Application of Spectral Element Method Combining Dilatation Theory to Sound Generated by a Co-rotating Vortex Pair

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A spectral element method for the simulation of an acoustic field is applied to the case of a co-rotating vortex pair in both stationary medium and mean flow. Based on the dilatation theory, the second time derivative of the pseudo-pressure is used as the source of the inhomogeneous convected wave equation which is discretized by a spectral element method in space and the Newmark-β method in time marching. In addition, the nonreflecting boundary condition is adopted too. Then we compared the numerical results with the analytical solution. Numerical results are in good agreement with the analytical solution. Moreover, different grid spacings and time steps are investigated for evaluating the numerical accuracy. To study the frequency content of the sound, spectral analysis is also carried out. Finally, sound propagation in uniform flows and sheared mean flows are simulated and analyzed. This study shows the capabilities of the spectral element method combined with dilatation theory for the aeroacoustic problems.

1. INTRODUCTION

The acoustic analogy theory introduced by Lighthill to study jet noise is now over six decades, and it has been by far the most successful and versatile theory in dealing with aeroacoustic problems. Considering the pseudo-sound pressure, Ribner and Meecham offered a different approach to Lighthill’s acoustic analogy which posited fluctuating fluid dilatations as the acoustic source. One motivation for developing a new expression is that the pseudo-sound term is much easier to calculate relative to Lighthill’s stress tensor. According to this dilatation theory which also corresponds to the acoustic/viscous splitting technique derived by Hardin and Pope, Hurdle, Ribner and Crawley studied the aerodynamic noise generating from a jet engine. Combining large eddy simulation and dilatation theory, Flemming used a hybrid approach to study the combustion noise of a turbulent flame. Hiramoto et al. investigated the sound generated in a separated shear flow by flow visualization and fluctuating static pressure measurements, and the results showed that the dilatation theory’s source term and the vortical structure are closely correlated. Escobar et al. presented a study on vortex sound propagation by using finite element method and compared dilatation theory with Lighthill’s acoustic analogy theory. The results showed that the dilatation theory can get good solutions easily. Based on the dilatation theory, Papageorgakopoulos and Tsangaris developed a numerical discretization scheme for acoustic wave equation and solved some benchmark problems.

Computational Aeroacoustics (CAA) is not the same as Computational Fluid Dynamics (CFD). Needs of accurate and efficient numerical solvers in CAA motivated the development of low-dispersion and low-dissipation schemes. Varieties of finite difference schemes occupy a dominant position in CAA and some of them can provide certain accuracy. However, the spectral element method (SEM) can provide high resolution and good flexibility with a low number of elements. Numerically, SEM has the advantage of low dispersion and diffusion alongside exponential convergence in the polynomial order. Therefore, the SEM has been widely used in wave propagation studies and CAA.

The motivation for simulating the sound generated by a co-rotating vortex pair in this work is that various vortices occupy only a very small portion in a flow but play a key role in organizing the flow, as “the sinew and muscles of the fluid motion” because they, at low Mach numbers, are the only source of aeroacoustic sound and noise. The other reason is that it has analytic solutions. Moreover, many other workers have used it to verify the validity of the proposed numerical schemes for CAA.

In the present work, we present an analysis methodology, described in Section 2, which aims to supply another computational tool for CAA. The inhomogeneous convected wave equation based on the dilatation theory was solved numerically by using the high resolution SEM. In order to demonstrate the capabilities and limits of this method, we have studied a benchmark 2D vortex sound propagation problem in detail.

The paper is organized as follows. In Section 2 we provide a description of the inhomogeneous convected wave equation with dilatation theory. The governing equation obtained will then be discretized by SEM in space and Newmark-β method in time. In Section 3, the sound generated by a co-rotating vortex pair is investigated in detail. Finally, Section 4 contains the conclusions of our work.

2. GOVERNING EQUATION

As well known, the original Lighthill’s acoustic analogy completely ignores the mean flow-sound interaction effects. Then Phillips and Lilley make a correction for the Lighthill’s equation. Phillips’ equation takes into account partially the interaction of the mean flow with the sound. Thus, the equa-