabla \varepsilon_p(r_s). $$  (8)

The linearized equations of the conservation of mass and state are given as:

$$\frac{\partial \rho'(r_s, t)}{\partial t} + \rho \nabla \cdot v(r_s, t) = 0, \rho(r_s, t) \approx c^2 \rho'(r_s, t); $$  (9)

where \( \rho'(r_s, t) \) is the fluctuating quantity of the medium density. Only considering harmonic steady state conditions, the pressure \( p(r_s, t) \) and vibration velocity \( v(r_s, t) \) can be written as \( p(r_s, t) = p(r_s)e^{-\imath \omega t} \) and \( v(r_s, t) = v(r_s)e^{-\imath \omega t} \). Their approximations are given as \( \hat{p}(r_s, t) = \hat{p}(r_s)e^{-\imath \omega t} \) and \( \hat{v}(r_s, t) = \hat{v}(r_s)e^{-\imath \omega t} \). Thus, the following equation is obtained.

$$\nabla \cdot \varepsilon_v(r_s) = \frac{ik}{\rho c} \varepsilon_p(r_s). $$  (10)

Substituting Eq. (8) into Eq. (10) yields:

$$\nabla^2 \varepsilon_p(r_s) + k^2 \varepsilon_p(r_s) = 0. $$  (11)

It can be noted that \( \varepsilon_p(r_s) \) satisfies the Helmholtz equation. Equation (8) can be rewritten as:

$$ik\rho c v(r_s) \cdot n_s = \frac{\partial \varepsilon_p(r_s)}{\partial n_s}. $$  (12)

Thus, this boundary condition is satisfied by \( \varepsilon_p(r_s) \). Because both \( p(r) \) and \( \hat{p}(r) \) satisfy the Sommerfeld radiation condition, the following equation at infinity is obtained.

$$\lim_{r \rightarrow \infty} r^\alpha \left( \frac{\partial \varepsilon_p(r)}{\partial r} + ik\varepsilon_p(r) \right) = 0; $$  (13)

where \( r \) denotes a cylindrical or spherical polar radius, and \( \alpha \) is equal to \( 1/2 \) in the two-dimensional case (2D) and equal to \( 1 \) in three-dimensional case (3D). Hence, \( \varepsilon_p(r_s) \) satisfies the Sommerfeld radiation condition at infinity.

Analogous to the pressure wave, an error wave of the pressure prediction that corresponds to the velocity reconstruction error appears to be radiated from the radiator. Obviously, \( \varepsilon_p(r_s) \) is small when \( \varepsilon_v(r_s) \) is small. Thus, \( \varepsilon_v(r_s) \) can be used to reflect the prediction accuracy of the acoustic pressure. As a result, the optimal locations can be determined by minimizing the velocity reconstruction error. In WSM, the non-uniqueness problems of monopole equivalent source at the eigen-frequencies that correspond to the locations of the sources can be eliminated by adjusting the locations. Therefore, the non-uniqueness problems can be avoided in the optimal locations.

3.2. Equivalent Source Location Determination

To accurately predict the acoustic pressure, the dipole matrix \( D \) should be diagonally dominant. This condition requires that each source must be paired with the corresponding node on the boundary surface, and that the distance between the source
and node is less than that from any other source to the node. However, only a few structures, such as the sphere and infinite circular cylinder, can satisfy this condition. To reduce the optimization parameters for determining the optimal locations, equivalent sources are located on an auxiliary surface that is retracted from the actual structure boundary by scale coefficient $Sc (0 < Sc < 1)$. The auxiliary surface has a similar geometric shape to the radiator. The coordinates of the sources are obtained by multiplying $Sc$ with the surface nodes. Thus, the parameters are reduced to one.

To determine the optimal $Sc$, a method is proposed by minimizing the normalized velocity reconstruction error, which is defined as:

$$
\varepsilon_v = \frac{\int |v(r_s) - \hat{v}(r_s)|dS}{\int |v(r_s)|dS} \times 100\% \approx \frac{\sum_{j=1}^{N} S_j |v(r_{sj}) - \hat{v}(r_{sj})|}{\sum_{j=1}^{N} S_j |v(r_{sj})|} \times 100\%. \quad (14)
$$

To calculate $\varepsilon_v$, the boundary surface is divided into two meshing patterns: modes A and B. In Eq. (14), $S_j$ is the area of the $j$th node in mode $B$. Here, it should be noted that the nodes of mode A do not coincide with those of mode B. The nodes of mode A are used to obtain equivalent sources, and the prescribed velocities on the nodes are used to calculate the acoustic pressure. According to Eq. (6), the constructed velocities $\hat{v}(r_{sj})$ on the nodes of mode $B$ are interpolated using the prescribed velocities on the nodes of mode A for each $Sc$. For each $Sc$, $\varepsilon_v$ is obtained by substituting $\hat{v}(r_{sj})$ and the prescribed velocities $v(r_{sj})$ on the nodes of mode $B$ into Eq. (14). Then $Sc$ that corresponds to the minimum $\varepsilon_v$ is selected as the optimal value. The radiated acoustic pressure is predicted in the determined auxiliary surface. The designed objective function is:

$$
\left\{ \begin{array}{l}
\text{Objective function} = \min(\varepsilon_v(Sc, k)) \\
\text{ } \quad Sc \in (0, 1), k = k_1, k_2, \ldots, k_n.
\end{array} \right. \quad (15)
$$

### 3.3. Frequency Threshold Criterion

It is known that the acoustic wavelength decreases with the increase in vibration frequency. Therefore, for a certain number of equivalent sources, the threshold of frequency within which the prediction accuracy of acoustic pressure in WSM is acceptable is limited. Besides that, the threshold is distinguishing when the equivalent sources are located on different auxiliary surfaces. Hence, it is indispensable to give the threshold in the optimal auxiliary surface to ensure the predicted results are acceptable for the given number of equivalent sources.

When the sources are located on the determined auxiliary surface, the radiated acoustic pressure in the exterior region to radiator can be expressed as:

$$
p(r) = i\rho_0\omega_\tau \int_{\sigma} q(r_\sigma) g(r, r_\sigma) d\sigma(r_\sigma); \quad (16)
$$

where $\sigma$ is the determined auxiliary surface and $\delta_\tau$ is a constant value for thickness.\(^{10}\) The discretization of Eq. (16) can be achieved by subdividing the auxiliary surface into a set of small quadrilateral or triangular elements. The integral over the auxiliary surface is approximated by summations of integrals over each element. The spatial coordinates and acoustic variables within an element can be related to the nodal values by shape functions. Then, the appropriately weighted Gaussian quadrature formula is used to calculate the element integrals. Using the isoparametric surface element, the global coordinate $r_\sigma$ and the source strength $q(r_\sigma)$ on each element can be approximated by:

$$
r_\sigma = \sum_{h=1}^{M} N_h r_h \quad \text{and} \quad q = \sum_{h=1}^{M} N_h q_h; \quad (17)
$$

where $N_h$ is the known shape function; $r_h$ is the local coordinate; $q_h$ is the nodal values of the source strength; and $M$ is the number of element nodes.

The discretized form of Eq. (16) based on the isoparametric transformation is given as:

$$
p(r) = i\rho_0\omega_\tau \sum_{i=1}^{K} \sum_{h=1}^{M} H_{ih}(r) q_h; \quad (18)
$$

where $K$ is the number of elements and $H_{ih}(r)$ is given as:

$$
H_{ih}(r) = \int_{\sigma_i} g(r, r_\sigma) N_h d\sigma(r_\sigma). \quad (19)
$$

Equation (19) represented in the $s-t$ coordinate, for quadrilateral element and triangular element respectively, are the form of:

$$
H_{ih}(r) = \int_{-1}^{1} \int_{-1}^{1} g(r, r_\sigma(s, t)) N_h(s, t) J(s, t) ds dt; \quad (20)
$$

$$
H_{ih}(r) = \int_{-1}^{1} \int_{-1}^{1} g(r, r_\sigma(s, t)) N_h(s, t) J(s, t) ds dt; \quad (21)
$$

where $J(s, t)$ is the Jacobian of the transformation. For two-dimensional Gaussian quadrature formula, the upper bound of error is given as:\(^{20}\):

$$
\left| \int_{-1}^{1} \int_{-1}^{1} H(s, t) ds dt - \sum_{l=1}^{m} \sum_{j=1}^{n} w_l w_j H(s_l, t_j) \right| \leq 2(E_1 + E_2); \quad (22)
$$

where $m$ and $n$ denote the number of Gauss integral points in $s$ and $v$ direction, respectively; $w_l$ and $w_j$ are the Gauss weighting factor for the corresponding Gauss points $s_l$ and $s_j$; $E_1$ and $E_2$ are the estimated relative errors.

The primary variations of the integrands in Eq. (20) and Eq. (21) are determined by the terms that can be characterized by $e^{-ikr}/r^p$, $p = 1$ or 2. Therefore, the following integral can be representative of the actual integrals to estimate the Gaussian quadrature error bounds:

$$
H = \int_{\sigma_i} e^{-ikr} d\sigma, \quad p = 1, 2, 3. \quad (23)
$$
Considering that the Jacobian is almost constant within an element and the distance from the field point to the element is not severely varied, $E_1$ and $E_2$ are determined as:\(^{20}\):

$$E_1 = \frac{\pi}{2(p-1)!} \left( \frac{kL_1}{4} \right)^{2n} \sum_{l=0}^{2n} \frac{(l + p - 1)!}{l!(2m - l)!} \left( \frac{1}{kr_{\text{min}}} \right);$$

$$E_2 = \frac{\pi}{2(p-1)!} \left( \frac{kL_2}{4} \right)^{2n} \sum_{l=0}^{2n} \frac{(l + p - 1)!}{l!(2m - l)!} \left( \frac{1}{kr_{\text{min}}} \right);$$

where $L_1 = \max(L_{12}, L_{34})$ and $L_2 = \max(L_{23}, L_{41}); r_{\text{min}}$ is the minimum distance between the field point on the boundary and the element. To ensure convergence of the Gaussian quadrature, $k < \min(4/L_1, 4/L_2)$. For triangular element, the same criteria can be obtained. Let $L_{\text{max}}$ denote the maximal size of all the elements on the optimal auxiliary surface. To make the prediction accuracy of acoustic pressure acceptable, the criterion should be satisfied.

$$k < 4/L_{\text{max}}.$$

### 4. NUMERICAL SIMULATION

#### 4.1. Validation of Equivalent Source Location Determination Method

In what follows, acoustic radiations from a uniformly pulsating sphere with radius $a = 1$ m and vibration velocity amplitude $v = 1$ m/s, and a radiator, as shown in Fig. 2, are presented to illustrate the use of the method. The acoustic medium is air with its density $\rho = 1.21$ kg/m$^3$, and the speed of acoustic $c = 343$ m/s. For the two radiators, the numbers of elements on the boundary for modes $A$ and $B$ are shown in Table 1, where TE and QE represent triangular and quadrilateral elements, respectively. Although there is no known analytical acoustic pressure for the designed radiator, an exact solution can be obtained using the substitute velocity boundaries, which is equivalent to that from a point source, that is, a simulation point source, in the structure surface. The prescribed normal velocity on the boundary surface is generated by placing an acoustic point source in the radiator. If the radiator vibrates under such a boundary condition, the radiated pressure must be equal to that generated by the point source. A monopole source that is a simulation point source is set at a fixed location with coordinate $(0.15 \text{ m}, 0 \text{ m}, 0 \text{ m})$ for the designed radiator.

<table>
<thead>
<tr>
<th>Mode A</th>
<th>Mode B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>82 TE</td>
</tr>
<tr>
<td></td>
<td>83 TE</td>
</tr>
<tr>
<td>Designed</td>
<td>774 TE+400QE</td>
</tr>
</tbody>
</table>

Table 1. The numbers of elements on the boundary for modes $A$ and $B$.

In this paper, $\varepsilon_1(\%)$ and $\varepsilon_2(\%)$ is used to represent the relative errors of the real and imaginary components of the predicted acoustic pressure, respectively. The root-mean square norms of them are given by:

$$\varepsilon_{\text{real}} = \sqrt{\frac{1}{M} \sum_{m=1}^{M} \varepsilon_{1m}^2}, \quad \varepsilon_{\text{imag}} = \sqrt{\frac{1}{M} \sum_{m=1}^{M} \varepsilon_{2m}^2};$$

where $\varepsilon_{1m}$ and $\varepsilon_{2m}$ represent the relative errors of the real and imaginary components of the acoustic pressure at node $x_m$, respectively. To study the effect of the equivalent source locations on the accuracy of the predicted result, $\varepsilon_{\text{v}}, \varepsilon_{\text{real}}$ and $\varepsilon_{\text{imag}}$ on boundary nodes of the two radiators are plotted for different $k$ and $S_c$ values in Figs. 3 and 4.

These results show that the locations have a remarkable influence on the accuracy of the predicted acoustic pressure, and that the errors change with $S_c$. Clearly, it can be seen that $\varepsilon_{\text{v}}, \varepsilon_{\text{real}}$ and $\varepsilon_{\text{imag}}$ show the almost identical trend, the prediction accuracy increases when $\varepsilon_{\text{v}}$ decreases, and good prediction accuracy is obtained in the optimal auxiliary surface corresponding to minimum $\varepsilon_{\text{v}}$, hence $\varepsilon_{\text{v}}$ is a good indicator of the prediction accuracy of the acoustic pressure. It is also observed that the prediction accuracy is relatively more sensitive to the source locations for complex radiator. From the data of the errors, the accuracy of the predicted values is acceptable at $0.19 \leq S_c \leq 0.37$ for the designed radiator.

To further test the effectiveness of the proposed method, the predicted acoustic pressure in optimal auxiliary surface is compared to those in other retracted surfaces on an arbitrary selected field point, $P_1 = (0 \text{ m}, 0 \text{ m}, 1 \text{ m})$ and $P_2 = (0.15 \text{ m}, 0 \text{ m}, 0.5 \text{ m})$ for the sphere and designed radiator, respectively. Figures 5 and 6 show the comparison results, where the calculation frequency $k$ satisfies the requirement of $k \leq 4/L_{\text{max}}$.

The comparison results show that the numerical solutions in the optimal auxiliary surface are consistent with the exact solutions, and the prediction accuracy is significantly improved in the optimal auxiliary surface. Furthermore, it is shown again that the prediction accuracy is relatively more sensitive to the locations for complex radiator. According to analysis, the maximum errors of the predicted results are $1.4 \times 10^{-8} \%$ and $5.6 \times 10^{-3} \%$ for the sphere and designed radiator, respectively. However, in other retracted auxiliary surfaces, the prediction accuracy is very poor and the predicted result is unacceptable. Therefore, the optimal auxiliary surface must be determined.

#### 4.2. Validation of Frequency Threshold Criterion

As previously analyzed, for a certain number of equivalent sources, the prediction accuracy is only good in a certain frequency range. Hence, it is necessary to judge the frequency range in which the prediction accuracy is acceptable. Figure 7 shows the plots of $\varepsilon_{\text{real}}$ and $\varepsilon_{\text{imag}}$ on boundary field nodes calculated in optimal auxiliary surfaces for different $k$. Obvi-
Figure 3. Plots of $\epsilon_v$, $\epsilon_{\text{real}}$ and $\epsilon_{\text{imag}}$ on boundary nodes of uniformly pulsating sphere for different $k$ and $S_c$ values.

Table 2. $k_{\text{threshold}}$ and maxima of $\epsilon_{\text{real}}$ and $\epsilon_{\text{imag}}$ on boundary field points calculated in optimal auxiliary surfaces when $k < k_{\text{threshold}}$.

<table>
<thead>
<tr>
<th>Surface</th>
<th>$k_{\text{threshold}}$</th>
<th>$\epsilon_{\text{real}}$</th>
<th>$\epsilon_{\text{imag}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>900</td>
<td>$2.2 \times 10^{-3}$</td>
<td>1.00</td>
</tr>
<tr>
<td>Designed</td>
<td>270</td>
<td>0.70</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Figure 4. Plots of $\epsilon_v$, $\epsilon_{\text{real}}$ and $\epsilon_{\text{imag}}$ boundary nodes of designed radiator for different $k$ and $S_c$ values.

5. EXPERIMENTAL INVESTIGATION

To further validate the proposed method, the sound radiation of two radiators, namely, a cylindrical shell and a clamped plate are investigated experimentally, the physical data of which are shown in Table 3. The cylindrical shell with flanges of 1.5 cm thickness welded at its both ends is fixed on a still frame by bolt to approximate the clamped boundary condition, as shown in Fig. 8a. Both ends of the cylindrical shell are sealed by two aluminum alloy end plates with a thickness of 2 cm. This material is chosen because of its high stiffness and low weight. An ordinary 10 inch loudspeaker is positioned on an end plate to generate an interior sound field, which excites the cylindrical shell, as shown in Fig. 8c. The resulting structural vibration causes an exterior sound field around the cylin-
Figure 5. Variation of pressure in the selected field point with $k$ for uniformly pulsating sphere.

Table 3. Physical data of two radiators.

<table>
<thead>
<tr>
<th></th>
<th>Cylindrical shell</th>
<th>Clamped plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (m)</td>
<td>0.60</td>
<td>0.50</td>
</tr>
<tr>
<td>External diameter (m)</td>
<td>0.35</td>
<td>1.08.2018</td>
</tr>
<tr>
<td>Width (m)</td>
<td>0.50</td>
<td>1.05.2018</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>1.05.2018</td>
<td>1.08.2018</td>
</tr>
<tr>
<td>Density (kg/m$^3$)</td>
<td>7900</td>
<td>7800</td>
</tr>
<tr>
<td>Young’s modulus (N/m)</td>
<td>$1.95 \times 10^{11}$</td>
<td>$2.1 \times 10^{11}$</td>
</tr>
<tr>
<td>Poisson Ratio</td>
<td>0.247</td>
<td>0.30</td>
</tr>
<tr>
<td>Loss factor</td>
<td>$10^{-4} \sim 10^{-3}$</td>
<td>$10^{-5} \sim 10^{-3}$</td>
</tr>
</tbody>
</table>

The surfaces of the cylindrical shell and the clamped plate are divided evenly into 30 and 25 elements, respectively. The nodes of elements are used to obtain equivalent sources, and the normal velocities on the nodes are used to calculate the source strengths and the sound pressure. The constructed velocities $\hat{v}(r_{sj})$ on the centers of the elements are interpolated using the measured velocities on the nodes. $\varepsilon$ is obtained by substituting $\hat{v}(r_{sj})$ and the measured velocities $v(r_{sj})$ on the centers into Eq. (14). The normal velocities and the radiated acoustic pressure in the selected field points as depicted in Fig. 9 are obtained by means of velocity and pressure measurements, as shown in Fig. 10. For the cylindrical shell, the auxiliary surface is retracted from the boundary of the shell along x-axis, as shown in Fig. 9a. Considering that the stiff frame is well sealed, only the exterior side of the clamped plate generates sound field into the infinite domain. Thus, the auxiliary surface can be located on an auxiliary plane at a certain distance from the exterior side, as shown in Fig. 9b.

The measurements are made from 100 Hz to 700 Hz for the cylindrical shell and 100 Hz to 500 Hz for the clamped plate with increment of 10 Hz, the lower limit being imposed by the semi-anechoic laboratory. The loudspeaker and the exciter are driven with sinusoidal signal. The initial values of input voltage and current for the loudspeaker are 15 V and 1.6 A, and for the exciter 1.5 V and 2.7 A, where both the input voltages remain constant when the frequency is increased. The mass ratio of the used accelerometer to the cylindrical shell and clamped plate is less than 1%. Thus, the accelerometer has little influence on the vibration of the radiator. The measured and computed sound pressure levels are plotted in Figs. 11 and 12 for two radiators, where $p_{amp}$ represents the amplitude of the acoustic pressure.

The results show that the prediction accuracy is significantly improved in the optimal auxiliary surface compared with the fixed auxiliary surface, and the computed pressures in the op-
optimal auxiliary surfaces are consistent with the measured values. The detailed data of the calculation accuracy are shown in Table 4. From the data of the relative errors, the average and maximum errors corresponding to the cylindrical shell are less than 3.7% and 6.5%, respectively. The accuracy of the computed acoustic pressure is good. For the clamped plate, all of the accelerometer bases have been bonded to the surface of the clamped plate when the vibration velocity and acoustic pressure are measured. Its surface shape is changed by these bases. When the radiated pressure is computed, the surface is still treated as a plane. Its surface area is smaller than that of the cylindrical shell. Thus, the accuracy is lower than that corresponding to the shell. The average error is still acceptable, although the accuracies in very few frequencies are poor. In addition, there is still some reflection of sound wave. These cause discrepancy between the computed and measured values in several frequencies.

6. CONCLUSIONS

The selection of the auxiliary surface is the key to obtain accurate prediction results in the wave superposition method. The relationship between the surface velocity reconstruction error and acoustic pressure prediction error has been theoretically derived in this paper. Representative numerical examples show that the normalized velocity reconstruction error is a good indicator of the prediction error of acoustic pressure. The prediction accuracy can be significantly improved in the optimal auxiliary surface compared with the fixed auxiliary surface. With the criterion satisfied, the prediction accuracy in the optimal auxiliary surface is acceptable and significantly improved. The experimental results show that the prediction error of acoustic pressure in the determined optimal auxiliary surface is greatly reduced. Thus, the proposed method is good at reducing the prediction error and is feasible.

ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China ("Research on structure sound radiation matrix deriving, computing and optimizing", Grant No.51079118; "Research on computational auditory scene analysis based internal combustion engine noise sources identification”, Grant No.51279148).
Figure 10. Acoustic pressure measurement (a) Cylindrical shell (b) Clamped plate.

Figure 11. Amplitude of the acoustic pressure of the cylindrical shell at the selected field points (a) Field point 1 (b) Field point 2.

REFERENCES


Free Vibration of Angle-ply Laminated Conical Shell Frusta with Linear and Exponential Thickness Variations

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(Received 19 October 2015; accepted 26 August 2016)

Free vibration of laminated conical shell frusta of variable thickness is studied using spline approximation. This problem includes first order shear deformation and considers shells as antisymmetric angle-ply orientation. The governing differential equations of the shells are resolved in terms of displacement functions and rotational functions. These functions are approximated using splines and the method of collocation is adopted for simultaneous algebraic equations. These equations become generalized eigenvalue problems and are solved numerically to avail eigenfrequencies and the corresponding eigenvectors. The variation of frequencies is analysed with respect to the cone angle, aspect ratio, material properties, number of layers, and thickness variation.

1. INTRODUCTION

To build a stable and sturdy construction in engineering industries, it is necessary to study the vibration analysis of the structures. This means that the structures should be designed with consideration to factors such as frequency parameter, materials, and orientation in order to construct highly reinforced structures. Thin shells play an important role as structural elements in industries because of their great range of desirable properties such as high degrees of reserved strength and structural integrity. Large-span roofs, water tanks, aircraft, and submarine are all examples of shell structures that can be found in engineering industries. Shell structures made up of composite materials have been used significantly as they possess high specific stiffness, better damping, and shock absorbing characteristics.

The study of conical shells enables engineers to meet the demand of industries. A number of analytical and numerical studies were conducted on the static and free vibration analysis of conical shells. Free vibration of laminated conical shells was studied by Wu and Wu using asymptotic differential quadrature (DQ) solutions. Tong developed a solution using power series method to solve free vibration of orthotropic conical shells using the Donnell-type classical shell theory with and without shear deformation effects. Shu conducted a free vibration analysis on composite laminated conical shells using generalized quadrature method (GDQ) where the method approximate a spatial derivative of a function with respect to a coordinate at a discrete point as a weighted linear sum of all the functional values in the whole domain. GDQ was also used by Ng et al. to investigate the orthotropic influence of composite materials on frequency characteristics for a rotating thin truncated circular symmetrical cross-ply laminated composite conical shell with different boundary conditions. Civalek presented discrete singular convolution method (DSC) for the vibration analysis of conical shells. Civalek continued this research, determining the frequencies of the free vibration of laminated conical shell including shear deformation theory. Liew et al. and Liew and Zhou used element-free kp-Ritz method to study the free vibration of conical shells and functionally graded conical shells respectively.

Studies on vibration of conical shells were conducted by Viswanathan et al. for free vibration of cross-ply and angle-ply laminated truncated shells using spline method. A study on free vibration analysis of multiple delaminated angle-ply composite conical shells using finite element method (FEM) was performed by Dey and Karmakar using QR iteration algorithm. In order to study the free vibration of simply supported circular cylindrical shells, a semi-analytical procedure was introduced by Farshidianfar and Olizadech for simply-supported boundary condition. A modified Fourier series was used by Jin et al. to analyse the vibration of truncated conical shells with general boundary conditions and the effect of elastic restraint parameters, semi-vertex angle and the ratio of length to radius.

Malekzadeh and Daraie presented a study on the dynamic behaviour of functionally graded (FG) truncated conical shells subjected to asymmetric internal ring-shaped moving loads.
He used FEM alongside Newmark’s time integration scheme to discretize the equations of motion in the spatial and temporal domain, respectively to solve the problem. Ma et al. used Fourier-Ritz method to solve the problem for free and forced vibration analysis of coupled conical-cylindrical shells with arbitrary boundary conditions.\(^\text{18}\) Free vibration analysis of fibre reinforced composite (FRC) conical shells resting on Pasternak-type elastic foundation was investigated by Zaroumi et al. using Galerkin and Ritz methods.\(^\text{19}\) A study on conical shells was conducted by Heydarpour et al. in which they used the applied differential quadrature method (DQM) to solve free vibration analysis of rotating functionally graded carbon nanotube-reinforced composite truncated conical shells based on the first order shear deformation theory of shells.\(^\text{20}\)

The vibration analysis of conical shells with variable thickness has been studied by some researchers using a few methods. Irie et al. analysed free vibration of truncated conical shell with variable thickness through use of the transfer matrix approach.\(^\text{21}\) Sankaranarayanan et al. conducted a study on vibrations of laminated conical shells of variable thickness based on classical thin shell theory using Rayleigh-Ritz procedure.\(^\text{22}\) Takahashi et al. used Ritz method to study the vibration of conical shells with variable thickness.\(^\text{23}\) Sivadas and Ganesan studied the free vibration of cantilever conical shells with variable thickness by using semi-analytical finite element method.\(^\text{24}\) Later, they performed a study on the vibration of laminated conical shells with variable thickness using the same method.\(^\text{25}\) Kang proposed a three-dimensional (3D) method of analysis for determining the free vibration frequencies of complete conical shells with linearly varying thickness.\(^\text{26}\) Selahi et al. developed a hybrid method based on 3D elasticity theory for transient analysis of FG truncated conical shell with variable thickness.\(^\text{27}\) Also, Mehdi et al. conducted a thermo-elastic analysis of axially functionally graded rotating thick truncated conical shells with varying thickness using multi-layer method (MLM).\(^\text{28}\)

This study investigates free vibration of antisymmetric angle-ply laminated conical shell frusta with linear and exponential variation in thickness under first order shear deformation theory (FSDT). The spline function approximation technique used here is preferable to other methods, since, in this study, a chain of lower order approximations can yield greater accuracy than a global higher order approximation. Also, polynomials of high degrees when applied to a large number of given data points tends to exhibit more numerous undulations than a curve drawn by spline. Hence, spline function is a more adaptable approximating function than a polynomial involving a comparable number of parameters. Another disadvantage of polynomial approximations is that, if the function to be approximated is badly behaved anywhere in the interval of approximation, then the approximation is poor everywhere. This global dependence on local properties is avoidable with the use of splines. This conjecture was made and tested by Bickley over a two-point boundary value problem with a cubic spline.\(^\text{29}\) The layers of conical shell are considered to be thin, elastic, and specially orthotropic or isotropic. Stress-strain relations and strain-displacement relations are substituted into the equilibrium equation of the conical shell to obtain the governing differential equations in terms of mid-plane displacement compo-

\[2. \text{THEORETICAL FORMULATION}\]

Consider a composite laminated truncated conical shell with an arbitrary number of layers that are perfectly bonded together. The orthogonal coordinate system \(x, \theta, z\) is fixed at its reference surface, which is assumed to be at the middle surface. The radius of the cone at any point along its length is \(r = x \sin \alpha\). The radius at the small end of the cone is \(r_a = a \sin \alpha\), the other end is \(r_b = b \sin \alpha\) and 1 is the length of the cone.

The displacement components are assumed to be in the form

\[
\begin{align*}
  u(x, \theta, z, t) &= u_0(x, \theta, t) + z \psi_x(x, \theta, t); \\
  v(x, \theta, z, t) &= v_0(x, \theta, t) + z \psi_\theta(x, \theta, t); \\
  w(x, \theta, z, t) &= w_0(x, \theta, t);
\end{align*}
\]

\[(1)\]

where \(u_0\), \(v_0\), and \(w_0\) are the displacements of the shell in the mid-plane, \(\psi_x\) and \(\psi_\theta\) are the shear rotations of any point on the middle surface of the shell.\(^\text{11}\) The stress-resultants and moment-resultants are given as

\[
\begin{align*}
  (N_x, N_\theta, N_{x\theta}, Q_{xz}, Q_{z\theta}) &= \int_z (\sigma_x, \sigma_\theta, \tau_{x\theta}, \pi_{xz}, \pi_{z\theta}) \, dz; \\
  (M_x, M_\theta, M_{x\theta}) &= \int_z (\sigma_x, \sigma_\theta, \tau_{x\theta}) \, dz. 
\end{align*}
\]

\[(2)\]

The equations of stress-resultants and moment-resultants are obtained as follows:

\[
\begin{pmatrix}
  N_x \\
  N_\theta \\
  N_{x\theta} \\
  Q_{xz} \\
  Q_{z\theta}
\end{pmatrix} = \begin{pmatrix}
  A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\
  A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\
  A_{10} & A_{20} & A_{60} & B_{10} & B_{20} & B_{60} \\
  B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\
  B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\
  B_{10} & B_{20} & B_{60} & D_{10} & D_{20} & D_{60}
\end{pmatrix}
\begin{pmatrix}
  \frac{\partial u_0}{\partial x} \\
  \frac{\partial u_0}{\partial \theta} \\
  \frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial z} \\
  \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial z} \\
  \frac{\partial w_0}{\partial x} + \frac{\partial \psi_x}{\partial z} \\
  \frac{\partial \psi_\theta}{\partial x} + \frac{\partial w_0}{\partial z}
\end{pmatrix}.
\]

\[
\begin{pmatrix}
  \frac{1}{x \sin \alpha} u_0 + \frac{1}{x \sin \alpha} \psi_x + \frac{1}{z \sin \alpha} \frac{\partial u_0}{\partial \theta} + \frac{1}{z \cos \alpha} \frac{\partial \psi_x}{\partial \theta} \\
  \frac{1}{x \sin \alpha} \psi_x + \frac{1}{x \sin \alpha} \frac{\partial u_0}{\partial \theta} + \frac{1}{z \cos \alpha} \frac{\partial \psi_x}{\partial \theta} - \frac{1}{z} \frac{\partial \psi_x}{\partial \theta}
\end{pmatrix}.
\]

\[(3)\]
and

\[
\begin{bmatrix}
Q_{0z} \\
Q_{xz}
\end{bmatrix} = K \begin{bmatrix}
A_{44} & A_{45} \\
A_{45} & A_{55}
\end{bmatrix} \begin{bmatrix}
ψ_0 + \frac{1}{x \sin α} \frac{∂w}{∂θ} - \frac{1}{x \tan α} v₀ \\
ψ_x + \frac{1}{x} \frac{∂w}{∂x}
\end{bmatrix};
\]

(4)

where we define the elastic coefficients \( A_{ij}, B_{ij}, \) and \( D_{ij} \) as extensional stiffness, bending-extensional coupling stiffness, and bending stiffness respectively and \( K \) is the shear correction factor. The value for the shear correction factor \( K \) is chosen from the lamination scheme. The procedure for finding the values of shear correction factors have been explored by Whitney. The elastic coefficients \( A_{ij}, B_{ij}, \) and \( D_{ij} \) corresponding to layers of uniform thickness with superscript 'c' are assumed to be in the form

\[
\begin{align*}
A_{ij} &= A_{ij}^c g(x); \\
B_{ij} &= B_{ij}^c g(x); \\
D_{ij} &= D_{ij}^c g(x); \\
A_{ij}^c &= \sum_k Q_{ij}^{(k)} (z_k - z_{k-1}), \\
B_{ij}^c &= \frac{1}{2} \sum_k Q_{ij}^{(k)} (z^2_k - z^2_{k-1}), \\
D_{ij}^c &= \frac{1}{3} \sum_k Q_{ij}^{(k)} (z^3_k - z^3_{k-1}) \quad \text{for } i, j = 1, 2, 6; \\
A_{ij}^c &= K \sum_k Q_{ij}^{(k)} (z_k - z_{k-1}) \quad \text{for } i, j = 4, 5;
\end{align*}
\]

(5)

and \( z_{k-1} \) and \( z_k \) are boundaries of \( k \)-th layer.

In this study, the thickness variation of the \( k \)-th layer of the shell is assumed in the form of

\[
h_k(x) = h_{0k} g(x);
\]

(6)

where \( g(x) = 1 + C_l(x - x_a/l) + C_c \exp(x - x_a/l) \), \( C_l = 1/η - 1, \) \( η \) is the taper ratio, \( h_{0k} \) is a constant thickness of the \( k \)-th layer, \( l = b - a \) is the length of the cone, \( x_a \) is the distance from origin to \( x = a \) (small end of the cone), and \( C_l \) and \( C_c \) are the coefficients of linear thickness in variation. The thickness of the shell becomes uniform when \( g(x) = 1 \).

The displacement components \( u₀, v₀, \) \( w \) and shear rotations \( ψ_x, ψ_y \) are assumed in the separable form given as

\[
\begin{align*}
u₀(x, θ, t) &= U(x) e^{i ω t}; \\
v₀(x, θ, t) &= V(x) e^{i ω t}; \\
w(x, θ, t) &= W(x) e^{i ω t}; \\
ψ_x(x, θ, t) &= Ψ_X(x) e^{i ω t}; \\
ψ_y(x, θ, t) &= Ψ_θ(x) e^{i ω t};
\end{align*}
\]

(7)

where \( ω \) is the angular frequency of vibration, \( t \) is the time, and \( n \) is the circumferential node number. The non-dimensional parameters are written as follows:

\[
X = \frac{x - a}{l}, \quad a ≤ x ≤ b \quad \text{and} \quad X ∈ [0, 1];
\]

\[
λ = ωl \sqrt{\frac{L_1}{A_{11}}} \quad \text{– frequency parameter};
\]

\[
γ = \frac{h}{r_a}, \quad γ' = \frac{h}{a} \quad \text{– ratios of thickness to radius and to length};
\]

\[
β = \frac{a}{b} \quad \text{– length ratio};
\]

\[
δ_k = \frac{h_k}{h} \quad \text{– relative layer thickness of the } k\text{-th layer}.
\]

The thickness \( h_k(X) \) of the \( k \)-th layer at \( X \) distance from the smaller end of the cone can be written as

\[
h_k(X) = h_{0k} g(X);
\]

(9)

Substituting Eqs. (3) and (4) into the equation of motion of conical shells and applying the condition of antisymmetric in angle-ply laminates (i.e., \( A_{16}, A_{26}, A_{45}, B_{11}, B_{12}, B_{22}, B_{66}, D_{11}, \) \( D_{26} \), and \( D_{26} \) are identically zero), the differential equations in terms of displacement functions and rotational function are obtained. Then, applying Eq. (7) into the obtained differential equations and using the non-dimensional parameters given in Eq. (8), we get a new differential equation in terms of \( X \) written in matrix form as

\[
\begin{bmatrix}
L_{11} & L_{12} & L_{13} & L_{14} & L_{15} \\
L_{21} & L_{22} & L_{23} & L_{24} & L_{25} \\
L_{31} & L_{32} & L_{33} & L_{34} & L_{35} \\
L_{41} & L_{42} & L_{43} & L_{44} & L_{45} \\
L_{51} & L_{52} & L_{53} & L_{54} & L_{55}
\end{bmatrix}
\begin{bmatrix}
U \\
V \\
W \\
Ψ_X \\
Ψ_θ
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}.
\]

(10)

The differential operators \( L_{ij} \) of the matrix are given in Appendix A.

### 3. SOLUTION PROCEDURE

#### 3.1 Spline Collocation Method

The displacement functions \( U(X), V(X), W(X) \) and shear rotational functions \( Ψ_X, Ψ_θ \) are approximated by the cubic
Tables 3, 4, and 5 provide the fundamental frequency parameters for two-layered and four-layered antisymmetric angle-ply shells under C-C and S-S boundary conditions, respectively. The tables show the effect of taper ratio and exponential variation of thickness on the fundamental frequency parameter for these shell configurations.

Tables 3 and 4 include data for C-C and S-S boundary conditions, respectively. Each table contains columns for taper ratio, exponential parameter, and frequency parameters at different thickness ratios.

Table 6 presents the exponential variation of thickness for two-layered and four-layered antisymmetric angle-ply shells under C-C and S-S boundary conditions.

The spline functions are expressed as:

$$U(X) = \sum_{i=0}^{2} a_i X^i + \sum_{j=0}^{N-1} b_j (X - X_j)^3 H(X - X_j);$$

$$V(X) = \sum_{i=0}^{2} c_i X^i + \sum_{j=0}^{N-1} d_j (X - X_j)^3 H(X - X_j);$$

$$W(X) = \sum_{i=0}^{2} e_i X^i + \sum_{j=0}^{N-1} f_j (X - X_j)^3 H(X - X_j);$$

$$\Psi_X(X) = \sum_{i=0}^{2} g_i X^i + \sum_{j=0}^{N-1} p_j (X - X_j)^3 H(X - X_j);$$

$$\Psi_\Theta(X) = \sum_{i=0}^{2} l_i X^i + \sum_{j=0}^{N-1} q_j (X - X_j)^3 H(X - X_j);$$

where $H(X - X_j)$ is the Heaviside step function and $N$ is the number of sub-intervals within the range $[0, 1]$ of $X$.

3.2. Boundary Conditions

Two boundary conditions are used to analyse the problem:

(i) Clamped-Clamped (C-C) (both the ends are clamped)

(ii) Simply-Supported (S-S) (both ends are simply supported)

Each boundary condition gives 10 equations on spline coefficients. Gathering them with those obtained earlier, we get $5N + 15$ homogeneous equations, with the same number unknown. Hence, the system of equations can be written in the form

$$[P][q] = [A][Q][q].$$

The condition that the differential equations given by Eq. (10) are satisfied by these splines at the knots, a set $5N + 5$ homogeneous equations in $5N + 15$ unknown spline coefficients, $a_i, c_i, e_i, g_i, l_i, b_j, f_j, p_j, q_j$ (i = 0, 1; j = 0, 1, 2, ..., N − 1) is obtained.

International Journal of Acoustics and Vibration, Vol. 23, No. 2, 2018
Here $[P]$ and $[Q]$ are square matrices, $[q]$ is a column matrix, and $\Lambda = 1/\lambda^2$, where $\lambda$ is the frequency parameter. $\Lambda$ or $\lambda^2$ is the eigenparameter and $[q]$ is the eigenvector for this generalized eigenvalue problem.

**4. RESULT AND DISCUSSION**

In this work, convergence study for the frequency parameter $\lambda$ has been calculated to choose the number of subintervals $N$ of the spline function. As shown in Table 1, the program is performed for $N = 2$ ($N$ being number of knots) and onwards. It is seen that $N = 18$ would be enough to achieve low change in percentage. In order to verify the results, the reduced case of constant thickness is compared with the value of fundamental frequency parameter $\lambda$ obtained by Daneshjou et al., Lam and Hua, Irie et al., and Jin et al. presented in Table 2.

It is seen that $\eta = 0.75$ and $\gamma' = 0.5$ for two-layered and four-layered conical shells under C-C boundary condition: linear variation in thickness.

Here $[P]$ and $[Q]$ are square matrices, $[q]$ is a column matrix, and $\Lambda = 1/\lambda^2$, where $\lambda$ is the frequency parameter. $\Lambda$ or $\lambda^2$ is the eigenparameter and $[q]$ is the eigenvector for this generalized eigenvalue problem.

The thickness variation is assumed to be in linear and exponential.

Table 3 depicts the influence of taper ratio $\eta$ on the fundamental frequency parameter $\lambda$ for two-layered and four-layered antisymmetric angle-ply shells under C-C boundary condition. The value of $\eta$ ranges for $0.5 \leq \eta \leq 1.9$. The parameters such as circumferential node number $n = 1$, length ratio $\beta = 0.5$ and ratio of thickness to radius $\gamma = 0.05$ are fixed. In Table 3, it is observed that the value of $\lambda$ for ply-angle $30^\circ/−30^\circ$ increases and then decreases multiple times throughout $0.5 \leq \eta \leq 1.9$. The frequency values for ply-angle $45^\circ/−45^\circ$ increase up to $\eta = 1.1$ and then decrease afterwards. The frequency for two-layered shell with ply-angle $60^\circ/−60^\circ$ shows a steady decrease throughout $0.5 \leq \eta \leq 1.9$. The value of $\lambda$ increases up to $\eta = 1.3$ and then decreases with the increase of $\eta$ for four-layered shell with ply-angle $30^\circ/−30^\circ/30^\circ/−30^\circ$ and $45^\circ/−45^\circ/45^\circ/−45^\circ$ in the case of ply-angles $45^\circ/−45^\circ$ and $60^\circ/−60^\circ$, the value of $\lambda$ decreases as $\eta$ decreases. The influence of $\eta$ towards the fundamental frequency parameter $\lambda$ for two-layered and four-layered antisymmetric angle-ply shells under S-S boundary condition is demonstrated in Table 4. Tables 5 and 6 show the effects of exponential variation of thickness on the fundamental frequency parameter for two-layered and four-layered antisymmetric angle-ply shells under C-C and S-S boundary conditions respectively. In Tables 4–6, the variation of fre-
Figure 2. Effect of cone angle on fundamental frequency parameter of two-layered and four-layered conical shells under S-S boundary condition: linear variation in thickness.

In Table 3, the frequency values are higher for two-layered shells as compared to the corresponding values of four-layered shells. Moreover, the values of the fundamental frequency parameter are lower for S-S boundary condition as compared to C-C boundary condition.

In Figs. 1a–1d, the value of frequency parameter is lower for S-S boundary condition as compared to the corresponding values of four-layered shells. Moreover, the values of the fundamental frequency parameter shows the same pattern as shown in Table 3. In Tables 3–6, the frequency values are higher for two-layered shells as compared to the corresponding values of four-layered shells. The results in Fig. 2 show the same vibrational pattern with the corresponding case of two-layered shell in Fig. 3. However, the frequency values for four-layered shells are higher than the corresponding frequency values for two-layered shells.

Figures 5a and 5b depict the variation of fundamental angular frequency with respect to the length ratio β for two-layered and four-layered conical shells respectively. In Fig. 5a, conical shell with ply-angle 60°/−60° for length ratio 0.1 until 0.65 has the highest angular frequency values when compared with the other two ply-angles but changes into the lowest from 0.65 until 0.8. Conical shell with ply-angle 60°/−60°/60°/−60° in Fig. 5b also shows the same pattern since it has the highest frequencies from 0.1 until 0.55 but switches into the lowest from 0.55 until 0.8. As observed, the angular frequencies for four-layered shells are higher than corresponding two-layered shells.
Figures 7 and 8 indicate the variation of frequency parameter $\lambda$ for two-layered and four-layered conical shells with respect to cone angle $\alpha$ for exponential variation in thickness by fixing the parameters $C_e$, $\beta$ and $\gamma'$. Figure 7 relates to the shells with C-C boundary condition while Fig. 8 uses S-S boundary condition. Based on Figs. 7 and 8, the value of $\lambda$ is higher for higher angles for both conical shells under C-C and S-S boundary conditions, but the frequency parameter values for S-S boundary condition are lower than corresponding values for C-C boundary condition. The fundamental angular frequency $\omega$ for two-layered antisymmetric angle-ply conical shells with the influence of length ratio $\beta$ when $\alpha = 30^\circ$ for exponential variation in thickness under C-C boundary condition is depicted in Fig. 9. Figure 9a shows the variation of angular frequencies with thickness variation coefficient $C_e = -0.2$ whereas Fig. 9b depicts the distribution of frequencies when $C_e = 0.2$. From Figs. 9a and 9b, it is seen that different value of exponential variation in thickness does not significantly af-
Figure 5. Effect of length ratio on fundamental angular frequency of two-layered and four-layered conical shell when $\alpha = 60^\circ$ is under C-C boundary condition: linear variation in thickness.

Figure 6. Effect of length ratio on fundamental angular frequency of two-layered and four-layered conical shell for $\alpha = 30^\circ$ and $\alpha = 60^\circ$ under C-C boundary condition: linear variation in thickness.

The effect of length ratio $\beta$ on the fundamental angular frequency $\omega$ of conical shells is shown in Fig. 10. The shells have cone angle $\alpha = 30^\circ$, $C_e = 0.2$, $n = 1$, $n = 2$ and $\gamma = 0.05$. The fundamental frequency $\omega$ is higher for four-layered shell compared to the values of $\omega$ for two-layered shell with the corresponding angles.

Figures 11a and 11b describe the influence of length ratio $\beta$ on angular frequency $\omega$ of two-layered and four-layered conical shells respectively. The thickness variation coefficient $C_e = 0.2$ and other parameters are held fixed. The vibrational behaviour of these shells is similar to the case in Fig. 5. Figure 12 depicts the effect of length ratio $\beta$ on angular frequency $\omega$ of two-layered and four-layered conical shells for cone angle $\alpha = 30^\circ$ and $\alpha = 60^\circ$ under S-S boundary condition. Analysing from Fig. 12, we can see that the angular frequencies for conical shells with cone angle $\alpha = 30^\circ$ are lower when compared to the corresponding values of $\alpha = 60^\circ$. 

International Journal of Acoustics and Vibration, Vol. 23, No. 2, 2018
5. CONCLUSION

The effect of linear and exponential thickness in variation, cone angle, length ratio, circumferential node number, different lamination materials, ply-angles, and two different boundary conditions on the free vibration of conical shells using spline approximation technique is analysed. Antisymmetric angle-ply laminations for two-layered and four-layered shells are considered. It is concluded from the results that the value of the frequency parameter strictly decreases for certain value of cone angle and become steady afterwards with the increase of cone angle. Also, the angular frequency values remain steady for certain value of length ratio and strictly increases afterwards as the length ratio increases. Further, S-S boundary condition results in lower value of frequency parameter as compare to C-C boundary condition. The results presented in this paper may be fruitful for designers in related fields for designing the conical shell structure according to their needs.
Figure 9. Effect of length ratio on fundamental angular frequency of two-layered conical shells when $\alpha = 30^\circ$ under C-C boundary condition: exponential variation in thickness.

Figure 10. Effect of length ratio on fundamental angular frequency of two-layered conical shells when $\alpha = 30^\circ$ under C-C boundary condition: exponential variation in thickness.

Figure 11. Effect of length ratio on fundamental angular frequency of two-layered and four-layered conical shells when $\alpha = 60^\circ$ under C-C boundary condition: exponential variation in thickness.
Figure 12. Effect of length ratio on fundamental angular frequency of two-layered and four-layered conical shells for $\alpha = 30^\circ$ and $\alpha = 60^\circ$ under S-S boundary condition: exponential variation in thickness.

ACKNOWLEDGEMENTS

This work was supported by MOHE, Project Vote No. 05H78 under Research management Centre (RMC), Universiti Teknologi Malaysia, Malaysia and also supported by My Brain 15. Malaysia.

REFERENCES


APPENDIX A

The differential operators $L_{ij}$ of the matrix are

\[ L_{11} = \frac{d^2}{dX^2} + \left( \frac{g'}{g} + p \right) \frac{d}{dX} + S_2 \frac{g'}{g} p - S_3 p^2 + S_{10} n^2 p^2 \cot^2 \alpha + \lambda^2; \]  
(A.1)

\[ L_{12} = (S_2 + S_{10})np \cot \alpha \frac{d}{dX} + \left( S_2 \frac{g'}{g} - S_3 p - S_{10} p \right) np \cot \alpha; \]  
(A.2)

\[ L_{13} = S_2 p \cot \alpha \frac{d}{dX} + \left( S_2 \frac{g'}{g} - S_3 p \right) p \cot \alpha; \]  
(A.3)

\[ L_{14} = 2S_1 n p \cot \alpha \frac{d}{dX} + S_1 \frac{g'}{g} np \cot \alpha; \]  
(A.4)

\[ L_{15} = S_1 n^2 \frac{d^2}{dX^2} + \left( S_5 \frac{g'}{g} - S_{16} p \right) \frac{d}{dX} + \left( S_{16} p + S_{16} n^2 p \cot^2 \alpha - S_5 \frac{g'}{g} \right) p; \]  
(A.5)

\[ L_{21} = (S_2 + S_{10})np \cot \alpha \frac{d}{dX} + \left( S_2 \frac{g'}{g} + S_{10} p + S_3 p \right) np \cot \alpha; \]  
(A.6)

\[ L_{22} = S_{10} n^2 \frac{d^2}{dX^2} + S_{10} \left( \frac{g'}{g} + p \right) \frac{d}{dX} + \left( -S_6 \frac{g'}{g} - S_{10} p - kS_{13} \cot \alpha + S_3 n^2 p \cot^2 \alpha \right) p + \lambda^2; \]  
(A.7)

\[ L_{23} = (S_3 + kS_{13})p^2 n \cot \alpha \cot \alpha; \]  
(A.8)

\[ L_{24} = S_{15} \frac{d^2}{dX^2} + \left( S_5 \frac{g'}{g} + S_{16} p + 2S_{15} p \right) \frac{d}{dX} + S_{16} \left( \frac{g'}{g} + p + n^2 \cot^2 \alpha \right) p; \]  
(A.9)

\[ L_{25} = 2S_{15} n p \cot \alpha \frac{d}{dX} + \left( kS_{13} \cot \alpha + S_{16} \frac{g'}{g} n \cot \alpha \right) p; \]  
(A.10)

\[ L_{31} = -S_2 p \cot \alpha \frac{d}{dX} - S_3 p^2 \cot \alpha; \]  
(A.11)

\[ L_{32} = (-S_3 - kS_{13})p^2 n \cot \alpha \cot \alpha; \]  
(A.12)

\[ L_{33} = kS_{14} \frac{d^2}{dX^2} + kS_{14} \left( \frac{g'}{g} + p \right) \frac{d}{dX} + \left( -S_3 \cot^2 \alpha + kS_{13} n^2 \cot^2 \alpha \right) p^2 + \lambda^2; \]  
(A.13)

\[ L_{34} = kS_{14} \frac{d}{dX} - S_{16} p^2 n \cot \alpha \cot \alpha + kS_{14} \frac{g'}{g} + kS_{14} p; \]  
(A.14)

\[ L_{35} = -S_{16} p \cot \alpha \frac{d}{dX} + \left( kS_{13} n \cot \alpha + S_{16} p \cot \alpha \right) p; \]  
(A.15)

\[ L_{41} = 2S_{15} np \cot \alpha \frac{d}{dX} + S_{15} \frac{g'}{g} np \cot \alpha; \]  
(A.16)

\[ L_{42} = S_{15} \frac{d^2}{dX^2} + \left( S_{15} \frac{g'}{g} - S_{16} p \right) \frac{d}{dX} + \left( -S_{15} \frac{g'}{g} + S_{16} p + S_{16} n^2 \cot^2 \alpha \right) p; \]  
(A.17)

\[ L_{43} = -kS_{14} \frac{d}{dX} + S_{16} p^2 n \cot \alpha \cot \alpha; \]  
(A.18)

\[ L_{44} = S_{17} \frac{d^2}{dX^2} + S_{15} \left( \frac{g'}{g} + p \right) \frac{d}{dX} + \left( S_6 \frac{g'}{g} - S_9 p^2 + S_{12} \cot^2 \alpha - kS_{14} \right) + \lambda^2 p_1; \]  
(A.19)

\[ L_{45} = \left( S_8 n n \cot \alpha + S_{12} n \cot \alpha \right) p \frac{d}{dX} + \left( S_6 \frac{g'}{g} - S_9 p - S_{12} p \right) n \cot \alpha; \]  
(A.20)

\[ L_{51} = S_{15} \frac{d^2}{dX^2} + \left( S_5 \frac{g'}{g} + 2S_{15} p + S_{16} p \right) \frac{d}{dX} + S_{16} \left( \frac{g'}{g} + p + n^2 \cot^2 \alpha \right) p; \]  
(A.21)

\[ L_{52} = 2S_{15} n p \cot \alpha \frac{d}{dX} + \left( S_{16} \frac{g'}{g} n \cot \alpha + kS_{13} \cot \alpha \right) p; \]  
(A.22)

\[ L_{53} = S_{16} p \cot \alpha \frac{d}{dX} + \left( S_6 \frac{g'}{g} \cot \alpha - kS_{13} n \cot \alpha + S_{16} p \cot \alpha \right) p; \]  
(A.23)

\[ L_{54} = \left( S_{12} + S_6 \right) n \cot \alpha \frac{d}{dX} + \left( S_{12} \frac{g'}{g} + S_{12} p + S_{16} p \right) n \cot \alpha; \]  
(A.24)

\[ L_{55} = S_{15} \frac{d^2}{dX^2} + S_{15} \left( \frac{g'}{g} + p \right) \frac{d}{dX} - \left( S_{12} \frac{g'}{g} + S_{12} \cot^2 \alpha - S_9 p^2 n^2 \cot^2 \alpha + kS_{13} \right) + \lambda^2 p_1; \]  
(A.25)

The quantities $S_i$ ($i = 1, 2, \ldots, 14$) are defined by

\[ S_2 = \frac{A_{12}}{A_{11}}; \quad S_3 = \frac{A_{22}}{A_{11}}; \quad S_4 = \frac{D_{11}}{S_{11}}; \quad S_5 = \frac{D_{12}}{S_{11}}; \quad S_6 = \frac{D_{12}}{S_{12}}; \quad S_7 = \frac{A_{66}}{A_{11}}; \quad S_8 = \frac{D_{16}}{S_{11}}; \quad S_9 = \frac{A_{14}}{A_{11}}; \quad S_{10} = \frac{A_{66}}{A_{11}}; \quad S_{11} = \frac{B_{16}}{A_{11}}; \quad S_{12} = \frac{D_{16}}{S_{11}}; \quad S_{13} = \frac{A_{14}}{A_{11}}; \quad S_{14} = \frac{A_{15}}{A_{11}}; \quad S_{15} = \frac{B_{16}}{A_{11}}; \quad S_{16} = \frac{B_{16}}{A_{11}}; \quad (A.26) \]
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