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Towards an International Year of Sound

The United Nations (UN) observes designated days, weeks, years, and decades to mark particular events, or topics, in order to promote, through awareness and action, the objectives of the UN. Although the majority of observances have been established by resolutions of the UN General Assembly, some have been designated by UN specialized agencies, such as UNESCO, UNICEF, FAO, etc., when they concern issues that fall within the scope of their competencies.

Initiated by the International Commission on Acoustics (ICA), the International Year of Sound (IYS) is planned to be a global initiative to highlight the importance of sound in all aspects of life on earth. Since this initiative strongly agrees with the main objective of our own organization, the International Institute of Acoustics and Vibration (IIA V) is one of the founding supporters of the IYS providing seed funding. The IYS will consist of coordinated activities at all levels, aiming to stimulate the understanding and awareness throughout the world of the important role that sound and hearing plays in all aspects of society. Specifically, the IYS will focus on how sound is an integral part of society and has inspired art, music, literature, and philosophy across the centuries. The year will also promote the applications of science and technology in dealing with sound and vibration and using it for the benefit of mankind. Considerable work and negotiations with international bodies including the United Nations Educational, Scientific and Cultural Organization (UNESCO) have been necessary to achieve the IYS proposal officially. ICA will be seeking the support from the all member organizations in this quest.

There are already some important acoustics-related dates that are celebrated around the world, and they may serve as a basis for the IYS. Probably the oldest is the International Noise Awareness Day (INAD) that was established by the Center of Hearing and Communication (known years ago as the League for the Hard of Hearing) in 1996 to encourage people to do something about bothersome noise where they work, live, and play. This idea, which began as a local initiative, has become truly international. The INAD is commemorated each year on the last Wednesday of April. Many scientific societies all over the world have started during the last decade to organize INAD specific events in their countries. These mainly address young people, who are among the most sensitive persons in our society, but also the general public by spreading of newsletters, and the organization of lectures, seminars and workshops.

On the other hand, 2018 was the 15th year of La Semaine du Son (the week of sound) in Paris (21-27 January), a week having events dealing with each of the following five themes: health, environmental sound, recording techniques and sound diffusion, relationship between pictures and sound, and music and sound expression. While the concept commenced in France, there are now events in several other countries. The five main themes of La Semaine du Son became the basis for the development of the Charter of Sound that was presented to UNESCO on 18 Jan 2016. Following on from the declaration of the Charter, the Executive Committee of UNESCO adopted the resolution 39C/49 “The importance of sound in today’s world: promoting best practices” on 31 October 2017.

More recently, the World Health Organization (WHO) has promoted the World Hearing Day, celebrated on 3 March each year since 2007 to raise awareness on how to prevent deafness and hearing loss and promote ear and hearing care across the world. The day was previously known as the International Ear Care Day. Each year, WHO decides the theme and develops a brochure on the topic based on the best available scientific evidence. This year, the theme will be “Hear the future”, and the day will draw attention to the anticipated increase in the number of people with hearing loss around the world in the coming decades. In recent years, an increasing number of UN member states and other partner agencies have joined the World Hearing Day by organizing a range of activities and events in their countries.

Originally, ICA submitted a prospectus where 2019 was the preferred candidate for the IYS. As the president of the IIAV, I attended two ICA board meetings held in Boston, Massachusetts, in 2017 (26 and 28 June). There, the ICA president informed us about the IYS status. The plan for the year 2019 become difficult since we would have a strong competition with the proposal of the International Year of the Periodic Table of Chemical Elements, IYPT 2019, which will coincide with the 150th anniversary of the discovery of the Periodic System by Dmitry Mendeleev in 1869, and the Centenary of the International Union of Pure and Applied Chemistry. In fact, IYPT 2019 was proclaimed by the UN 74th Plenary Meeting on 20 December 2017. Now, the targeted year for the International Year of Sound will be 2020 (IYS 2020). ICA has already secured the domain “sound2020.org” in anticipation of year 2020 being named the IYS.

Undoubtedly, the declaration of an IYS will provide a focus that will encourage transfer of knowledge on creating, controlling, hearing, and using sound in nature, in the built environment and in all aspects of our life. I invite all acousticians to support the ICA efforts in achieving this objective and help spreading the word among your peers.

Jorge P. Arenas
President IIAV
Editor-in-Chief IJAV

Editor-in-Chief IJA V
Evaluation of Psychoacoustic Annoyance and Perception of Noise Annoyance Inside University Facilities

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The levels of noise produced in university facilities by students, increases the noise annoyance. The quality of life and the academic performance of university students could be inuenced by this factor. Unfortunately, as far as methodology is concerned, there are no regulations or standards that allow for the correct evaluation of noise annoyance at educative facilities. In this work a method for the evaluation of noise annoyance and an indicator of noise annoyance are presented. In order to obtain a numerical value, a percentage index, and a verbal index that represents the noise annoyance in a specific area at university facilities, psychoacoustic annoyance (PA) and evaluation of perception of noise annoyance has been related. Resulting from this correlation an indicator of noise annoyance was proposed. The results were associated with this indicator. The method and the proposed indicator allow for deeper evaluation of noise annoyance and facilitate the development of appropriate actions against noise.

1. INTRODUCTION

The impact of noise on the quality of life of people is a topic of interest of many researchers. The works on this topic mainly focused on studying noise annoyance produced by urban sources and its inuence on quality of life, and the physical and psychological health of the general population.\textsuperscript{1–5} However, noise annoyance inside educational facilities at the university level is a topic that has not been deeply studied. Topics such as background noise levels in classrooms, speech intelligibility, isolation, and acoustic conditioning speciﬁcally inside classrooms are the main study areas.\textsuperscript{6–9} There are several factors that inuence the increase of noise in these places. Inside university facilities, the students themselves are the main source of noise. Throughout the day, students are involved in many activities inside the university facilities. At the university, usually, there are places with a large mass of students. This is why noise increases signiﬁcantly and, consequently, the noise annoyance as well.\textsuperscript{10} This problem is worse mainly in places with inexistent acoustic treatment like common halls and hallways. Typically, noise annoyance is a parameter evaluated with objective and subjective methods, but currently, there are no regulations to evaluate this parameter. This is the reason why different methods to determine annoyance are used in many studies.\textsuperscript{1,2,4,11–20} However, the method described by Fastl and Zwicker\textsuperscript{21} is used in many studies. This method is based on the calculation of psychoacoustic parameters such as loudness, roughness, sharpness, and uctuation strength. These four parameters result in another parameter that evaluates the annoyance known as psychoacoustic annoyance (PA). On the other hand, another typical method is based on listening tests in combination with the application of questionnaires.\textsuperscript{22–25} Different kinds of questionnaires and different scales of evaluation were used in the existing studies.\textsuperscript{1,3,5,26} However, to standardize the evaluation of the response to noise, the International Commission of the Biological Effects of Noise (ICBEN) established a method based on a survey that was developed in different languages.\textsuperscript{27} This survey is the result of the collaboration of different researchers from around the world and consists of two questions; each one of them has different scales (a 5-point verbal scale and an 11-points numerical scale), which will be explained later. Because of the importance of the effects of noise on health, quality of life, and school performance of university students, the noise sources and acoustic conditions in areas where students usually develop their daily activities should be studied. In addition, it is essential to develop the mechanisms that study and develop strategies and regulations applicable to students, an important population group.
This could help protect students from the effects of noise on their physical and psychological health, and consequently improve their school performance. In this work, a methodological proposal to evaluate noise annoyance at university facilities is presented. The results were correlated and represented in a graphic colour scale.

2. METHOD

This work involved a study of the noise conditions in university facilities. The methodology proposed requires an acoustic analysis using data obtained from the recordings of specific sound environments. This was done using the method of evaluation of the PA proposed by Fastl and Zwicker. To obtain two related noise indexes, the results of the two methods mentioned previously were correlated. These indexes, called "percentage noise annoyance index" (PNAI) and "verbal noise annoyance index" (VNAI), indicate a conclusive percentage value and a verbal value incorporated and represented in a graphic color scale that helps to identify the level of noise annoyance in a specific place.

2.1. Participants

A total of 33 students (17 male and 16 female) between 19 and 34 years old participated in the experiment. Twelve of the participants were degree students, 9 were master students, and 12 were doctorate students. Before the trial, the current state of participants’ auditory health was evaluated according to the International Organization for Standardization Acoustics. The participants did not exceed 20 dB in the frequency range of 125–8 kHz.

2.2. Selection and Recording of Sound Environments

The audio recordings completed at the places where students usually conduct their study activities have been used for this experiment. These locations include: a classroom during an exam, a classroom during a normal lesson, a hallway adapted for studying, a library, a computer room, and a common hall also adapted for studying. The sound environments were recorded using a Zoom H4n Handy Recorder previously calibrated in controlled acoustic conditions (anechoic chamber). The file format used was a WAV file at 16-bit, 44.1 kHz, and the length of each sound environment recorded was two minutes long. The output level also was calibrated in an anechoic chamber using the Bruel and Kjær head and torso simulator (HATS) Type 4128D and Pulse platform. The audios were reproduced using Pro Tools D9 software for Mac, a sound card Mackie Onyx Satellite, and headphones (model AKG K142 HD). The listening tests were carried out inside an audiometric chamber. The total time spent on each test lasted 12 minutes.

2.3. Evaluation of PA

The recordings and their subsequent analysis were performed using the psychoacoustic analyzer Brél and Kjær Sound Quality type 7698. Psychoacoustic metrics, such as loudness (N), specic loudness (N(L)), sharpness (S), roughness (R), and vocalization strength (FS), were obtained. These parameters are related to describe quantitatively the PA from the information obtained in the laboratory, the method proposed by Fastl and Zwicker. The PA value is obtained using the following equation:

\[
PA = N_5(1 + \sqrt{W_5^2 + W_{FS}^2 + W_{R}^2}).
\]

Where \(N_5\) is the percentile 5 of loudness, \(W_S\) is the component where sharpness (S) is included,

\[
W_S = \begin{cases} 
0, & \text{if } S \leq 1.75 \\
0.25(S - 1.75) \log N_5 + 10, & \text{if } S > 1.75
\end{cases}
\]

\(W_{FR}\) is the modulation component where fluctuation strength (FS) and roughness (R) are included.

\[
W_{FR} = \frac{2.18}{N_5^{0.4}} (0.4(FS) - 0.6R).
\]

2.4. Perception of Noise Annoyance

The method to evaluate the perception of noise annoyance is normally through listening tests and surveys. This questionnaire consisted of two questions related to noise annoyance based on the questions recommended by ICBEN. Likewise, two answer scales are proposed for these two questions: a 5-point verbal scale and 0–10 numeric scale (11-point scale). These scales have been used because a slight difference of perception between a numerical scale and a verbal scale could exist. The participants were questioned about the noise conditions inside the facilities recorded previously. The questionnaire was applied to the students while they were listening to the recording inside an audiometric room and they did the test only once. They were asked to take enough time to listen to the recordings before starting to answer the questions to have a better sensation of the sounds presented. For the 5-point verbal scale (see Table 1), the question was, “In this recording, how much does the environmental sound bother, disturb, or annoy you: extremely, very, moderately, slightly, or not at all?”

Table 1. Verbal answer and numeric scale for the annoyance question (5-point scale).

<table>
<thead>
<tr>
<th>Verbal answer recommended</th>
<th>Numeric scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Not at all</td>
<td>0</td>
</tr>
<tr>
<td>b) Slightly</td>
<td>21.93</td>
</tr>
<tr>
<td>c) Moderately</td>
<td>47.34</td>
</tr>
<tr>
<td>d) Very</td>
<td>73.39</td>
</tr>
<tr>
<td>e) Extremely</td>
<td>97.72</td>
</tr>
</tbody>
</table>

2.5. Numerical, Percentage, and Verbal Noise Annoyances Indexes

To have a variable that allows the classification of sound environment according to the level of annoyance, two correlated...
variables have been proposed: one percentage index and one verbal index. Resulting from the mean of the results between the evaluation of the PA and the evaluation of perception of noise annoyance, a numerical index is obtained and called numerical noise annoyance index (NNAI). Because NNAI is numerical data represented on a scale of 0 to 100 points, it is possible to transpose and express it directly in percentage terms. Thus it is possible to obtain an index which is called percentage noise annoyance index (PNAI). Furthermore, a verbal index called verbal noise annoyance index (VNAI) is directly related to the numerical ranges given on the scale of reaction to noise proposed by ICBEN\(^2\) and according to the numerical value obtained from the mean of the results between the evaluation of psychoacoustic annoyance (PA) and that of the perception of noise annoyance (Table 2).

### 3. RESULTS

#### 3.1. Evaluation of PA

The evaluation of the PA was performed according to the method described previously in section 2.3. This research is not focused on the particular study of each one of the psychoacoustic parameters because the main variable to be studied is the PA formulated by Fastl and Zwicker.\(^{21}\) However, as described previously, to obtain this variable, it is necessary to obtain known psychoacoustic parameters such as loudness (\(\text{N}\)), specific loudness (\(\text{N}'\)), sharpness (\(S\)), roughness, and fluctuation strength (\(\text{FS}\)). To obtain these parameters, the six sound clips were recorded with HATS. The recordings and the subsequent analysis of the clips were performed using the psychoacoustic analyser Brel and Kjaer Sound Quality Type 7698. Table 3 summarizes the results of this analysis for all clips recorded, and Fig. 2 shows the one-third octave spectrum of all environments as reported by Tristn et al.\(^{29}\) The equivalent total level (\(L_{\text{Aeq,T}}\)) of these sound environments oscillates from 68.1 and 83.5 dBA. For loudness, it is possible to observe that the minimum and maximum values correspond to the classroom during an exam (16.55 sones) and the common hall (46.75 sones), respectively. For sharpness, the minimum level was observed in the common hall (1.31 acum), and the maximum level was seen in the classroom during an exam (1.65 acum). In the case of roughness, it has been possible to observe the minimum and maximum values in the computer room (1.57 asper) and the classroom during an exam (1.70 asper), respectively. The minimum value of fluctuation strength was registered in the common hall (1.28 vacil), whereas the computer room is the place with the maximum value (2.07 vacil). Finally, the minimum and maximum values of specific loudness (\(\text{N}\)) correspond to the classroom during an exam (0.75 sones/Bark) and the hallway (2.25 sones/Bark), respectively.

Thus, in evaluating and performing according to the method described in III, the results show that the sound environment with a higher value of PA corresponds to the hallway (PA = 64.99) and the common hall (PA = 67.09). On the other hand, the lowest values of PA correspond to the classroom during an exam (PA = 25.96) and the hallway (PA = 23.34). The computer room obtained a PA value of 39.6, and the library, with a PA value of 33.5, is also an environment with low PA (Fig. 3).

#### 3.2. Evaluation of the Perception of Noise Annoyance

##### 3.2.1. Relation Between ICBEN 5-Point verbal scale and 11-Point Numerical Scale.

The results show that participants perceive that the common hall (63.04 points) and the hallway (57.56 points) are the most annoying places. Conversely, the classroom during an exam (5.31 points) and the classroom during a normal lesson
(14.06 points) are the less annoying places. Table 4 summarizes the results of the question related to the ICBEN 5-point verbal scale. On the other hand, according to the results for the ICBEN 11-point numerical scale, the classroom during an exam is the place with the lowest annoyance (4.54 points). On the other hand, the common hall is the place that participants perceive as the most annoying place of all the environments (67.27 points). A summary of the descriptive statistics is shown in Table 4.

Thus, to determine if obtained results between both 5- and 11-point scales (referred to as ICBEN 5 and ICBEN 11, respectively) are directly related, a statistical analysis was performed. According to the Kolmogorov-Smirnov test, the differences between both 5- and 11-point scales of all sound environments (except for the classroom during an exam) present a normal distribution; therefore, a t-test for related samples was carried out. For the classroom during an exam, a repeated-measures ANOVA was performed. The related measures t-test revealed that in the case of the classroom, no significant differences (t(32) = 0.166, p = 0.869) between ICBEN 5 (14.0612.31) and ICBEN 11 (14.5511.75) were found. Likewise, for the common hall, no significant differences (t(32) = 1.753, p = 0.089) between ICBEN 5 (14.0612.31) and ICBEN 11 (14.5511.75) were found. Furthermore, no significant differences (t(32) = 0.696, p = 0.491) for the computer rooms between ICBEN 5 (31.1712.41) and ICBEN 11 (33.3310.80) were found. The mean of the differences of the classroom among the exam results from the two tests, ICBEN 5 (5.319.54) and ICBEN 11 (4.545.64), were not significantly different (t(32) = 0.558, p = 0.581). Differences in the hallway between ICBEN 5 (57.5622.95) and ICBEN 11 (64.8527.17) also were not significant (t(32) = 0.147, p = 0.147). In the case of the library, it also was possible to observe no significant differences (t(32) = 0.435, p = 0.667) between ICBEN 5 (27.7323.48) and ICBEN 11 (23.3317.79). Finally, for the classroom during an exam, the repeated-measures ANOVA test shows no significant differences (F(1,32) = 0.311, p = 0.581). Furthermore, the comparison and the mean value between both 5- and 11-point scales of each environment are shown in Table 4.

Thus, to determine if obtained results between both 5- and 11-point scales (referred to as ICBEN 5 and ICBEN 11, respectively) are directly related, a statistical analysis was performed. According to the Kolmogorov-Smirnov test, the differences between both 5- and 11-point scales of all sound environments (except for the classroom during an exam) present a normal distribution; therefore, a t-test for related samples was carried out. For the classroom during an exam, a repeated-measures ANOVA was performed. The related measures t-test revealed that in the case of the classroom, no significant differences (t(32) = 0.166, p = 0.869) between ICBEN 5 (14.0612.31) and ICBEN 11 (14.5511.75) were found. Likewise, for the common hall, no significant differences (t(32) = 1.753, p = 0.089) between ICBEN 5 (14.0612.31) and ICBEN 11 (14.5511.75) were found. Furthermore, no significant differences (t(32) = 0.696, p = 0.491) for the computer rooms between ICBEN 5 (31.1712.41) and ICBEN 11 (33.3310.80) were found. The mean of the differences of the classroom among the exam results from the two tests, ICBEN 5 (5.319.54) and ICBEN 11 (4.545.64), were not significantly different (t(32) = 0.558, p = 0.581). Differences in the hallway between ICBEN 5 (57.5622.95) and ICBEN 11 (64.8527.17) also were not significant (t(32) = 0.147, p = 0.147). In the case of the library, it also was possible to observe no significant differences (t(32) = 0.435, p = 0.667) between ICBEN 5 (27.7323.48) and ICBEN 11 (23.3317.79). Finally, for the classroom during an exam, the repeated-measures ANOVA test shows no significant differences (F(1,32) = 0.311, p = 0.581). Furthermore, the comparison and the mean value between both 5- and 11-point scales of each environment are shown in Table 4.

### 3.3. Numerical, Percentage, and Verbal Noise Annoyances Indexes

A numerical variable was obtained from the comparison of the means carried out between the results derived from the mean of both 5- and 11-point annoyance scales and evaluation of the PA. This variable has been called numerical noise annoyance index (NNAI). A graphical comparison of both variables is shown in Fig. 4. Likewise, Table 5 summarizes the results obtained from each sound environment and the allocation of the verbal noise annoyance index (VNAI), which directly depends on the numerical noise annoyance index (NNAI) obtained. Also, the percentage noise annoyance index (PNAI) is shown. In addition, has been changed to verify that the numerical noise annoyance index (NNAI) is a significant value, a Pearson correlation was used to check whether a significant correlation exists between both annoyance scale variables. The Pearson's r correlation shows a clear positive correlation between both variables: \( r = 0.999, p < 0.01 \) (Fig. 5).

### 4. INDICATOR OF NOISE ANNOYANCE LEVEL

An indicator of the level of noise annoyance represented in a colour scale is proposed. This scale of annoyance is graphically represented in an indicator, where the colour levels depend on the numerical noise annoyance index (NNAI). The relationship exists between both annoyance scale variables. The Pearson’s r correlation shows a clear positive correlation between both variables: \( r = 0.999, p < 0.01 \) (Fig. 5).
percentage noise annoyance index (PNAI) which is derived for the numerical noise annoyance index (NNAI) and also with the verbal noise annoyance index (VNAI) (see section 3). Considering the importance of unifying criteria for the evaluation of noise annoyance, the ranks of this colour scale are based on the 5-point scale described previously (Table 1) in reaction modier given by ICBEN. 21

5. CONCLUSIONS

In this work, a method to evaluate noise annoyance in university facilities has been proposed. This method was based on the comparison and correlation of two known methods to evaluate noise annoyance, the PA, 15 and the evaluation of perception of noise annoyance through listening tests, on the basis of the method described by ICBEN. 21 The results from the evaluation of perception of noise annoyance show that between the answers of both rating scales (5-point and 11-point), there were no statistically signicant differences; this means that there is an important relation between the answers of both scales. Thus, derived from the mean of both tests, it has been viable to determine a numerical value that represents noise annoyance in a specic place. Furthermore, the recordings of six sound environments inside university facilities were studied. Psychoacoustic parameters were obtained, and it was possible to determine a specic value for the PA. A numerical variable, named numerical noise annoyance index (NNAI), was obtained from the comparison of the means between the results derived from both 5- and 11-point annoyance scales and the PA. This variable gives us a numerical value to represent the level of noise annoyance in a specic sound environment. During this study, it has been possible to observe that classrooms and libraries are not the only places used for study activities. Many other places, such as hallways, halls, and gardens, are employed as study places in an improvised way and used for study activities and for socialization with other students. The results of this study show that these places are the most annoying. The data collected from this study provides important information about the conditions where students develop their study activities. It makes the development of the most adequate strategies against noise easier. It is well known that acoustic conditioning is the most expensive action. Consequently, nding cheap and reliable alternatives is necessary. Thus, a colour indicator of noise annoyance has been proposed. The variables from this methodology have been summarized. This indicator serves as a tool to facilitate the classication of a particular sound environment according to their level of noise annoyance and also for designing strategies against noise. The method and the colour indicator proposed in this work will allow a deeper evaluation of noise annoyance and also facilitate the development of strategies and appropriate actions against noise.

6. AKNOWLEDGEMENTS

The rst author would like to acknowledge the Mexican Council of Science and Technology, (CONACYT) for the financial support given to develop this project, as well as Universidad Autonoma de San Luis Potos and Universidad Politcnica de Madrid.

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Active-suspension Design for a Special Road-Rail Vehicle Based on Vehicle-Track Coupled Model Using Genetic Algorithm

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In this paper, a PID controller for a special road-rail vehicle is developed. The dynamic model is derived to properly consider the vehicle and track vibrations. This model contains the effects of track elasticity which makes it more reliable and precious compared to traditional models. The vehicle model contains a two dimensional 10 DOF vehicle model, and the track model consists of 40 sleepers. In addition, the effect of the sleeper number on car-body vibration is investigated. The results show that the present vehicle-track coupled model is more efficient in controller design compared to traditional ones. Finally, tuning of controller gains is performed with the aid of a genetic algorithm, in order to achieve a well-organized active suspension.

NOMENCLATURE

- $C_1$: Damping coefficient of primary suspension.
- $C_2$: Damping coefficient of secondary suspension.
- $CF$: Cost function in genetic algorithm.
- $e$: Control error.
- $F$: Force matrix of system.
- $F_i, F_j$: Shear forces exerted on nodes $i$ and $j$.
- $I_b$: Half-mass moment of inertia of bogie.
- $I_c$: Half-mass moment of inertia of car-body.
- $K_1$: Stiffness coefficient of primary suspension.
- $K_2$: Stiffness coefficient of secondary suspension.
- $l_1$: Semi-longitudinal distance between bogies.
- $l_2$: Semi-longitudinal distance between wheelsets in bogie.
- $M$: Mass matrix.
- $M_b$: Half-mass of bogie.
- $M_c$: Half-mass of car-body.
- $M_i, M_j$: Moments about $z$-axis exerted on nodes $i$ and $j$.
- $M_w$: Half-mass of wheelset.
- $MO$: Maximum overshoot of controller.
- $N$: Total number of rail-sleeper supporting points.
- $NJ$: Number of rail nodes.
- $q$: Time-variant state vector.
- $u$: Actuator force.
- $V$: Vehicle forward speed.
- $Z_b$: Displacement of bogie.
- $Z_c$: Displacement of car-body.
- $Z_r$: Displacement of rail.
- $Z_w$: Displacement of wheelset.
- $\theta_b$: Pitch angle of bogie.
- $\theta_c$: Pitch angle of car-body.

1. INTRODUCTION

Rail transportation is one of the most common forms of transportation in the world. In order to provide a comfortable trip, using active suspension systems in these vehicles has become one of the most effective technologies in the transportation industry.

So far, there exist mainly two theories on railway vehicle dynamics. One is the traditional theory of railway vehicle dynamics, and the other is the vehicle-track coupled theory. The traditional theory of railway vehicle dynamics usually focuses on the railway vehicle itself without considerations of the dynamic behavior of the track. In the vehicle-track coupled theory, the track is treated as an elastic structure. Vibrations of the vehicle can be transmitted to the track via the wheel-rail contact and excite vibrations of the elastic track structure, which can in reverse influence the vibrations of the vehicle. Therefore, the vibrations of the vehicle and the track are essentially coupled with each other.

For various research purposes, different types of models have been presented. To study the feasibility for improving ride quality using magnetorheological dampers, a nine DOF model of railway vehicles was developed, including vertical, pitch and roll motions of car-body and trucks. Also, a 17 DOF model of a semi-active suspension system was used to improve the ride quality on train. In a study done by Sezer and Atalay, a 54 DOF model was established to design a fuzzy controller to reduce the vertical, lateral, and angular vibration of a rail vehicle containing a body, three bogies, and six axles. Track models presented in the aforementioned papers were considered rigid.

Investigating safety limits against derailment, several models have been proposed to consider the track vibration. A coupling model of vertical and lateral vehicle-track interaction was proposed by Zhai et al. in which the vehicle subsystem was demonstrated as a multi-body system with 37 DOF, and the track substructure was modeled as a discretely supported system of elastic beam. In addition, the vibration of the train as well as the track was discussed, where the rail was modeled based on the finite element method. Another similar work was done by Uzzal et al., in which the dynamic model was presented with the aid of partial differential equations. More-
over, in order to review the vibration of a vehicle and track to a single rail irregularity, Zakeri et al.\textsuperscript{9} presented general equations of a vehicle-track coupled system in a matrix form using finite element method. Furthermore, a road-rail fire-fighting vehicle was considered to demonstrate the effectiveness of the vehicle-track coupled model.\textsuperscript{11}

As mentioned in the above literatures, most studies have examined the vibration of railway vehicles, and the track is assumed as a rigid body that may lead to significant errors in the mathematical models; therefore, it is inaccurate. In some of the literatures reviewed above, the vehicle-track coupled model has been used. However, the sheer stress component between the ballast masses is completely ignored. Although both of these assumptions have been considered in a few of the studies, no active suspension is involved there.

In this paper, an active suspension system is applied to a vehicle-track coupled model in order to control the vertical and pitch vibrations of the car-body. This model includes a 10 DOF model for the vehicle and a track, including a rail, a sleeper, and a ballast, as well as consideration of the sheer stress components of the ballast. A new code is developed in MATLAB to study the vibration response of the system. Moreover, an active PID controller is designed to reduce the vibration. Tuning the corresponding parameters of the controller, a genetic algorithm code is established. The results show that using the active controller reduces the acceleration of the car-body by more than 60%. Furthermore, the effects of the vehicle speed on efficiency of the active controller indicate that the controller performance is gradually reduced by increasing the vehicle speed.

2. MODELLING

The vehicle system model used in this study consists of a car-body supported by two bogies, each of which has two axels. Without considering the yaw and roll motions of the car-body and bogies, the total DOF of the model is 10: the vertical displacement of the car-body, the pitch angle of the car-body with respect to its center of mass, the vertical displacements of bogies, the pitch angles of bogies with respect to their center of mass, and the vertical displacements of wheel sets. In this paper, vehicle equations proposed by Li et al.\textsuperscript{10} were used which are deduced according to the Newton’s laws of motion.

Based on the model presented by Li et al.,\textsuperscript{10} it is assumed that the track includes a rail mass, sleepers, and ballast masses. With the aid of finite element method, the rail model is established with applying the Euler-Bernoulli beam element type presented by Liu\textsuperscript{11} (Fig. (1)). Other track equations related to sleepers and ballast masses are utilized from the track equations presented by Li et al.\textsuperscript{10}

The vehicle and track coupled model is shown in Fig. (2). It is proved in Section 3 that the rail is long enough to neglect the boundary effects, so every deflection in both ends of the rail is assumed to be zero. It should be mentioned that the vehicle is placed at the center of the track. Finally, the corresponding vibration equation of the vehicle-track coupled model shown in Fig. (2) can be written in standard form as:

\[
M \ddot{q} + C \dot{q} + K q = F. \tag{1}
\]

The matrices \(M, K,\) and \(C\) are assembled according to (2) as follows:

\[
\begin{bmatrix}
\text{Carbody} & 0 & 0 & 0 \\
0 & \text{Rail} & R/S & 0 \\
0 & R/S^T & \text{Sleeper} & S/B \\
0 & 0 & S/B^T & \text{Ballast}
\end{bmatrix}
\tag{2}
\]

where the sub-matrices \(R/S, S/B\) represent the interaction between rails and sleepers, sleepers and ballast respectively. In Fig. (2), it is apparent that dynamic contact force between wheel and rail is assumed as non-linear equation.\textsuperscript{9}

3. NUMERICAL PROCEDURE

A numerical code is developed in MATLAB to investigate the vibration of vehicle-track interaction. It should be noted that the specifications of the vehicle and track are the same as what is used by Li et al.\textsuperscript{10} Matrices of the mass, stiffness and damping are square, and all have \(10 + 2NJ + 2N\) elements, where \(NJ\) is the number of rail nodes on which an external force is exerted, and \(N\) is the number of sleepers. So, \(N\) plus four points that wheels touch rail. Figure (3) shows the effect of these parameters on the vibration of the car-body. Also, it can be depicted that by increasing the number of sleepers and nodes, the acceleration response is getting more improved into real one. It also illustrates that by increasing the number of sleepers and nodes, the dominant period of response increases. Since displacement diagrams for the higher number of sleepers and nodes have less concavity, it can be concluded that increasing the number of sleepers and nodes results in the decrease of the car-body acceleration. Decreasing the car-body acceleration can be expressed by the fact that by adding the length of the rail, the effect of the boundary condition is diminished, and the track is more reasonable in conditions.

Since overloads on the wheel are predicted when the rigid track (traditional model) is considered, further acceleration (concavity of response) on the car-body is observed. In order to take away from the traditional model, the number of sleepers is improved in the presented model. Therefore, the track model becomes softer, and the applied force between wheel and track...
approaches the actual one. In other words, the overestimated acceleration on the car-body is removed by omitting the overloads in the present model. As mentioned above, choosing the values of $N$ and $NJ$ as 40 and 44, respectively, leads to a reliable model.

4. VERIFICATION OF THE MODEL

The present model is verified here with the aid of two viewpoints. Firstly, the body response of the present model to an irregularity shown in Fig. (6) is obtained assuming a great number for the stiffness of the track. Then, the response is compared with that of the model reported by Li et al.\textsuperscript{10} As depicted in Fig. (7), an excellent agreement is easily observable.

Secondly, the present model is compared with the models presented by Uzzal et al.,\textsuperscript{9} and Sun and Dhanasekar.\textsuperscript{12} In the work done by Uzzal et al.,\textsuperscript{9} dynamic analysis of a railway vehicle moving on an elastic track is performed. A five DOF model is assumed for the vehicle that contains vertical displacement of the vehicle car-body, bogie’s pitch and vertical displacement, and vertical motion in two axles. The track model is similar to the one previously depicted in Fig. (2). Using the vehicle and track characteristics of that model into present model, the results of this comparison are illustrated in Fig. (6).

As shown in this figure, there is a good agreement between these two models. In other words, in both models, the contact force between the wheel and track falls, due to onset of contacting the wheel flat irregularity with the track. Then, the contact force increases because the wheel suddenly impacts the track. The dominant period of oscillation of the contact force is approximately 10 milliseconds. However, a slight difference between two models was observed 30 milliseconds later, due to the differences between finite element method and partial differential solution. This difference influences computation of the damping matrix. In other words, it slightly affects the wheel-track damping force. Nevertheless, this has a negligible effect on car-body vibration.

Another comparison has been done using the 10 DOF vehicle model used by Sun and Dhanasekar,\textsuperscript{12} as well as the experimental work performed by Newton and Clark.\textsuperscript{13} In the work done by Sun and Dhanasekar,\textsuperscript{12} a track model with four-layer is considered, and the Hertz contact coefficient is deduced from Johnson.\textsuperscript{15} However, in present work, the recent model of contact phenomenon presented by Zhai\textsuperscript{2} is developed. It can be observed from Fig. (7) that when the wheel flat irregularity touches the track, the contact force reduces to zero. This means the wheel and the rail separate for a while. When they meet again, an enormous peak force is induced between them. This phenomenon is reasonably predicted in these three methods. Bothnumerical models show some disagreements with the experimental data. This is due to the discrepancy that occurred as a result of disagreement between the real and calculated damped
ing values of track components. In addition, the results of Sun and Dhanasekar\textsuperscript{12} give the maximum force much higher than that of the experimental model\textsuperscript{,13} whereas the present work gives the maximum force much more reliable.

These two comparisons confirm the reliability of the present model in simulating the vehicle-track coupled dynamics, especially for designing the controller, in which the reduction of maximum contact force is of high importance.

5. COMPARING VEHICLE-TRACK COUPLED MODEL WITH 10 DOF MODEL REGARDING VEHICLE BODY RESPONSE

This section discusses the importance of modelling accuracy in controller designing. At first, in order to illustrate the importance of considering track vibrations, car-body response of the 10 DOF model (traditional model) was compared to the response of the vehicle-track coupled model (present model). Then, in the next section, suitable PID controller gains are calculated based on the vehicle-track coupled model using the genetic algorithm.

Applying the irregularity shown in the Fig. (4) to the models, vertical displacement of the car-body for the traditional and the present model was obtained, as illustrated in Fig. (8). Significant differences are observed comparing the results of these two models. This is due to the fact that in the traditional model, the stiffness of the track is not considered. In present work, as the elasticity of the track is fully considered, it can be imagined that the vehicle travels on a softer bed. Apparently, applying spring elements to a vibration system in series-form causes decreasing stiffness; consequently, oscillations decrease, and thus displacements increase. Moreover, the amplitude of the acceleration of car-body (concavity of displacement graph) decreases. In other words, it can be seen that present model differs from the 10 DOF model in a significant quantity; therefore, the results of the present model is of a higher accuracy to design PID controller for the road-rail vehicle.

6. ACTIVE SUSPENSION SYSTEM

6.1. Active Suspension System Modeling

Using active secondary suspension is an efficient way to reduce car-body vibrations.\textsuperscript{14} Secondary active suspension im-
where $M_c$ is the half-mass of the car-body, $I_c$ is the half-mass moment of inertia of the car-body, $K$ and $C$ represent the stiffness coefficient and damping coefficient of suspension elements, and the subscripts 1 and 2 are used to identify primary and secondary suspension elements.

Respectively, the vertical displacement of the front and rear bogies are written as:

$$M_{b1}\ddot{z}_{b1} + C_2 + 2C_1\dot{z}_{b1} + K_2 + 2k_1z_{b1} - C_2\dot{z}_c + l_1\dot{\theta}_c = -K_1\dot{z}_{w1} - C_1\dot{z}_c - K_1\dot{z}_{w1} - C_1\dot{z}_{w2};$$

$$M_{b2}\ddot{z}_{b2} + C_2 + 2C_1\dot{z}_{b2} + K_2 + 2k_1z_{b2} - C_2\dot{z}_c - l_1\dot{\theta}_c = -K_1\dot{z}_{w3} - C_1\dot{z}_c - K_1\dot{z}_{w3} - C_1\dot{z}_{w4};$$

where $M_{b1}$ and $M_{b2}$ are the half-mass of the bogies, $I_{b1}$ and $I_{b2}$ are the half-mass moment of inertia of the bogies, $l_1$ and $l_2$ represent, respectively, the semi-longitudinal distance between bogies and the semi-longitudinal distance between wheelsets in bogies.

Considering the above relationships, the governing equations of the system can be written in a matrix form as:

$$M\ddot{q} + C\dot{q} + K = F + u;$$

where $u$ is the actuator force. For the PID controller $u$ is followed as:

$$u(t) = K_pe(t) + K_i\int_0^t e(t)dt + K_De(t);$$

where $K_p$, $K_i$, and $K_d$ are, respectively, proportional, integral, and derivative gains of the PID controller. The variable $e(t)$ is the control error.

### 6.2. Obtaining the Gains of the PID Controller and Evaluating Results

First, in order to find out the gains of the controller, the irregularity shown in Fig. (4) is applied to the system, and vibration of the car-body is measured. Then, the controller gains are optimized by using the genetic algorithm to minimize the car-body vibrations. The objective function of the genetic algorithm is to reduce integral of absolute error and maximum overshoot, written as:

$$CF = MO + \int_0^t e(t)dt;$$

where $CF$ is cost function and $MO$ is maximum overshoot.

Parameter specifications for the genetic algorithm are listed in Table 1. One of those specifications is a crossover fraction. A crossover fraction is the fraction of individuals in the next generation, other than elite children, that are created by crossover (that is, mating). The rest are generated by mutation. A crossover fraction of one means that all children other than elite individuals are crossover children. A crossover fraction of zero means that all children are mutation children. Choosing the best crossover fraction leads to better optimization, so the effect of the crossover fraction was investigated for both actuators that can be seen in Fig. (10). Figure (11) shows how fitness values are converged by assuming the best crossover fraction for minimizing cost function. Furthermore, 100 was selected as the number of generations because increasing this parameter does not result in a noticeable change in fitness values in either optimizations for vertical or angular actuators. Optimized controller gains are listed in Table 2.

The responses of car-body acceleration in both active and passive suspension systems are compared in Fig. (12). The effect of applying the active suspension system on the acceleration of the car-body is clearly observable from these figures. The reduction percentage of maximum vertical and pitch accelerations of the car-body are respectively 58% and 73%. In addition, by assuming ±0.02 m/s² as marginal acceleration for vertical acceleration and ±0.002 rad/s² for pitch acceleration, the set time for those accelerations are 0.053 s and 0.051 s while for the passive suspension system, those were 1.3 s and 2.9 s. Therefore, the new presented active control can effectively enhance the ride quality of the vehicle.

In Fig. (13), the effect of vehicle speed is investigated for speeds $1.2V$ and $0.8V$ as well, where $V$ is the previous speed amounting to 27 km/h. As illustrated, by increasing the vehicle speed, the maximum car-body acceleration increases, and the dominant period decreases. By increasing vehicle speed, the acceleration of irregularity increases and its period decreases, so the acceleration of car-body rises, and the dominant period of car-body acceleration lowers. In Fig. (14), the forces of actuators for different vehicle speeds are depicted. These figures justify this phenomenon. The higher the speed, the worse the actuator can reflect. As a result, maximum acceleration increases.

The percentage of maximum acceleration reduction of the car-body is given in Table 3. As listed, by increasing the vehicle speed, the efficiency of active suspension is reduced. The loss of efficiency is due to increasing frequency and input force that increases control error, so it reduces the efficiency of active suspension. In other words, the designed controller performs better at low speeds.

Based on the above discussion, the designed PID controller for the suspension of the road-rail fire-fighting vehicle reduces car-body vibration very well and improves the ride quality.

**Table 1. Genetic algorithm specifications.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Characteristic value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of variables</td>
<td>3</td>
</tr>
<tr>
<td>Population Size</td>
<td>20</td>
</tr>
<tr>
<td>Crossover fraction</td>
<td>For $u_1$ For $u_2$</td>
</tr>
<tr>
<td>Generations</td>
<td>100</td>
</tr>
<tr>
<td>Stall generation</td>
<td>100</td>
</tr>
<tr>
<td>Function tolerance</td>
<td>1e-10</td>
</tr>
<tr>
<td>Constraints tolerance</td>
<td>1e-10</td>
</tr>
<tr>
<td>Select function</td>
<td>Roulette</td>
</tr>
<tr>
<td>Crossover function</td>
<td>Scattered</td>
</tr>
</tbody>
</table>

**Table 2. Optimized controller gains.**

<table>
<thead>
<tr>
<th>$u_1$</th>
<th>$u_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3535659</td>
<td>68314788</td>
</tr>
<tr>
<td>13313522</td>
<td>14352104</td>
</tr>
<tr>
<td>32983</td>
<td>258263</td>
</tr>
</tbody>
</table>

**Table 3. Maximum acceleration reduction (%).**

<table>
<thead>
<tr>
<th>Pitch Acc.</th>
<th>Vertical Acc.</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8V</td>
<td>65 %</td>
<td>78 %</td>
</tr>
<tr>
<td>1.2V</td>
<td>58 %</td>
<td>73 %</td>
</tr>
</tbody>
</table>
Figure 10. The effect of crossover fraction on best fitness value for: (a) actuator No. 1 and (b) actuator No. 2.

Figure 11. Convergence of fitness value for: (a) actuator No. 1 and (b) actuator No. 2.

Figure 12. Car-body vertical and pitch acceleration.

Figure 13. Car-body vertical and pitch acceleration for different speeds.
REFERENCES


Estimation of Viscous Damping Parameters of Fibre Reinforced Plastic Plates using Finite Element Model Updating

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A combined numerical-experimental technique has been implemented using the finite element model updating technique to estimate a set of proportional viscous damping parameters for determining the global responses of fibre reinforced plastic (FRP) plates over a chosen frequency range of interest. The experimentally determined frequencies and mode shapes are used to update the homogenised anisotropic in-plane material constants, before estimating the damping parameters from the directly observed frequency response functions (FRFs). Gradient based inverse sensitivity method has been implemented for the parameter estimation. Existing FRP structures may degrade considerably due to environmental effects over the long period of existence—changing the material and damping properties significantly compared to their initial values after fabrication—thus requiring updating. The estimated viscous damping parameters using the current technique reproduces higher values of modal damping factors for FRP plates. For nonviscous damping, estimation of a set of relaxation factors produces a more realistic estimate of modal damping factors. Increased value of the relaxation factors make the model coincide with the viscous one. A numerically simulated plate problem has been presented along with the experimental validation.

1. INTRODUCTION

Dynamical responses of Fibre Reinforced Plastic (FRP) plates are functions of geometry, material properties, existing boundary conditions, and damping. The initial material properties used for modelling the dynamical responses of such plates come from the material characterisation tests on separately prepared samples. However, development of an accurate damping matrix is far more difficult, even at the initial stage of existence of the plate, because the global damping matrix is usually formulated directly from the global stiffness and mass information. It is extremely difficult to form a damping matrix from constituent level damping parameters, taking care of all the complex individual phenomena and their combinations, such as material damping, joint friction, and environmental effects. Gaul provided excellent guidelines to understand the damping of materials and structural members, including laminated parts. Making assumptions about small damping allows linear damping models to approximate the actual nonlinear damping behaviour. In proportional damping—where the viscous effects are dominant—the damping matrix is formed by weighted sum of mass and stiffness matrices. Thus, proportional damping or Rayleigh damping is mathematically expressed as

\[ C = a_0 M + a_1 K. \]  

Here, \( M, C, \) and \( K \) are the mass, viscous proportional damping, and stiffness matrices of the FRP plate considering the plate as a multiple degrees of freedom system. The coefficients \( a_0 \) and \( a_1 \) are the Rayleigh damping coefficients. The Rayleigh damping model has enjoyed an almost universal acceptance because the modal transformation of the proportional damping matrix—respect to the undamped normal mode shapes—produces a diagonal damping matrix. However, a more generalised viscous proportional damping matrix has been proposed by Caughey and Caughey and Kelly

\[ C = M \sum_{n=0}^{N-1} a_n [M^{-1} K]^n. \]  

In fact, the value of \( n \) can be chosen as an integer between \(-\infty\) and \(+\infty\), but practically this is taken very low (e.g., \( n = 2 \)) for Rayleigh damping. Woodhouse emphasised the importance of accurate predictions of physical mechanisms for estimating a proper damping matrix. Adhikari incorporated frequency variations of damping factors to determine a generalised proportional damping model. Adhikari and Phani obtained a generalised proportional damping matrix from a single driving point frequency response function (FRF) only.

Several investigators have used measured FRFs for damping identification. Arora et al. implemented a two-step damping identification methodology in which the stiffness and mass matrices are updated first from the measured FRFs, followed by identification of the damping matrix. Friswell identified a generalised viscous proportional damping matrix along with the stiffness matrix from measured FRFs. More recently, Pan and Wang used a FRF based iterative model updating approach to identify an exponential damping model. Kausel proposed methodology to construct families of nonproportional damping matrices.

Extensive investigations have been carried out by Dalenbring on laminated aluminium and Plexiglass plates, constrained viscoelastic layers of aluminium and carbon fibre epoxy laminate, and bonded aluminium polymethyl methacrylate plates to identify damping parameters. Dovstam formulated a linear three-dimensional material damping modelling technique using an augmented Hooke’s law (AHL) in the frequency domain, and recently investigated interface damping in built-up structures.
Literature related to modelling of damped FRP structures is limited. Crocker and Li presented reviews on damping of composites including sandwich composites.\textsuperscript{24} Gelman et al. used a FRF-based diagonalization of damping matrix, Akroot et al. investigated the vibroacoustic behaviour of thin film-laminated glass panels in the presence of fluid layers, and Wu and Huang studied the natural frequency and the damping of laminated composites plates.\textsuperscript{25–27}

Identification of damping using a finite element model updating is an active area of research.\textsuperscript{28} All existing methods of damping estimation usually assume availability of accurate mass and stiffness matrices; in reality, these requirements are hard to achieve for existing FRP structures, since uncertainties will grow both for stiffness and damping parameters as the structure deteriorates over time. As a result, most of the proposed techniques in literature for estimation of damping are not directly applicable to real existing structures unless a stiffness identification exercise is carried out prior to or in conjunction with the damping identification. Since the phenomenon of damping is complex, a proper, regularised, well-posed inverse approach is the only solution to forming a reliable damping matrix from measured modal or FRF responses.

The objective of the present investigation is to apply a model updating technique to estimate a set of globally equivalent viscous damping parameters and stiffness parameters for pultruded FRP plates using the Inverse Eigensensitivity Method (IEM) so that the predicted responses match exactly with observed responses.\textsuperscript{29} The procedure includes detailed experimental modal testing and subsequent data analysis, precise finite element modelling with appropriate discretization, correlating finite element results to the experiment, and updating selected parameters using a robust gradient-based optimization technique.\textsuperscript{30} The methodology is established by a numerically-simulated example, followed by a real experimental case study on a pultruded FRP plates under free-boundary conditions. A set of relaxation parameters has been defined to model nonviscous damping for FRP plates. Increased values of these relaxation factors make the model coincide with the viscous one.

2. MATHEMATICAL FORMULATION

2.1. Estimation of Viscous Damping Parameters

The equation of motion of the FRP plate in time domain can be written as

\[ M \ddot{x}(t) + C \dot{x}(t) + Kx(t) = f(t). \]  

Here, \( x(t) \) is the displacement vector. The undamped eigenvalues and eigenvectors are computed from the eigensolutions of the equation

\[ K \phi = \omega^2 M \phi. \]

The undamped eigenvalues, eigenvectors and the damping modes can be related through the expression of acceleration FRFs as

\[ H_{pq}(\omega) = -\omega^2 \sum_{n=1}^{N} \frac{\phi_{pn} \phi_{qn}}{(\omega_n - \omega + 2i\omega_n \zeta_n)}; \]

where \( H_{pq} \) is acceleration response at point \( q \) due to excitation at point \( p \).\textsuperscript{31} The modal damping factor \( \zeta_n \) for \( n \)th mode of the FRP plate can be expressed as

\[ \zeta_n = \frac{1}{2} \left( \frac{a_0}{\omega_n} + a_1\omega_n + a_2\omega_n^3 + \ldots + a_{N-1}\omega_n^{2N-3} \right). \]

For proportional damping, \( N \) can be taken as 2. Thus, Eq. (6) reduces to

\[ \zeta_n = \frac{1}{2} \left( \frac{a_0}{\omega_n} + a_1\omega_n \right). \]

Here, \( a_0 \) and \( a_1 \) are the Rayleigh damping coefficients. When \( N > 2 \) in Eq. (6), the damping matrix is a fully populated matrix,\textsuperscript{32} whereas the stiffness and mass matrices still remain sparse and banded. Substituting the values of Rayleigh damping coefficients, Eq. (5) can be expressed as

\[ H_{pq}(\omega) = -\omega^2 \sum_{n=1}^{N} \frac{\phi_{pn} \phi_{qn}}{(\omega_n - \omega + i\omega (a_1\omega^2 + a_0))}. \]

Out of all three system matrices, the mass matrix has less uncertainty. Moreover, the magnitude of the coefficients of the damping matrix is an order lower in comparison to the mass and stiffness matrices in a consistent formulation. Thus, for estimating the damping parameters of the FRP plate, a two-stage model updating approach needs to be implemented by updating the stiffness matrix in the first phase, then updating modal damping coefficients—only parameters \( a_1 \) and \( a_0 \)—in the damping matrix during the second phase.

The objective function can be formed as the summation of the weighted differences of the measured and computed FRFs (used to estimate damping parameters) or the frequencies and mode shapes (used to estimate stiffness parameters) of the FRP plate. The objective function can be expressed as

\[ E = \sum_{p=1}^{n} w_{pp} \| H_{exp} - H_{mu} \|^2; \quad \text{or} \]

\[ E = \sum_{p=1}^{n} w_{pp} \| (\omega, \phi)_{exp} - (\omega, \phi)_{mu} \|^2, \]

where \( w_{pp} \) are the weights and \( p \) is the number of frequencies or FRFs considered sensitive enough with respect to the elastic parameters or modal damping coefficients. The sensitivities of the frequencies, mode shapes, or the FRFs with respect to the material parameters or damping parameters are computed using first-order finite difference approximation

\[ S = \left[ \frac{\partial(\omega, \phi \text{ or } H)}{\partial r_n} \right]. \]

Here, \( r_n \) is the material or damping parameters for which sensitivity is calculated. The relationship between changes in measured modal properties (e.g., frequencies, mode shapes, or FRFs) and the changes in the parameters to be updated (e.g., material constants, \( a_0 \), and \( a_1 \)) of the FRP plate can be related through the first-order sensitivity matrix

\[ \{ \Delta f \} = [S]\{ \Delta r \}. \]

Suitable changes \( \{ \Delta r \} \) are made to the initial guesses of the parameters within reasonably selected bounds, and the finite element model of the pultruded FRP plate is updated iteratively.
minimizing the objective function in a weighted least-square sense. Such changes implement the IEM

\[
\{r\}_{p+1} = \{r\}_p + (\Delta r)^T P.
\] (12)

The similarity of experimental modes with the numerically computed modes is quantitatively verified during each step of iteration through Modal Assurance Criteria (MAC); Values close to 1 indicate good correlation. The experimental FRFs are compared with the numerical FRFs using Signature Assurance Criteria (SAC). The above correlations are vector correlations, whereas the FRF magnitudes are compared using Cross Signature Assurance Criteria (CSAC) and Cross Signature Scale Factor (CSF).

2.2. Estimation of Relaxation Factors from Updated Viscous Damping Parameters

For nonviscous damping, the equation of motion (Eq. (3)) can be written as

\[
M\ddot{x}(t) + \int_0^t G(t-\tau) x(\tau) d\tau + K x(t) = 0.
\] (13)

Here, the dissipative force depends on the time-history of motion. This can be represented by a convolution integral between the velocities and a decaying kernel function. One promising expression for \(G(t)\) is exponential damping model

\[
G(t) = C \mu e^{-\mu t}.
\] (14)

Here, \(C\) is the damping matrix with real coefficients and \(\mu\) is the relaxation factor. The viscously-damped system can be expressed as a special case of exponential nonviscous damping when \(\mu \to \infty\). The Laplace transformation of the dynamical equation can be expressed as

\[
s^2 M z + s L(s) z + K z = 0.
\] (15)

Here, \(L(s)\) is the Laplace transformation of \(G(t)\), \(z\) is the complex mode, and \(s\) is the complex frequency of the FRP plate. Assuming small damping and first-order perturbation theory, the \(q^{th}\) complex natural frequency and mode shape can be approximated as

\[
\lambda_q = \pm \omega_q + i \frac{\mu_0}{2} \omega_q \phi_q + i \sum_{n=1}^{N} \frac{\omega_q \omega_n \phi_n}{(\omega_q^2 - \omega_n^2)} \phi_n.
\] (16)

The imaginary part of the above equation can be written as

\[
L'_{nl}(\omega_q) = \phi_n^T L(\omega_q) \phi_l;
\]

\[
L''_{nl}(\omega_q) = \frac{\mu C \lambda'}{(\mu - i \omega_q)} \frac{\mu C}{(\mu - i \omega_q)} = \frac{\mu^2 C}{(\mu - i \omega_q)} - i \frac{\mu \omega_q C}{(\mu - i \omega_q)}.
\] (17)

where

\[
C'_{nl} = \phi_n^T C \phi_l.
\] (18)

In the above equation, \(\phi_l\) is the real part of the complex mode shapes. Assuming that

\[
G(t) = (a_0 M + a_1 K) e^{-\mu t} = C_0 \mu e^{-\mu t} + C_1 \mu e^{-\mu t};
\] (19)

the Laplace transform of the equation can be written as

\[
L'_{nl}(\omega_q) = \left\{ \frac{\mu^2 C_{nl}^2}{(\mu^2 + \omega_q^2)} + \frac{\mu^2 C_{nl}^2}{(\mu^2 + \omega_0^2)} \right\} - i \left\{ \frac{\mu \omega_q C_{nl}^2}{(\mu^2 + \omega_q^2)} + \frac{\mu \omega_q C_{nl}^2}{(\mu^2 + \omega_0^2)} \right\}.
\] (20)

Here

\[
C_{0, nl} = a_0 \phi_n^T M \phi_l;
\]

\[
C_{1, nl} = a_1 \phi_n^T M \phi_l;\]

\[
\lambda_q = \pm \omega_q + \frac{a_0}{2} \frac{\mu_0 \omega_q}{(\mu_0^2 + \omega_q^2)} + \frac{a_1}{2} \frac{\mu_1 \omega_q}{(\mu_1^2 + \omega_q^2)} + i \frac{a_0}{2} \frac{\mu_0^2}{(\mu_0^2 + \omega_q^2)} + \frac{a_1}{2} \frac{\mu_1^2}{(\mu_1^2 + \omega_q^2)}.
\] (22)

The complex natural frequency corresponding to the elastic modes can be written as

\[
\lambda_q = \omega_q + i \omega_q \xi_q.
\] (23)

Here, \(\xi_q\) is the modal damping of \(q^{th}\) mode. Comparing the imaginary parts of the above two equations, the value of the modal damping factor can be correlated to the relaxation parameter as shown

\[
\xi_q = \frac{a_0}{2 \omega_q} \frac{1}{(1 + \frac{\omega_q}{\omega_i})} + \frac{a_1 \omega_q}{2} \frac{1}{(1 + \frac{\omega_q}{\omega_i})};
\] (24)

where \(a_0\) and \(a_1\) are values of updated damping parameters with respect to the measured FRFs. When the value of \(\mu_0\), \(\mu_1\) are comparatively higher, then the exponential damping closely matches with the Rayleigh damping.

3. NUMERICALLY SIMULATED EXAMPLE

Consider a rectangular FRP composite plate of dimensions \(400 \text{ mm} \times 300 \text{ mm}\) and with a thickness of \(10 \text{ mm}\). This numerically-simulated example deals with the estimation of the in-plane stiffness properties and global damping parameters of a pultruded FRP composite plate. A \(12 \times 12\) mesh division using the isoparametric element (S8R) in ABAQUS was found to be adequate for the convergence of eigenproperties within selected frequency ranges, and is used throughout this present investigation. The experimental frequency, mode shapes, and FRF data are generated using the same finite element model. The in-plane material elastic constants are taken as \(E_x = 35 \text{ GPa}, E_y = 30 \text{ GPa}, G_{xy} = 5 \text{ GPa}\); the Poisson’s ratio is taken as \(v_{xy} = 0.17\). Here, \(x\) and \(y\), respectively, are the longitudinal and transverse in-plane directions of the FRP plate. The data is similar to the actual experimental case study reported later. The transverse shear moduli, \(G_{xz}\) and \(G_{yz}\) both are assumed to be 5 GPa. The direction \(z\) implies direction...
perpendicular to the plane of the plate. Since the in-plane Poisson’s ratio can only be estimated properly when sufficient numbers of torsional modes are included as information, this factor was removed from scope of the experimental investigation. The mass density is assumed to be 1500 kg m⁻³. The assumed experimentally measured modal damping parameters and natural frequencies are given in Table 1.

The parameter estimation exercise was carried out using the inverse eigensensitivity method (IEM) to estimate the material parameters first. The convergences from a few initial values to the correct material constants are shown in Fig. 1. As expected, changes in modal damping values had little effect on the estimation of the in-plane material constants.

In Fig. 1, s₄₀,37,₁₀ is a set of initial values of the material constants $E_x = 40$ GPa, $E_y = 37$ GPa, and $G_{xy} = 10$ GPa used in the model updating exercise. The solutions were unique in all cases.

The estimate of the damping coefficients $a_0$ and $a_1$ is sensitive to the FRF magnitudes. Simulated “experimental” frequencies and modal damping parameters from Table 1 were used to choose sets of initial values of $a_0$ and $a_1$ for use in the model updating program. Equation (7) gives a set of initial values of $a_0$ and $a_1$, which differ considerably depending upon the modes selected, as shown in Table 2. The first three sets show the variations of these two parameters due to the incorporation of varying number of modes—at most, eight modes here. To check the robustness of this FRF based inverse algorithm, two additional arbitrary sets of $a_0$ and $a_1$ values were chosen, which did not immediately correspond to any combination of modes; this is shown in Table 2.

Figure 2, shows the convergence curves for both the parameters which are found to be monotonic and unique. The values are $a_0 = 2.53E1$ and $a_1 = 4.27E-6$ respectively. The updated mass-proportional damping coefficient converged to a higher value while the stiffness proportional damping coefficient converged to a somewhat lower value. Similar instances have been reported in current literature, demonstrating that a higher value of mass-proportional damping coefficients indicates a trend towards more viscous damping. Also, similar observations have been made for FRP beams.

The regenerated FRF curves—as shown in Fig. 3—continually matched as additional modes were added into the simulation, indicating that the updated parameters of the identified damping matrix are globally representative. Thus, the response prediction of the plate became much more accurate. The degree of improvement was judged by comparing improved values of MAC and SAC between the simulated “experimental” observation and the observations made using the updated model.

Figure 4 shows the FRF curve with 2% random noise in FRF data; here, the damping parameters were updated to the values $a_0 = 2.55E1$ and $a_1 = 4.3E-6$, which can be compared with the previous values obtained in the FRF data without noise. Thus, the noise sensitivity of the algorithm was observed to be low.

4. EXPERIMENTAL INVESTIGATION

A rectangular FRP composite plate of the same size—400 mm × 300 mm, with a thickness of 10 mm, as described in the numerically simulated example—was fabricated out of woven roving glass fibres and epoxy matrix using the pultrusion process. In fact, the plate has been extracted from a
much bigger plate fabricated for the purpose of experimentation on beam samples as well as for the plate. The average thickness and the mass density of the plate was 10.12 mm and 2012 kgm$^{-3}$ respectively. Modal testing was carried out on the FRP plate under free-boundary conditions, using impact excitation imparted through an impact hammer; Responses were picked up by an accelerometer. The Frequency Response Functions (FRFs) were directly measured, and frequencies and mode shapes were extracted using modal analysis software MEScope. Such modal testing is standard in current practice—details of similar modal testing on pultruded FRP composite beam may be obtained from investigations carried out earlier.

Figure 5(a) shows the modal test setup, in which the free boundary condition was realised by suspending the FRP plate using soft elastic strings from a rigid support. Figure 5(b) shows the position of the accelerometer (point 16), and a few selected points of excitations of the plate with the impact hammer. All 49 points were used as excitation points.

The finite element modelling of the plate was performed using the same shell element (S8R) with adequate discretization to get converged values of the modal properties. The numerical FRFs were synthesized from the undamped modal properties, using the modal damping factors from experimental observation. The mode shapes and initial MAC values for correlations between modes are not shown for brevity. Differences in modal and FRFs between the experimental observation and the finite element model were resolved through model updating after selecting appropriate sets of parameters. The most likely parameters affecting the results are 1) the in-plane material constants affecting the resonant frequency shifts and 2) the damping factors affecting the response amplitude sharpness of the FRF curves of the pultruded FRP plate.

First, the material parameters were updated using the IEM considering the experimental modal data, viz. frequencies and mode shapes. The convergence curves from a set of selected initial values of the parameters are shown in Fig. 6.

Frequencies obtained using updated elastic material parameters along with the experimental frequencies are shown in Table 3. The experimental modal damping factors are also shown.

The final MAC values indicate good correlations between the updated mode shapes and the observed mode shapes. Since MAC is a global vector correlation function at resonant fre-
Figure 4. Comparison of actual FRF (with 2% noise) and regenerated FRF using updated parameter values.

Figure 5. (a) Experimental setup for modal test, (b) Grid points for measurement of FRFs of the plate.

Figure 6. Typical convergence curves for material parameters from selected initial values.

Table 3. Updated and measured eigenvalues and damping.

<table>
<thead>
<tr>
<th>Updated freq. (Hz)</th>
<th>Exp. freq. (Hz)</th>
<th>Measured damping (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>164.11</td>
<td>164.15</td>
<td>3.247</td>
</tr>
<tr>
<td>256.53</td>
<td>256.59</td>
<td>1.8351</td>
</tr>
<tr>
<td>418.57</td>
<td>420.1</td>
<td>1.7126</td>
</tr>
<tr>
<td>481.92</td>
<td>482.27</td>
<td>1.1702</td>
</tr>
<tr>
<td>576.47</td>
<td>577.30</td>
<td>1.2725</td>
</tr>
<tr>
<td>710.41</td>
<td>710.9</td>
<td>1.0058</td>
</tr>
</tbody>
</table>

Next, the damping parameters were updated. The initial values of these parameters were selected again from Eq. (7), following the methodology outlined in the numerically simulated example. The two sets of estimated damping coefficients shown in Table 4, along with two arbitrary values, were used men, and can be attributed to the size effects of the beam sample. The updated parameters obtained from the updating exercise of the plate specimen are believed to be more accurate. Table 3 shows the regenerated frequencies, experimentally observed frequencies, and the experimental modal damping factors.

The average material constants determined experimentally, were $E_x = 33.05$ GPa, $E_y = 31.80$ GPa, $G_{xy} = 5.73$ GPa, and $v_{xy} = 0.15$; the updated material constants were found to be $E_x = 35.64$ GPa, $E_y = 32.31$, and $G_{xy} = 7.12$ GPa respectively. The updated material parameters differed only slightly from the results obtained by updating a beam specimen, and can be attributed to the size effects of the beam sample. The updated parameters obtained from the updating exercise of the plate specimen are believed to be more accurate. Table 3 shows the regenerated frequencies, experimentally observed frequencies, and the experimental modal damping factors.

The average material constants determined experimentally, were $E_x = 33.05$ GPa, $E_y = 31.80$ GPa, $G_{xy} = 5.73$ GPa, and $v_{xy} = 0.15$; the updated material constants were found to be $E_x = 35.64$ GPa, $E_y = 32.31$, and $G_{xy} = 7.12$ GPa respectively. The updated material parameters differed only slightly from the results obtained by updating a beam specimen, and can be attributed to the size effects of the beam sample. The updated parameters obtained from the updating exercise of the plate specimen are believed to be more accurate. Table 3 shows the regenerated frequencies, experimentally observed frequencies, and the experimental modal damping factors.
to test the robustness of the algorithm.

FRFs using a typical set of trial values and updated values of damping parameters are shown in Fig. 8. The regenerated responses using the updated damping parameters matched well with the experimentally obtained FRFs and added confidence to the updating procedures. In Fig. 9, the comparison between the regenerated FRFs using modal damping values and the updated damping parameters were found to be almost exactly matching. The SAC value also approaches 1, indicating favourable global correlations.

The corresponding values of the real and imaginary parts of the FRFs computed using updated Rayleigh damping coefficients are shown in Fig. 10(a) and 10(b) respectively.

The convergence curves of the damping parameters are shown in Fig. 11, and were found to be monotonically converging in all cases, with the final updated parameters being $a_0 = 72.3$ and $a_1 = 0.38E-5$ respectively. Altogether, 6 modes were considered. It was immediately apparent that the parameters converged to values much different from the initial estimate in Eq. (7); this can be attributed to the effect of incorporating more than two frequencies at a time.

As a verification exercise, experimentally observed FRFs that had not been used for updating were compared with corresponding regenerated FRFs using the updated parameters. The correlations were also found to be favourable, though these results are not shown for brevity. The results of this exercise help confirm the robustness of the current algorithm.

The question not yet resolved is the acceptable number of modes to include in the determination of a globally representative set of damping parameters. Convergence of the damping parameters with increasing numbers of modes—up to 8 modes in the current model updating exercise—indicated fewer variations amongst themselves as shown in Table 5. There were difficulties in accurately measuring higher modes during this investigation, thus results are hence not shown here.

A higher value of the mass-proportional coefficients in

---

**Table 4.** Initial value of damping coefficients from experimentally obtained modal damping.

<table>
<thead>
<tr>
<th>Mode considered for average of $\omega_1$ and $\zeta_1$</th>
<th>Mode considered for average of $\omega_1$ and $\zeta_1$</th>
<th>$a_0$</th>
<th>$a_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial_1</td>
<td>1.2</td>
<td>3.4</td>
<td>10.69</td>
</tr>
<tr>
<td>Trial_2</td>
<td>1.25</td>
<td>3.45</td>
<td>12.54</td>
</tr>
<tr>
<td>Trial_3</td>
<td>--</td>
<td>--</td>
<td>100</td>
</tr>
<tr>
<td>Trial_4</td>
<td>--</td>
<td>--</td>
<td>90.00</td>
</tr>
</tbody>
</table>

**Table 5.** Updated damping coefficients for different frequency ranges.

<table>
<thead>
<tr>
<th>Frequency range up to</th>
<th>Updated $a_0$</th>
<th>Updated $a_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>450 Hz</td>
<td>7.97E1</td>
<td>2.29E-6</td>
</tr>
<tr>
<td>650 Hz</td>
<td>7.43E1</td>
<td>3.93E-6</td>
</tr>
</tbody>
</table>

---

Figure 7. (a) MAC value of different mode, (b) CSF and CSAC along frequency.

Figure 8. Comparison between experimental FRF and FRF using Rayleigh damping parameters.

Figure 9. Comparison between experimental FRF and FRF using updated damping parameters.
Rayleigh damping indicates that the equivalent viscous damping effect is predominating and can be modeled conveniently with assumptions about velocity proportional damping. On the other hand, higher values in stiffness proportional coefficients indicate that damping is mainly due to internal material damping, and may be nonviscous in nature. In this experimental investigation, the updated mass proportional damping coefficient $a_0$ was indeed found to be of a much higher order than the stiffness proportional damping coefficient. Subsequently, the updated damping parameters $a_0$ and $a_1$ can be used to compute the modal damping coefficients. In the present investigation, the modal damping factor was overestimated in comparison to the measured one (Fig. 12). Estimations of relaxation factors corresponded to the measured modal damping. The value of $\mu_0$ was found to be 95 s$^{-1}$ and the value of $\mu_1$ was estimated as 150000 s$^{-1}$, which closely resembles viscous damping in the present case study. Additionally, estimated lower values of $\mu_0$ reaffirm the viscous damping model. For a structure in which internal material damping force is dominant, the damping coefficients of the present model can conveniently be adjusted through model updating to produce accurate vibration responses.

5. CONCLUSIONS

A finite element model updating algorithm was implemented to estimate proportional damping parameters of a fibre reinforced plastic plate over a large frequency range from experimentally-observed modal and frequency response functions. In most practical cases of existing fibre reinforced plastic structures, it will be mandatory to update the uncertain material parameters first, followed by estimation of damping parameters. The number of frequencies to include will be decided depending upon the frequencies of interest, but includes all frequencies contributing effectively to the response functions. For updating material parameters, information from frequencies and mode shapes is convenient; alternatively, for updating damping parameters, frequency response functions are more appropriate.

In this study a numerically-simulated example was presented, followed by a detailed experimental investigation of an FRP composite plate under free-boundary conditions for the correct estimation of the elastic parameters and damping coefficients together. The results were verified using measured responses which are not used during updating, thereby eliminating...
The possibility of the updating methodology being biased towards limited measured observations. Finally, the updated material parameters were checked through static characterization tests.

The relationship between the viscous damping and nonviscous damping was also established through an estimation of a pair of relaxation parameters using model updating. The actual damping effect for this experimental case study was found to be predominantly viscous. The methodology used in this study can easily be applied to model the damping effects of plates where internal material damping is dominating, instead of the viscous effects by adjustment of the damping relaxation parameters through model updating.

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Improved Numerical Modelling of Fiber Reinforced Plastics I-Beam from Experimental Modal Testing and Finite Element Model Updating

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Fiber reinforced plastics (FRP) is increasingly being used in infrastructural applications like bridges, chemical plants etc., where the environment can limit the expected service life of structures made of conventional materials such as reinforced concrete, steel or timber. Advantages of FRP over conventional constructional materials are its high specific strength and specific stiffness, ease with which it can be moulded to various shapes, corrosion resistance, lower lifecycle cost, durability etc. Estimation of accurate dynamic responses of FRP structures is very important from their operation point of view. Such dynamic responses are functions of material properties, boundary conditions, geometry and applied loading. FRP being an anisotropic and layered composite material, a large number of elastic material property parameters are to be determined. Moreover, its structural fabrication and material fabrication at constituent level being one unified process, the actual existing material property parameters may vary considerably from those specified in established standards or determined from characterisation tests. The present approach attempts at establishing a non-destructive technique based on experimental modal testing and finite element model updating to estimate the elastic material parameters of an ‘I’ beam made of FRP, thereby making the prediction of dynamic responses more accurate. Static load test on the beam and characterisation tests on samples cut from actual structure are conducted to assess the performance of this updating exercise. The current approach can also be used to non-destructively monitor degradations of elastic material properties over time and thus can be used for health monitoring of existing FRP structures.

1. INTRODUCTION

Use of Fiber Reinforced Plastics (commonly known as FRP) in weight sensitive and high performance applications, such as aerospace, marine, biomedical and sports, is well established due to its high specific strength (strength to weight density ratio) and high specific stiffness (elastic modulus to weight density ratio). Although FRP is being used for repair material to original concrete or steel structures for a long time, its use as main structural members of bridge or building etc. is relatively recent and is economic only in special circumstances. Examples of such applications are structures built in highly corrosive environment of chemical plants or near seashore where conventional building or bridge components made of steel, timber or reinforced concrete have problems that can limit their expected service life. Other applications include replacement of part of existing bridge made of conventional materials with prefabricated FRP structural components in extremely busy places where roadways cannot be closed for long. There are examples of structures made entirely of FRP, e.g. Aberfeldy footbridge over the River Tay in Scotland (Daniel and Ishai 2005). Application of FRP in infrastructure is promising due to its ability to be moulded to various shapes with very good surface finish, faster fabrication time, superior durability and fatigue strength, good thermal, electrical and acoustic insulation property etc. Davalos et al. (1995, 1996, 1997a, 1997b) have explored approaches to analyse and design pultruded FRP beams in bending and flexural-torsional buckling. A FRP box-girder bridge made of blade angle and T stiffened panels was analysed by Upadhyay and Kalyanaraman (2003). Kumar et al. (2004) tested FRP pultruded bridge deck, made of square hollow glass and carbon tubes of varying lengths. Wael (2010) presented a stability model for the local buckling of pultruded fiber reinforced polymer structural shapes subjected to eccentric compression. Esfandiar et al. (2010) have put forward a parameter estimation approach using response sensitivities with respect to the change of mass and stiffness, evaluated from the decomposed frequency response functions. Hollaway (2010) discusses the infrastructural applications of advanced polymer composite materials over the past four decades. The behavior and performance of ultra thick laminate component of T-sections manufactured with non-crimped fabrics (NCF) was investigated by Zimmermann et al. (2010) using a 3D modeling approach.
al. (2011) modelled a pultruded FRP hat section box girder. Contribution of shear deformation on the lateral buckling behavior of open cross-sections pultruded FRP beams were investigated by Ascione et al. (2011). Feo et al. (2013) presented experimental and numerical results of investigation on one of the major structural issues termed as the “influence zone” that defines the strength limit-state of pultruded fiber-reinforced polymer (PFRP) profiles. A state-of-the-art review of the analytical and numerical studies performed with the aim of predicting the strength, the load-deformation response and the failure mode of rehabilitated RC members was reported by Napoli et al. (2013). A carbon fiber based FRP floor panel, designed as a pultruded beam with an open cross-section, was investigated by Gao et al. (2013) and it was observed that the pultruded fabrication has relatively weak strength in the transverse matrix and through thickness direction. Cardoso et al. (2015) developed a simple accurate equation to determine the local buckling critical stress of pultruded GFRP I-sections. It appears that the current literature is still scanty about the dynamic behavior of such FRP structural forms.

Static and dynamic behaviour of FRP structures are functions of physical properties, existing boundary conditions and applied loading. Unlike isotropic materials, material fabrication and structural fabrication are a unified process in FRP composites. Moreover, manufacturing and curing processes differ considerably in various applications. Thus, the material property parameters may differ significantly from those specified nominally by the manufacturer or obtained from established standards or determined from quasi-static characterisation tests. In most cases, these characterisation test results also have substantial variations, making the material property as one of the most uncertain parameter in predicting dynamic behaviour. Moreover, the construction of FRP structural components is still very much dependent upon skill of labour and is another source of uncertainty.

Unlike isotropic materials, experimental quasi-static characterisation of FRP composite materials is time consuming as a large number of parameters need to be measured. It is even more difficult to access the global material properties of structural components from constituent level properties, such as from fiber and matrix properties. It would be best if a FRP structure can be tested as a whole and the overall existing material properties verified in situ through a well posed inverse problem. All subsequent global prediction of behaviour will match much more accurately with observations.

Correction of finite element models by processing dynamic test data is an active area of research. There are numerous examples of inverse problems for the determination of average material properties of composite materials from dynamic testing. Special attention is given in current literature for the estimation of the four in-plane elastic constants viz. $E_1$, $E_2$, $G_{12}$ and $\nu_{12}$ of orthotropic materials from experimentally measured natural frequencies and mode shapes. Early examples can be found with the work of De Wilde et al., Sol, Deobald and Gibson, Grédiac and Paris, Mota Soares et al. (1993) used plate shaped specimens to identify layered properties of composites. Larsson presented an iterative method to determine all four in-plane elastic constants of thin oriented strand board (OSB) from a single modal test. The distinct feature of this paper is that the shear modulus is estimated from twisting modes and the Poisson’s ratio is determined separately from compression mode. Cugnoni et al. identified both in-plane and transverse elastic constitutive properties of composite laminates from a single non-destructive test. Out of many optimization algorithms, gradient based optimization techniques are conveniently used to estimate the material parameters, due to their faster convergence. Collins et al. were the first to introduce one of the most popular gradient based approaches, viz. the Inverse Eigensensitivity Method (IEM) for model updating. Later, Chen and Garba modified Collin’s statistical approach into a matrix perturbation method to make it more convenient for practical use. Dascotte applied this approach to determine the in-plane elastic constants of vertically stiffened composite cylindrical shells. Mishra and Chakraborty have recently applied the same methodology to estimate the material properties of FRP plates under clamped boundary conditions. Also they have estimated the constituent level elastic properties for the materials used. In the present investigation, the same gradient based IEM implemented through commercially available software FEMtools has been explored. The current literature related to application of model updating to estimate elastic material parameters advocates small rectangular plate type of specimens in free-free boundary conditions. Application of model updating to real existing civil engineering structural forms is seen to be very rare.

An investigation has been carried out here to determine the representative average material parameters of a fabricated FRP ‘I’ beam from experimentally determined modal parameters and finite element predictions using model updating techniques. The ultimate goal is to prepare a precise numerical model to predict dynamic responses more accurately for any such FRP structure with the help of updated parameters. The results have been verified by quasi-static characterisation tests of samples cut from the actual structure later. Also, the entire FRP beam is tested under static load to confirm the overall stiffness and strength achieved. The actual procedure of modal testing and updating takes very less time as compared to the characterisation tests and gives opportunity for rapid assessment to monitor the structure’s health from time to time non-destructively. The effect of damping is neglected in the present investigation. This is justified because of the fact that damping shifts the resonant frequencies very little in the frequency axis while measuring the frequency response functions in modal domain. However, this is to be remembered that if the actual response magnitude is to be predicted, estimation of damping must be done as accurately as possible. Since damping depends upon material behaviour, looseness at supports, or on environmental effects such as existence of surrounding fluid etc., a global average damping model is used at system level with the aid of proper measurement. Velocity proportional equivalent viscous damping model is generally agreed upon to represent appropriate damping behaviour of structures. Since the present investigation is totally performed in frequency domain after doing the Fourier transform of the time responses, the issue of appropriate modelling of damping does not affect the results significantly. For this reason, most of the current literature related to estimation of material parameters of layered composites apply the finite element model updating consid-
ering the modes to be undamped, although in actual the FRP types of structures have considerable damping. The modal testing has been performed both in free-free and simply supported boundary conditions to extract the free vibration behaviour in undamped modes here. However, it is suggested that proper damping model be included in future similar studies where reproduction of actual response prediction is necessary. Accurate prediction of actual response is absolute necessity for structural control and serviceability assessment related problems. This is also mandatory where health monitoring or condition assessment studies are conducted.

### 2. FABRICATION OF SAMPLE FRP BEAM

In this present investigation, hand lay-up procedure was adopted as it gives the opportunity to observe the lower limit of achievement in terms of strength and stiffness of such structural component. A FRP ‘I’ shaped beam of 2 m span was fabricated. Four numbers of steel channel sections of 70 mm × 200 mm were selected to act as the rigid formwork for the casting and to give superior surface finish, thereby giving less variations of geometric parameters. This is to be remembered that in actual practice, such smoothness may not be available for formworks. Thin Teflon sheets were used to prevent the direct contact between the FRP sheets and the steel formwork and also for the easy removal of formwork at a later stage. Araldite CY230 and Hardener HY951 were mixed in 9:1 proportion by weight to produce the Epoxy matrix. E-Glass fiber woven mats were laid layer by layer with matrix in between the layers in such a manner to get an I section of 160 mm × 220 mm with flange and web thicknesses of 10 mm and 8 mm respectively. For doing so, 24 number of FRP sheets having dimensions 2000 mm × 368 mm were laid layer by layer with matrix in between the layers to build two ‘C’ shaped sections of 12 layers each with half the web thickness (4 mm). These two FRP ‘C’ sections were put back to back immediately with the same matrix in between to get an I-section with flange thickness as half the desired web thickness (4 mm). The rest of the flange thickness (6 mm) was made immediately by laying 18 layers of FRP sheets (6 mm) of dimension 2000 mm × 160 mm at the top as well as at the bottom. The whole assembly was kept in position with the help of external supports as shown in Fig. 1.

After curing at room temperature, the formwork was removed with proper care. Some minor finishing works such as cleaning, grinding and polishing was necessary to achieve smooth surfaces of the fabricated FRP beam. The finished FRP beam has been shown in Fig. 2. The final geometrical properties of the beam are given in Table 1.

<table>
<thead>
<tr>
<th>Density (kg/m³)</th>
<th>Sectional Area (A)</th>
<th>Depth of Section (D)</th>
<th>Width of Flange (W)</th>
<th>Flange Thickness (Tf)</th>
<th>Web Thickness (Tw)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1625</td>
<td>4800 mm²</td>
<td>220 mm</td>
<td>160 mm</td>
<td>10 mm</td>
<td>8 mm</td>
</tr>
</tbody>
</table>

### 3. MODAL TESTING OF FRP BEAM

Modal testing was carried out on the ‘I’ beam in both free-free and simply supported boundary conditions. It is to be mentioned that for the free-free boundary conditions, the full 2 meter length was available as effective span, whereas, in simply supported case, the effective span reduces to 1.8 meter in order to provide adequate supports at both ends. Broadband excitation was imparted to the structure with the help of a roving impact hammer (B&K 8206-002) fitted with a force transducer at its tip and the responses at a single reference point were picked up by an accelerometer (DeltaTron 4507). The time domain force and response signals were Fourier transformed and stored in a spectrum analyser (B&K 3560-C-L4). Pulse LabShop™ software was used for computing the frequency response functions (FRFs) and the software MS’ScopeVES™ was used for estimating the modal parameters, i.e. the frequencies and mode shapes using multi degrees of freedom curve fitting algorithm. These are the modal parameters from which the material elastic parameters will be updated using inverse problem.
3.1. Modal Testing under Free-Free Boundary Conditions

The FRP beam was freely suspended with a cluster of rubber bands as shown in Fig. 3 to nearly simulate the free-free boundary. The hanging points are a compromise with respect to the nodal points in most of the modes of interest-effort has been made to keep the supports as near to the node points as possible for better estimate of modal parameters. Figure 4 depicts the schematic diagram for the layout of the testing arrangement. Three experimental flexural mode shapes are shown in Fig. 5. The stiffness of the rubber bands were found to be 0.237 kN/m which is quite less than the structural stiffness of the beam which is of the order of approximately 5000 kN/m, as is later predicted from static testing of the entire beam. Again, the frequency of oscillation of this suspended system is of the order of fraction of 1 Hz, whereas the fundamental frequency of vibration starts from well above 200 Hz for the bending modes of the FRP beam. Hence, such arrangement of supports practically can replicate well the free boundary conditions. Also, it is to be mentioned that a bigger capacity hammer can increase the frequency range of interest but runs the risk of driving the structure to nonlinear responses. In the same manner, using steel tip can impart local deformation at the point of contact, although the frequency range increases. The current formulation of modal test is not suitable if nonlinearity is present.

For this particular investigation related to the test in free boundary conditions, the second mode was not measured with sufficient accuracy and therefore has not been presented here. The reason for this can be guessed if the locations of the supports are closely looked into – the support points of the second mode were not very close to the node points, thereby making the measurements somewhat noisy, as the signal to noise ratio deteriorate for the second mode while testing in free-free boundary conditions. When the excitation or the response points are very near to the node points, then also the measurement quality deteriorates. Selection of the support points were made thinking about all the modes in range. So, the excitation points were tried at various locations. However, in this case changing the measurement point could not improve the results much. The other modes were measured well and presented. In case all modes are to be measured accurately, then separate hanging points could be used for each mode while testing, avoiding the node points as excitation or response point for that mode. However the arrangement of the present investigation was restricted to a single modal test. It is suggested that for fruitful application of finite element model updating exercise, the modal data be extracted for each individual mode separately, either by changing the hanging position for supports or by deploying other techniques of excitation, such as a shaker in direct contact mode. However, the modal testing time will increase considerably in such cases.

3.2. Modal Testing under Simply Supported Boundary Conditions

The FRP beam was simply supported at two ends keeping a clear span of 1.8 m with special knife-edge fixtures to allow only rotation at supports Fig. 6. The first four experimental modes are shown in Fig. 7.

Figure 3. FRP beam suspended with rubber bands to simulate free-free boundary.

Figure 4. Schematic diagram of the modal testing arrangement.
Figure 5. Observed modes for the free-free beam.

Figure 6. Simply-supported FRP beam.

Figure 7. Observed modes for the simply supported beam.

Figure 8. Numerical mode shapes in simply supported boundary conditions.

Table 2. Comparison of frequencies of the FRP ‘I’ beam.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Free-Free boundary conditions</th>
<th>Simply supported boundary conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental (Hz)</td>
<td>Numerical (Hz)</td>
</tr>
<tr>
<td>First</td>
<td>219.9</td>
<td>246.93</td>
</tr>
<tr>
<td>Second</td>
<td>-</td>
<td>488.79</td>
</tr>
<tr>
<td>Third</td>
<td>574.0</td>
<td>671.09</td>
</tr>
<tr>
<td>Fourth</td>
<td>737.0</td>
<td>846.95</td>
</tr>
</tbody>
</table>

updated here from careful review of all geometric and material parameters, boundary conditions and their relative uncertainties.

5. STATIC TEST ON FRP BEAM

Once the dynamic testing was over, static test of the FRP beam was carried out to assess the load-deformation characteristics in linear range. The four point bending test was carried out using a universal testing machine (UTM) to impart a constant bending moment within most of its span in the middle (Fig. 9). The clear span of the beam was 1.6 m. Three dial gauges were put at one third, middle and two third positions to observe the deflection characteristics. The strain gauges were put along the depth of the beam at the mid span to ascertain the assumption of linear strain variation, i.e. plane sections remain plane after bending remains valid during testing. The load deflection curves are shown in Fig. 10.

Although it was not the primary objective, the beam was loaded further until failure and an ultimate load of 52 kN with a deflection of 14.7 mm at the centre before failure was recorded. The moment carrying capacity was estimated to be 13.87 kN·m. The average equivalent modulus of elasticity of the material was found out to be approximately 15.8 GPa. Dur-
6. UPDATING OF FINITE ELEMENT MODEL TO ESTIMATE MATERIAL PARAMETERS

6.1. Mathematical Formulation and Software Implementation of Updating Process

The modes computed from finite element analysis were paired with corresponding experimentally observed modes using Modal Assurance Criteria (MAC). Here, the main source of error is believed to be from the uncertainties in elastic material properties and they are selected as the parameters to be updated.

A sensitivity based inverse approach has been implemented through the commercially available model updating software FEMtools for the parameter estimation. The linearized first order approximation of the relationship between measurable (output) eigenvalues and eigenvectors and (input) average material parameters, can be represented through a Taylor series approximation:27

\[
\{f_i(r)\} \approx \{f_i(\hat{r})\} + \left[ \frac{\partial f_i(r)}{\partial r_i} \right]_{r=\hat{r}_i} \Delta r_i \quad (1)
\]

where, \(r_i\) represent the parameters to be identified (i.e. the in-plane elastic parameters), \(\hat{r}_i\) are the a-priori estimate of \(r_i\), \(f_i(\hat{r})\) is measured eigenvalues and eigenvectors of the 'I' beam, \(f_i(\hat{r})\) is modal properties of initial finite element model of the beam, and \(\left[ \frac{\partial f_i(r)}{\partial r_i} \right]_{r=\hat{r}_i}\) is first order sensitivity matrix (Jacobian matrix), of eigenvalues and eigenvectors of the beam with respect to the material parameters to be estimated.

The residual error vector at any iteration can be expressed as:

\[
\{ \Delta \} = \{ \Delta f \} - \{ S \} \{ \Delta r \} \quad (3)
\]

The IEM tries to minimize the above error function in least square sense, thereby implementing \(\{ \Delta r \}\) change to the parameters \(r_i\).

A new \(\{ r \}\) is generated at every iteration using the equation:

\[
\{ r \}_{i+1} = \{ r \}_i + \{ \Delta r \}_i \quad (4)
\]

The procedure is repeated with updated in-plane material parameter values, until the square of the error between the numerical and experimental modes falls within a predetermined margin of error: \(\varepsilon\).

\[
\{ \Delta \}^T \{ \Delta \} \leq \varepsilon. \quad (5)
\]

IEM requires that initial guesses are made for the parameters to start the proposed iterative algorithm. This is done by generating uniformly distributed random values of the in-plane elastic parameters within selected realistic upper and lower bounds. Material properties were estimated through a sensitivity based algorithm implemented by the commercially available software FEMtools using the finite element program ABAQUS as the external solver – ABAQUS is called by the FEMtools whenever there is an eigensolution needed. The flowchart of the software implementation has been shown in Fig. 11.

6.2. Implementation of Updating Algorithm to Estimate In-plane Material Parameters

The FRP laminated composite used in the present investigation has almost negligible difference in elastic properties in
longitudinal and transverse directions, thus the elastic material property parameters to be updated can be assumed to be same for the ‘I’ beam in longitudinal and transverse directions. The in-plane Poisson’s ratio can only be updated successfully from measured modal properties, if sufficient number of torsional modes can be excited during modal testing. However, in the present investigation only first few flexural modes were measured. Thus the value of Poisson’s ratio was taken nominally to be 0.26 for the numerical simulations and later almost the same value was found from the characterisation tests.

The parameters were estimated iteratively and the convergence plots of the material parameters from selected initial values with wide ranges of variations are presented in Figs. 12 and 13 for the free-free boundary conditions.

The convergences of $E_1$ as well as $E_2$ together were to a value of 16.97 GPa, with the shear modulus $G_{12}$ converging to 3.06 GPa for free-free boundary conditions. It is to be mentioned that the algorithm is framed to solve the inverse problem in an over-determined manner. Thus the number of frequencies and mode shape coordinates need to be sufficiently high with respect to the parameters to be estimated for unique solution in a least square sense. That’s the reason other higher modes were found to be acceptably accurate enough to participate in model updating, even when the second mode could not be measured with sufficient accuracy. Use of only the fundamental (first) mode runs the risk that contribution of all higher modes would not reflect in the parameter estimation. The second mode itself could have been measured with greater accuracy by shifting of the suspension cord a bit and accepting somewhat more error into the other modes. The updating exercise with simply supported boundary conditions showed a convergence of $E_1$ and $E_2$ to approximately 17.95 GPa and the shear modulus to approximately 3.0 GPa. The review of the modal testing procedure in both free-free and simply supported conditions suggests putting more confidence on the results obtained from the free-free condition. It appears that the simply supported boundary with knife edge support has provided at least some resistances to rotation, thereby making the prediction of Young’s modulus slightly higher than that obtained in free boundary conditions. It can only be verified by conducting characterisation tests for the above material constants. However, in all cases the convergences are found to be monotonic and unique from a varied set of initial starting values of parameters.

7. QUASI-STATIC CHARACTERIZATION TESTS TO CONFIRM MATERIAL PROPERTIES

Specimens have been prepared for the quasi-static characterisation tests by cutting the actual beam and the tests were conducted as per ASTM D3039/D3039M using a Universal Testing Machine (Tinius-Olsen, Super ‘L’ series UTM) to determine the in-plane elastic properties ($E_1$, $E_2$, $G_{12}$ and $\nu_{12}$) of the FRP laminate. The practical available lengths of samples in this existing structural component were slightly smaller than that specified in the above standard. Samples have been drawn from different locations of the FRP beam as shown in Fig. 14. Coupons at 45° orientations to the principal material axis were cut from the web only (as there were insufficient length available in flanges) and tested to determine the shear modulus and the formula used was taken from literature and is reproduced here:

$$G_{12} = \frac{1}{4 \frac{E_{45}}{E_1}} \left( 2 \frac{1 - \nu_{12}}{E_1} \right).$$

The Poisson’s ratio was determined by measuring the strains in both longitudinal and transverse direction with the help of two extensometers. The test results are given in Table 3.

<table>
<thead>
<tr>
<th>Sample Designation</th>
<th>Tested for</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Young’s modulus (E1)</td>
<td>15.5 GPa</td>
</tr>
<tr>
<td>S2</td>
<td>Young’s modulus (E1)</td>
<td>15.77 GPa</td>
</tr>
<tr>
<td>S3</td>
<td>Young’s modulus (E1)</td>
<td>16.92 GPa</td>
</tr>
<tr>
<td>S4</td>
<td>Young’s modulus (E1)</td>
<td>16.74 GPa</td>
</tr>
<tr>
<td>S5</td>
<td>Young’s modulus (E1)</td>
<td>15.93 GPa</td>
</tr>
<tr>
<td>S6</td>
<td>Poisson’s ratio</td>
<td>0.29</td>
</tr>
<tr>
<td>S7</td>
<td>Young’s modulus in 45°</td>
<td>9.87 GPa</td>
</tr>
<tr>
<td>S8</td>
<td>Young’s modulus in 45°</td>
<td>10.2 GPa</td>
</tr>
</tbody>
</table>

Table 3. Material parameters from characterisation tests.
An attempt has been made to produce a more accurate updated model of a FRP I-beam from measured dynamic responses by estimating the most uncertain in-plane material properties in free-free and simply supported boundary conditions. For this, a FRP beam was fabricated using the hand lay-up process which is very simple to replicate and is also very cost effective as compared to any mechanised process. The methodology of model updating is observed to work acceptably in the environment of modal and spatial sparsity, as selective modal coordinates and only a few selected frequencies were used for updating. The demand on number of modes to be measured can be drastically reduced provided quality mode shape data can be acquired. It was found from model updating and characterisation tests that if a set of global material parameters is to be agreed upon, the material constants obtained from model updating in free-free condition can be most appropriate. This is difficult to determine from characterisation tests, where substantial local variations in material and geometric properties exist. Once an accurate updated model of such FRP structure is made, this can be used for keeping track on its performance due to operational and live loads and can also be used for regular condition assessment and health monitoring. These goals are impossible to achieve with approximate models having uncertainties in material or other parameters.

9. CONCLUSIONS

REFERENCES


33 ME’scopeVES v4.0.0.96, Vibrant Technology Inc., (2007).

34 ABAQUS v6.9, Dassault Systmes Simulia Corp, (2009).


1. INTRODUCTION

Social and economic growth of a city, region or country is inevitably underpinned by rail mass transit. To stimulate sustainable productivity, an urban rail infrastructure is often built either underground or on the surface, surrounded by agglomerate buildings and public communities. Its network generally spans over a large distance. Ongoing operation and maintenance of such rail infrastructure systems are critical to public safety, in addition to the routine management of business risks and reliability. With a significant demand from the public to cater faster and more frequent train services (evidenced by the 24/7 rail services such as in London, Berlin, Sao Paolo, and many more to come), structural health monitoring of such rail infrastructure systems is one of the grand challenges in practice. Especially in underground or subway systems, track engineers commonly envisage the facts that the tunnel facilities, resources, and time windows permitted for railway staff to carry out the critical infrastructure inspection and maintenance is, indeed, extremely limited. On the other hand, any tiny period of train-free duration (e.g. 3 hours in Tokyo; 1–2 hours in Hong Kong; or down to 5 minutes in London) in a late night rather discourages on-track activities by the railway staff. This is due to the difficulties and the deficiency of time and access to facilities to carry out any meaningful inspection and maintenance tasks. As a result, the application and utilisation of an inspection train vehicle has been more demanding than ever.11,12,17

Rail corrugation is an irregularity on rail running surface, inducing large dynamic loads and vibrations onto adjacent railway track components as well as rolling stocks. Such a defect is a periodic, undulated or wave-like vertical alignment of the rail surface. The rail corrugations are typically caused by uneven wears, due to the variations of wheel-rail contact stresses. The wavelength and severity of corrugations is dependent on track structure, track geometry, traction system, rail vehicle behaviours, and wheel-rail interaction. The rail corrugations are the source of rapid track degradation, poor ride comfort, excessive vibration, and noticeable nuisance noise. Often such irregularities can be observed on the low rail of small-radius curves. A large number of research studies have been devoted into the fundamental causes and mitigation techniques.6,21,29,30 The effect of rail corrugation wavelengths on noise generations has been the main focus in most studies. It is noted that \( v = f \lambda \) (\( v \) is the train speed, \( f \) is the frequency, and \( \lambda \) is the wavelength). “Contact patch filter” has been found to be a mechanism that attenuates very high frequency effects above 2 kHz. Table 1 shows the frequency ranges associated with railway noises due to rail corrugations. To meet appropriate requirements of rail authority, maintenance and control of rail roughness at various wavelengths are generally carried out by rail grinding and lubrication methods, depending on various factors such as geometry, location, gradient, environments, etc.5,7,14,29 In Australia, an urban rail network suffers from rail corrugations on the low rail in curves. Such defects often regenerate secondary ground-borne vibrations in surrounding environments, such as nearby buildings and structures. In addition to noise issues, the defects incur costly track and train maintenance because the excessive vibrations from wheel/rail interaction undermine structural integrity of those railway assets.21,22 It is important to note that the rail corrugations could also be a source causing other types of rail defects, e.g. rail squats, shelling, etc.3,15,18

On this ground, monitoring and control of rail corrugations is mandatorily required in practice.19,31,32 It is, therefore, very important to develop an alternative monitoring methodology that could be practically applied to special railway tracks with restrictions of safe access and facilities.

The inspection or patrol train vehicle is generally equipped with sensors (i.e. accelerometers, gyroscope, ground penetrating radar, laser profiling, etc.) and high-speed cameras to...
supplement track inspection activities. This paper presents the utilisation of an inspection train vehicle to evaluate and monitor rail corrugation growth on curved tracks. It aims to provide an alternative on-board monitoring approach for helping track engineers in maintenance activity prioritisation. It highlights the integration of numerical train/track simulations, axle box acceleration data obtained from the calibrated track inspection vehicle “AK Car”, and data analytics in order to assess and monitor rail corrugation growth on curves. A case study is presented to demonstrate practical evidence and examine the technique to on-board monitor the rail corrugation defects on a sharp curve, where short-pitch rail corrugation on the low rail prevails.\textsuperscript{4, 8, 25, 26} The emphasis of the case study is placed on an Australian underground rail track system, where it is extremely difficult to reach for routine inspections by detailed walking method or by using any mobile handheld equipment (such as CAT trolley, laser board for roughness measurement, or roller wheel for surface profiling, etc.).

2. TRACK INSPECTION VEHICLE

Tracking recording data is the track geometry data obtained from an inspection vehicle. The accuracy, repeatability, and quality of the data depend largely on measurement method, sensor and instrumentation, train speed, and location identification.\textsuperscript{20} Figure (1) shows the AK Car and its analytical modelling. In this case study, the track inspection vehicle has been installed with axle-box accelerometers. The inertia data is then computed to provide track maintenance engineers with geometry data. Figure (2) shows the flowchart to derive geometrical data.\textsuperscript{27} In practice, the wheel has been retrofired frequently to remove any potential of irregularity over time. Wheel roughness has been monitored from vibration records over long distance and an adaptive frequency band filtering approach (similarly to adaptive noise cancellation method) has been applied to minimize any effect from wheel roughness and any irregularity. Inertia geometry calculation method, using the trapezoidal method, has been adopted and the method has been validated by field measurements and systems tests.\textsuperscript{28} The geometry data, which had been recorded using ”AK Car” Geometry Recording Vehicle, illustrates fundamental dynamic track parameters (top, line, gauge, cross level and twist) in each stage of rail track’s life cycle.

2.1. Gauge

Track gauge is defined as the distance between the gauge points on the face of each rail. The default gauge point is 16 mm from the azimuth (maximum $y$ point) on the rail surface. AK Car recognizes the known distance between laser cameras instrumented on its bogie, so that the resulting gauge measurement is the difference between the optimal gauge and the measured gauge.\textsuperscript{20} The standard rail gauge for the track in this study is 1 435 mm with 600 mm sleeper spacing.

2.2. Superelevation

Superelevation (or so-called crosslevel) is defined as the height of one rail above the other. The superelevation is calculated from all four of the inertial measurement package components installed on the AK Car body, consisting of two single-axis fibre optic rate gyroscopes (measuring roll and yaw in terms of turning angles); two accelerometers (measuring vertical and lateral accelerations); and a signal conditioning board.\textsuperscript{20}

2.3. Top

Track surface is defined as the evenness or uniformity of track in short distances measured along the top of the rails. Under load of the AK Car body, top surface of rails and vertical alignments can be measured by either a mid-chord offset or by a space curve method. In Australia, the AK Car uses the former method (calculating a versine at 1.8 m) by adopting

<table>
<thead>
<tr>
<th>Railway noise</th>
<th>Frequency ranges</th>
<th>Wavelength (at 40 km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Audible ground-bourne</td>
<td>25–250 Hz</td>
<td>40–400 mm</td>
</tr>
<tr>
<td>Structure-bourne</td>
<td>100–2000 Hz</td>
<td>5–100 mm</td>
</tr>
<tr>
<td>Wheel-rail rolling</td>
<td>100–5000 Hz</td>
<td>2–100 mm</td>
</tr>
<tr>
<td>Range of greatest human sensitivity</td>
<td>2–5 kHz</td>
<td>1–5 mm</td>
</tr>
</tbody>
</table>
Double Integration:
of interest are shown in Fig. (3). From Fig. (3), the section
measurement using laser board scanner or even CAT trolley) will
implies that any meaningful inspection (e.g. roughness mea-
( no train service) is about 3 hours (from 1 a.m. to 4 a.m.). This
anance. At the location of interest, the headway (the time gap
it is extremely difficult to access for inspection and mainte-
tracks, especially within the underground railway network. A
3. CASE STUDY

tainment will be calculated from the uniform curve trajectory or
data” represents the deviation from zero. In a curve, the devi-
on a tangent or straight track so that the AK Car “measured line
Cal variation in curvature of each rail of track. It should be zero
2.4. Line

Line or horizontal alignment smoothness is defined as the lo-
cal variation in curvature of each rail of track. It should be zero
on a tangent or straight track so that the AK Car “measured line
data” represents the deviation from zero. In a curve, the devi-
ation will be calculated from the uniform curve trajectory or
design alignment over a specified distance. In Australia, the
10 m chord has been adopted for the purpose of benchmarking.

2.5. Axle Box Acceleration

Corrugation roughness data can also be obtained by double
integration of axle box accelerations (ABA) with respect to
train/track receptance and signal filtering, similar to Top data.
Calibration and repeatability tests were previously carried out
to meet rail authority requirements.20, 23, 24 The wheelset recep-
tances have been used for the calibration of multi-body dynam-
ics. The ABA-based roughness has been validated by and in
very good agreement with field roughness measurements using
a laser board scanner.25 In this study, the roughness data rep-
resents the combined vertical rail surface quality for all wave-
lengths.

3. CASE STUDY

Rail corrugations have been a problem, especially in curved
tracks, especially within the underground railway network. A
pilot study was carried out on an urban rail network, where
it is extremely difficult to access for inspection and mainte-
nance. At the location of interest, the headway (the time gap
between trains) is 2.5 minutes and the daily track possession
(no train service) is about 3 hours (from 1 a.m. to 4 a.m.). This
implies that any meaningful inspection (e.g. roughness mea-
surement using laser board scanner or even CAT trolley) will
incur penalty (additional track downtime due to the compul-
sory requirement for extra time, resources and safety-related
facilities). The geometry and speed of the uni-directional track
of interest are shown in Fig. (3). From Fig. (3), the section
from km 0.400 to km 0.800 tends to undergo “unbalanced”
speed that usually causes “short- pitch” low rail corrugation,
compared with the adjacent section (km 0.750 to km 1.055).
At unbalanced speed, the train travels at the speed that induces
centripetal resultant force. Subsequently, the wheels acting on
the lower rail will carry more weight or load burden than those
on the outer rail. With additional dynamic and curving effects
imposed by wheel-rail interaction, the rail will suffer from ex-
cessive cyclic wheel loads and wear out. At this location, the
track structure consists of 60 kg rail, fastening system, and tim-
ber sleepers, embedded into concrete slab, as shown in Fig. (4).
Asset inspections for this track section are often carried out in
practice by using a combination of methods such as Engine
Ride Inspection, train driver report, as well as a Track Inspec-
tion Vehicle, i.e. “AK Car”, as shown in Fig. (1). Most rail cor-
rugations have been often detected firstly by Engine Ride In-
spection when experienced track engineers or staff can observe
and identify mild corrugations through visualisation and train
motions. Then, they could prioritise work, based on criticality
and severity of defects, to suit their Base Operating Conditions.
However, high turnaround of staff could lead to inconsistency
in track maintenance prioritisation. It is, thus, important to
adopt AK Car data for helping such tasks.

To evaluate and monitor the rail corrugation growth on
curves for this case study, the axle box acceleration and corru-
gation data were collected from AK Car archives for the runs
between July 2010 and August 2013. It is noted that the data
sampling rate is 10 kHz, resulting in good quality data for fre-
quency analysis up to 5 kHz, which is well above the wave-
lengths of interest (see Table 1). Figure (5) shows the raw data
derived from the corrugation data archive. These data represent
‘combined’ track and rail roughness measured based on AK
Car parameters. Note that these data have already been filtered
out to minimize wheel noise in high frequency range.26 Also,
using the smaller cord length filtering (e.g. 500 mm cord), the
track deflection effect is minimal and negligible.

Based on Fig. (5), although it is difficult to measure track
surface quality due to the transient nature of vibrations, it could
be observed that low rail (or Top Left/TopL) tends to be much
more cyclical than high rail (TopR) data. The “RMS ampi-
ditude” can generally inform the growth of roughness of track.
The higher the roughness; then, rail running surface control is
required (i.e. rail grinding, top of rail lubrication using friction
modifier). In this case, noticeably, the RMS amplitude has de-
creased over the time as the rail is worn out by running wheels.
But, the real question is that can rail or track engineers observe
if there is a clearer growth of rail corrugation? It is unclear at
this initial stage if any corrugation could be detected merely
from RMS amplitudes, since the overall RMS amplitudes de-
crease. For most cases, the data analyses of time-domain RMS
amplitude can be time-consuming and such ambiguous data
could lead to uncertainties. To evaluate rail corrugation growth
from these data, further analyses are thus required. Wavelength
analysis is necessary to ensure that correct rail surface irre-
gularity is emphasised. The rail corrugation defect on sharp
curves tends to be frequently associated with the wavelength
bands from 30 mm to 100 mm. In general, the rail roughness
(obtained from CAT measurement tool) can be plotted against
the logarithm of wavelength (mm). Equation (1) shows the
calculation method for decibel RMS of roughness amplitude
(R). The decibel RMS of roughness amplitude is based on the
measured rail roughness ($r$):

$$R = 20 \log_{10} \left( \frac{r}{r_0} \right) ;$$

(1)

where the reference $r_0 = 1$ micron ($= 0.001$ mm).

By the similar manner, the rail roughness obtained from AK Car corrugation data ($r$) can also be evaluated against logarithm of wavelength (mm).

Figures (6) and (7) show the roughness level using Top 500 and Top 1000 (Top 500: a filtered displacement between 0.25 m and 0.5 m; and, Top 1000: a filtered displacement between 0.5 and 1.0 m). These parameters have been chosen because they well correspond to sharp curve corrugations in this study (as also identified by Table 1). It is clear that rail corrugation exists at the wavelength band between 30 mm to 100 mm (0.03 m to 0.1 m). To monitor the rail corrugation growth, the data is re-plotted to evaluate the movement of roughness over time with respect to ISO3095 guidelines. Note that this guideline is generally advisory, and it does not imply obligation for track maintenance. Figures (8) and (9) clearly show that rail corrugation growth on low rail is evident over time. Rail or track engineers could also notice that high-rail wavelike irregularity also grows with a slower rate compared to that of the low rail. On this basis, it is evident that it is feasible to determine the growth rate of rail corrugation over a timeframe or per million gross tonnages (MGTs) of revenue services. It is, thus, recommended for maintenance practice that filtered RMS data be used for rail corrugation defect evaluation and monitoring. However, it is noted that the track maintenance limits based on RMS data depend largely on operational requirements.

For example, for asset damage control, track engineers may consider peak to peak roughness as: mild corrugation $>300$ microns; moderate corrugation $>500$ microns; and significant corrugation $>2000$ microns. For a noise control, track engineers may consider that Top 500 RMS should be less than 50 microns or 5 mm/100; or Top 1000 RMS may be limited by 100 microns or 10 mm/100. In principle, if a track section has the Top RMS exceeding a bespoke limit, rail engineers or staff should closely monitor and inspect for any structural damage of assets or for excessive nuisance noise. Based on this pilot study, rail engineers can make use of this data to efficiently plan for rail grinding or asset control strategy.
Figure 5. Corrugation growth from km0.400 to km0.800 (Top 500).
Figure 6. Roughness level using Top500 data (TopL — low rail; TopR — high rail).

Figure 7. Roughness level using Top500 data (TopL — low rail; TopR — high rail).
Figure 8. Roughness growth of low rail.

Figure 9. Roughness growth of high rail.
tion with risk management, track engineers can prioritise track maintenance, in order to:

- Reduce noise and vibration spectra in urban area (wave-lengths associated with natural frequencies of ground and built environments)
- Reduce potential damage to train and track components with respect to large corrugation amplitude
- Improve ride comfort to passengers by smooth rail grinding (rail surface metal removal), and
- Predict track maintenance interval for specific locations, of which track and operational parameters induce severe corrugations.

4. CONCLUSIONS

Significant demand for rail asset management with physical constraints has resulted in an application of vehicle-track interaction for assessment and monitoring of rail assets. In practice, rail corrugations are firstly detected by train drivers and track inspectors. The locations of mild to severe corrugations are often known in advance by infrastructure managers. However, monitoring the rail surface defects to prioritise maintenance frequency and tasks are often inadequate because detailed inspections (using CAT or track measurement trolley) require track closure and safety management, which are time and resources-consuming. This paper demonstrates an application of vibration-based data obtained from an inspection vehicle to assist track maintenance engineers predicting severity overtime and controlling rail corrugation growth on sharp curves. A case study using the inspection vehicle data on a selected track section with a very sharp curve radius was carried out as a demonstration.

This case study demonstrates that track engineers can use the inspection vehicle to monitor rail corrugation growth. Top 500 is found to be a stable data set, which could be used for corrugation monitoring. The efficiency of track maintenance can be improved by using such application integrated with risk management framework. The improvement will also enhance ride comfort and quality of lives for railway neighbours. The data analytics also suggest area of improvement. Higher quality and sampling rate of data collection are essential to the improvement of accuracy of rail roughness back-calculation at a micro-meter level. Automated inspection system, condition monitoring, and trend analysis are among the demanding and important future research topics related to railway structural monitoring and maintenance.

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Design Considerations and Establishment of a Low Frequency Hydrophone Calibration Setup using the Principle of Vibrating Water Column

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This paper describes the design and implementation of a low frequency hydrophone calibration system, using a vibrating water column. The hydrophone to be calibrated is immersed in the water column and the position of the transducer is kept constant while a hydrodynamic pressure field is generated in the water column by means of a shaker (similar to what is described in IEC 60565). F. Schloss et al. used a vibrating water column for hydrophone calibration in the frequency range from 10 Hz to 700 Hz. An interlaboratory comparison calibration was carried out by the Russian Metrological Institute of Technical Physics and Radio Engineering (VNIIFTRI) and Hangzhou Applied Acoustics Research Institute (HAARI) in the frequency range from 250 Hz to 1 kHz. The dimensions of the test vessel are important for deciding the frequency range of operation. Simulations were carried out for the selection of vessel material and dimension. To overcome limitations in the frequency range caused by resonance in the water column, the principle of operation was modified from absolute calibration to calibration by comparison. By using a single cylindrical vessel, the frequency range is extended to cover frequencies from less than 100 Hz to 1 kHz. The calibrated reference hydrophone type Bruel & Kjaer (B&K) 8104 is used in this calibration. Some results obtained from the use of the calibration system are also presented in this paper.

1. INTRODUCTION

There is an increasing requirement to perform measurements of sound in the ocean for both environmental monitoring and assessment. Most of the sound sources of environmental concern, including marine species, emit much of their energy in the 20 Hz to 1 kHz frequency range. Traceable calibrations of the hydrophones used are required to support these measurements. However, free field calibration of hydrophones in laboratory test tanks is not possible at such frequencies (due to limited tank dimensions), necessitating the use of calibration methods such as the one described herein. This calibration is fundamental for accurate measurements and to estimate the characteristics of hydrophones. The most significant parameter in specifying hydrophone performance is its free field sensitivity ($M_H$) expressed as a function of frequency. $M_H$ can be expressed as a quotient of the voltage and sound pressure:

$$M_H = \frac{v}{p}.$$  (1)

where $v$ is the electrical output voltage from the hydrophone and $p$ is the acoustic pressure acting on the hydrophone.

As the calibration frequency is decreased, the number of cycles in the usable time window is reduced. This effect defines a lower limiting frequency for measurements made using tone burst signals.

As an alternative to free field calibration in an acoustic water tank, a number of different methods for low frequency calibration of hydrophones exist. These include calibration by piezoelectric compensation, acoustic coupler reciprocity calibration and calibration with a pistphone. In the piezoelectric compensation method, the hydrophone is calibrated in an enclosed small water-filled chamber. This method, which works only for frequencies with a wavelength sufficiently larger than the largest dimension of the chamber, is useful for calibration in the 1 Hz to 5 kHz frequency range. For our required frequencies, more than one chamber is required in order to cover the full frequency range. Acoustic coupler reciprocity calibration is an absolute calibration method using the reciprocity principle. Though laboratory standard hydrophones may be calibrated with the highest accuracy, the coupler is not a good option to calibrate hydrophones that are large or to calibrate hydrophones near the resonance frequency. In a pistphone, a continuous sound pressure is generated in an enclosed small air-filled chamber by a vibrating piston. This technique may not be used at frequencies greater than the upper limiting frequency because of non-uniform pressure in the chamber.

The benefit of using water versus air is that, due to different propagation velocities in water and air, the wavelength in water is much longer than that in air; therefore the useable frequency range is expanded by the relation between the two velocities. An absolute pressure calibration of the hydrophone can also be performed at low frequencies by immersing the hydrophone in...
a column of liquid and oscillating the liquid column vertically. This can result in large pressure oscillations and eliminates any possible response of the hydrophone to its own acceleration. The hydrophone sensitivity is calculated from the acoustic pressure measured at a particular depth. Various limitations on the procedure, such as those due to fluid motion around the hydrophone and wave effects in the liquid column, are discussed. Strasberg et al. explained several advantages calibrating the pressure gradient hydrophones in a cylinder of liquid, wherein they are subjected to axial vibration (as opposed to horizontal orientation of the cylinder). In the vibrating water column method, the hydrophone under calibration is immersed in a water column; the position of the transducer is kept constant while the fluid column is moved sinusoidally. Schloss et al. used the vibrating water column method for the calibration of hydrophones, wherein the cylinder is placed over the vibrating shaker and half the cylinder is filled with water. The vibrating column method has also been used for interlaboratory comparison calibrations in the frequency range 250 Hz to 1 kHz by the VNIIFTRI and HAARI, China. Here the up and down motion of the cylinder is driven by the vibrating shaker and the hydrophone element is exposed to an oscillating pressure fluctuation. Bauer et al. developed the calibrator for pressure gradient hydrophone by using a very rigid tank and testing in the frequency range 3 Hz to 2.5 kHz. Here two calibrated pressure hydrophones are lowered to the same depth as a pressure gradient hydrometer, then the system is set into axial oscillation. The difference in pressure is then measured by differential amplifiers.

All of the above low frequency calibration techniques, either in air or water, are pressure calibration methods. While free field sensitivity is the significant performance parameter for hydrophones, at such low frequencies the pressure sensitivity and free field sensitivity are equivalent. Hence the vibrating water column based hydrophone calibration setup has been taken up.

The objective of the current work was to establish a system for calibration of hydrophones in the frequency range 100 Hz to 1 kHz (calibration at higher frequencies is performed in the acoustic tank by free field calibration method). Matjaz Prek studied the sound propagation and resonance conditions for different water filled pipe materials. An experimental study of sound propagation in liquid-filled ducts has established the utility of wall materials for sound attenuation. In a vertical vibrating water column, the hydrophone is exposed to two forces. Firstly, the gravitational force due to the small displacement of the water column. Secondly it is exposed to an hydrodynamic force according to Newton’s second law $F = m \cdot a$ (where $m$ is the mass of the vibrating water column above the hydrophone and $a$ is its acceleration).

Above a certain frequency the gravitational forces can be neglected, and the acoustic pressure inside the water column can be written as:

$$ p = \rho x h \omega^2; $$

where $\rho$ is the density of the liquid, $h$ is the height of the water column above the acoustical center of the hydrophone, $\omega$ is the angular frequency, and $x$ is the peak amplitude of vibration.

The sensitivity of the hydrophone can be obtained from the calculated pressure at the depth of the hydrophone and the measured open circuit voltage. The sensitivity of the hydrophone ($M$) is found as the ratio of the open circuit voltage ($V_o$) and the pressure ($p$). The absolute calibration suffers from limitations due to the resonant effects of the setup. To overcome the influence of the resonant phenomena, a reference hydrophone is introduced to measure the actual acoustic pressure.

## 2. METHODOLOGY

The vibrating water column method requires a test vessel (often cylindrical), hydrophone fixtures, a vibration shaker, and instrumentation for signal generation and signal acquisition. It is very important that the design of the hydrophone fixtures is such that no external vibrations are picked up by the hydrophone. The diameter of the test vessel must be large and sufficient water must be around the hydrophone in order to avoid the influence of the boundary effects. To choose the optimum test vessel, a simulation was carried out with different materials and wall thicknesses.

### 2.1. Modelling and Simulation

The acoustic pressure of the water field and the vibration of the structure around the water field are modelled and simulated using the software COMSOL Multiphysics. The pressure acoustics tool available in COMSOL Multiphysics is used to model the isolated pressure within the water. Initially, calibration using a water column as given in IEC 60565 was simulated. Then the behaviour of the pressure field in the water for different dimensions and structure materials were modeled and simulated. Finally the responses of the hydrophones with different dimensions and different water column levels were studied.

The acoustical impedance of water is high; hence the shell around the water column must be of such a nature that it can be considered rigid. If the shell is not sufficiently rigid it will influence the water pressure and vice versa. Several materials such as Aluminum, Steel and PVC (all with different thickness) have been considered for the calibration vessel. It was noticed that the frequency response of a PVC vessel has several peaks, which makes it unsuitable for calibration purposes. The resonance for all other materials is above 1 kHz. Based on the simulation, it was found that an aluminium cylinder with 20 mm wall thickness, because of its reduced weight when compared with a steel cylinder with the same dimensions, would be the most feasible choice for the calibration setup. The dimensions of the vessel were chosen to be small enough to keep the pressure field uniform throughout the vessel. During the study, different dimensions of the hydrophones were simulated. The simulations showed that the resonances can be shifted up in the frequency if the length of the water column is shorter. The shortest water column length is considered the most desirable in order to shift resonant frequency as high as possible. It is however expected that the hydrophone is positioned at the center of the water column in order to avoid the influence of boundary effects. As described in IEC 60565, the vibrating water column method is an absolute calibration with a very limited frequency range. The comparison calibration principle used here allows an extension of the frequency range up to 1 kHz or more.

## 3. DESIGN CONSIDERATIONS

The establishment of the setup, including mechanical structures and instrumentation, was carried out by Brel & Kjr (B&K), Denmark. The main elements of the calibration system...
include a cylindrical test vessel, vibration shaker, signal generator, power amplifier, data-acquisition system, accelerometer, and hydrophone fixture arrangement. The developed system is intended for the calibration of hydrophones in the frequency range of 100 Hz to 1 kHz utilizing the vibrating water column method. This method uses an open column of liquid sized such that the wavelength is larger than the length of the column. The cylindrical vessel with rigid walls that contains the liquid column is driven at the bottom by the shaker arrangement. Some of the challenges overcome by this design were: the selection of the vibration shaker (including static deflection of the shaker head due to the static load of the test vessel with water), the hydrodynamic force rating, standing waves of the water column (including mode shapes), and finally the selection of the vessel material with the required dimensions. The upper frequency of the calibration range depends on the dimensions of the device under test and the total height of the water column. The upper limit of the useful frequency range is smaller than a quarter of the wavelength of the sound in the liquid. The test vessel is an aluminium tube with 0.3 m diameter and 0.02 m wall thickness. If the payload is being carried by the shaker itself, the static payload from the water and vessel will result in a large deflection of the shaker head. To avoid this static deflection, the walls of the test vessel are made to rest on the foundation via a trunnion arrangement and, further, the static load of the water column is carried by the pressurized air. 

Only the bottom of the test vessel is driven by the shaker arrangement which oscillates the entire column of liquid vertically. An arrangement of O-rings provides the sealing between the walls of the test vessel and the vibrating bottom. A triaxial piezoelectric accelerometer is used for the measurement of the axial and transverse vibrations of the shaker with respect to the pressure created in the water column.

4. MEASUREMENT METHOD

The implemented system uses calibration by comparison and is a modification of an absolute method described in IEC 60565. A reference hydrophone (H) and the unit under test (T) are placed in a hydrodynamic pressure field (water), with the acoustical centers at the midpoint of the water column with the accuracy of ±2 mm. The sensitivity of the hydrophone under test is determined from the sensitivity of a known reference standard. The excitation signal is a sine/random signal with bandwidth limitation. The auto-spectra for the frequency response function (FRF) is measured by means of 1/24 octave CPB filters in this method.

The voltage sensitivity of unknown hydrophone SVT can be written as:

\[ SV_T = SV_H + [U_T/U_H] \]  \quad (3)

where:

- \( SV_H \) – Voltage sensitivity of reference hydrophone;
- \( U_T \) – Output voltage of hydrophone T;
- \( U_H \) – Output voltage of hydrophone H;
- \( M_H \) – Sensitivity level of reference transducer (dB re V/μPa);
- \( M_T \) – Sensitivity level of hydrophone T (dB re V/μPa).

The values of \( M_H \) for the reference hydrophone type, B&K 8104, are taken from the National Physical Laboratory (NPL), UK calibration certificate, and at 1/24 octave frequencies considered by interpolation where needed.

The response as calculated above needs to be corrected for (1) losses due to the capacitive load of the reference and device under test (2) filter and gain tests in the preamplifier, and (3) any narrow frequency range resonance phenomena in the vessel. All these factors are considered by a number of corrections as shown in the complete formula below.

\[ M_T = M_H + H_1 G_N + C_L + C_F + C_V \]  \quad (5)

where:

- \( M_T \) – Receiving sensitivity of the hydrophone under test as a function of frequency;
- \( M_H \) – Reference transducer sensitivity as a function of frequency;
- \( H_1 \) – Transfer function between under test hydrophone and the reference hydrophone \((U_T/U_H)\);
- \( G_N \) – Correction factor for preamplifier gain-if used;
- \( C_C \) – Correction for the capacitive load effect;
- \( C_L \) – Load error, caused by the resistive load for low frequencies;
- \( C_F \) – Correction for the filter response (wherever relevant);
- \( C_V \) – Vessel correction, a small correction for resonances in the test vessel. This is small correction for standing waves and it is frequency and hydrophone type dependent.

Each calibration may have its own individual set of corrections. In general, corrections due to the gain of the preamplifier \((G_N \text{ in dB})\) are the same for all the frequencies of operation. However, for better accuracy one can calibrate the amplifier to characterize the gain at each frequency point. The hydrophone sensitivity and frequency response must be stated as open circuit values at the end of the hydrophone cable (i.e., the value measured without any load). Hence, whenever the hydrophone is connected with the rest of the circuit, which is attempting to measure the output voltage, the capacitance due to the hydrophone and the extension cable must be corrected for. This correction is called \( C_L \) and its value depends on the capacitance of the load and the input capacitance of the measuring channel.

If both the transducers are considered identical, or nearly identical, the correction will be zero or close to zero. For hydrophones with a relatively low capacitance \((C < 4 \text{ nF})\), the effect of \( C_L \) should be corrected for. However, since both the unknown and the reference have the same loading effect, the transfer function (i.e., the ratio) is less influenced than a single channel. In that case the vessel dimension and other circumstances cause small glitches in the response. A correction \((C_V)\) can be used to correct these phenomena.


5. RESULTS AND DISCUSSION

COMSOL simulation demonstrated that the vibrating water column principle described in IEC 60565 can be extended to frequencies above $c/(16L)$ where $c$ is the speed of sound in water and $L$ is the length of the vibrating water column. The frequency range where this resonance gives a peak can be estimated based on dimensions and material constants. This is because the wave speed can change according to the shell and the exact water length. Figure 2 illustrates the simulation of the pressure in a water column with a height of 0.3 m for steel, aluminium and PVC cylinder materials of thickness 0.02 m. The graph for PVC shows that the first resonance occurs approximately at 350 Hz and multiple peaks are observed. Hence, PVC has not been considered for further modelling. Though the steel cylinder performance is equally good compared with the aluminium cylinder, steel has not been taken up due to its weight.

The simulation is also carried out for an 0.3 m diameter aluminium cylinder of 0.02 m wall thickness with different water column heights. If the water height is increased there is a shift in the resonance frequencies. As the water column increases the first resonant moves towards the low frequency. The response for water column heights of 0.3 m and 0.6 m are shown in Fig. 3.

Figure 4 shows the test vessel with the shaker beneath it. Figures 5 and 6 show the measured receiving sensitivity for hydrophones B&K type 8103 and Cetacean type C55, respectively, in the frequency range of 100 Hz to 1 kHz. The values are stated in dB re V/µPa.

The reference hydrophone type B&K 8104 (H) used in this system was calibrated at NPL, UK and the calibration uncertainty is directly included as one element in the calibration uncertainty. In NPL, UK, the low frequency range 25 Hz to 250 Hz was calibrated by the technique of air pistophone and the upper frequency range up to 1 kHz is calibrated by three transducer spherical wave reciprocity. The uncertainty budget is made in accordance with the Guide to the Expression of Uncertainty in Measurement (GUM). The sensitivity of the unknown hydrophone is calculated by Eq. (5). The major uncertainty is due to the reference hydrophone.

However, all major sources are considered for computing overall measurement uncertainty. The sources of uncertainty are uncertainty due to reference hydrophone, uncertainty for capacitive load correction, vessel correction, uncertainty from filter (if used), and accuracy of the data acquisition system (see Table 1). These uncertainty values are all dependent
on the hydrophone and frequency. All relevant uncertainties are summed using the root of summed squares for calculating combined uncertainty. Then the expanded uncertainty is computed from combined uncertainty (i.e., combined uncertainty is multiplied with the coverage factor $k = 2$ to a confidence level of 95%). The uncertainty table is given as below.

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>Standard uncertainty ($k = 2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Uncertainty for reference hydrophone</td>
<td>±0.6 dB</td>
</tr>
<tr>
<td>2. Uncertainty for capacitive load correction (depends on the hydrophone)</td>
<td>±0.2 dB</td>
</tr>
<tr>
<td>3. Correction for resonances in the vessel</td>
<td>±0.2 dB</td>
</tr>
<tr>
<td>4. Uncertainty from filter (if used)</td>
<td>±0.05 dB</td>
</tr>
<tr>
<td>5. Accuracy of the data acquisition system</td>
<td>±0.03 dB</td>
</tr>
</tbody>
</table>

6. CONCLUSIONS

Simulations using COMSOL were carried out for different water column heights and different vessel materials — such as steel, aluminium, and PVC — of different thicknesses. The simulation results demonstrated that the vibrating water column principle explained in IEC 60565 can be applied for calibration of hydrophones with certain limitations (due to the resonance phenomena of water column). To overcome these limitations the principle of comparison calibration with a reference hydrophone was used in order to extend the frequency range. After successful simulation, an aluminium vessel with $0.3 \text{ m}$ diameter and $0.02 \text{ m}$ wall thickness was manufactured with the trunnion arrangement to avoid static load deflection. The calibration procedure was prepared by considering all the correction factors, such as the capacitive load effect of the hydrophone cable, resistive load, correction for resonances in the test vessel, and correction for filter. The random signal with the bandwidth of 1.6 kHz was given to the shaker. The frequency response of the hydrophone under test and reference hydrophone are acquired by the data acquisition system. The sensitivity of the under test hydrophone was estimated from transfer function ($H_t$) along with corrections. Here the major sources of uncertainty are due to the reference hydrophone uncertainty. Though the calibration frequency range is 1 kHz, the overall uncertainty can be minimized by increasing the wall thickness of the vessel and decreasing the water column height for smaller dimensional hydrophones, whereby the upper frequency may be extended.

Further investigations and potential future improvements are given below:

- Closer investigation of the usable frequency range in the absolute mode.
- Use a set of very high input impedance hydrophone pre-amplifiers in order to avoid some of the present corrections; this can simplify the operation of the system.
- Investigation of the system performance at lower frequencies, down to around 30 Hz is expected to be possible.

ACKNOWLEDGEMENTS

The authors would like to express their sincere thanks to the director, National Institute of Ocean Technology for the support given for developing the vibrating water column based calibration system at their Acoustic Test Facility.

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Use of Feature Ranking Techniques for Defect Severity Estimation of Rolling Element Bearings

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Bearing are the most common components used in rotating machines. Their malfunction may result in costly shutdowns and human causalities which can be avoided by effective condition monitoring practices. In present study, attempt has been made to estimate the severity of defect in bearing components by a two step process. Initially, defects of various severities in all bearing components are classified. In the next step, if defect exist in any of the bearing components, i.e. inner race, outer race, and rolling elements, level of severity of defect is estimated.

Various statistical features are extracted from the raw vibration signals. Two machine learning techniques; support vector machine and artificial neural network, along with four feature ranking techniques; Chi-square, gain ratio, ReliefF and principal component analysis are used and employed for the analysis. Results show the potential of the proposed methodology in defect severity estimation and classification of rolling element bearings.

1. INTRODUCTION

Rolling element bearings are the backbone of almost all the rotating machinery. Studies show that around 40% of the failures in rotating machines are due to bearing faults. If the defect severity is diagnosed well in advance, bearing failure and thus machinery shutdowns can be reduced significantly by avoiding catastrophic failure. Various online health monitoring (OHM) techniques2 are available which respond as the fault initiates, but it is impossible to estimate the defect severity at fluctuating speed and load during operation. Having various techniques, vibration based condition monitoring techniques are preferred due to ease of use and higher responsive towards the faults.

Various remarkable vibration based fault diagnosis methodologies have been developed for bearings. However, in these studies, authors have not considered the severity of faults in their analysis. Classification and estimation of the specific defect is an important part of machinery maintenance systems. Faults with different severity levels in the same component give same characteristic frequency which makes the estimation of defect severity a challenging task. Inaccurate defect severity classification misleads the maintenance program. Various authors have classified the single level severity in rolling element bearing with 100% classification accuracy. Saxena and Saad, Wu et al.8 and Liu et al.9 have classified single level severity in rolling element bearing with 100% classification accuracy. Few attempts have been reported in the literature which attempts to classify various defect sizes in the same component with higher classification accuracy. Skewness, kurtosis, standard deviation, crest factor, and other statistical measures have been utilized by Sharma et al.10 and Amarnath et al.11 In order to increase the computational efficiency, feature ranking techniques are used to select the appropriate features which contain most significant information about the system. In their extensive study, Zhao et al.12 proposed various feature selection techniques; such as Chi-square, ReliefF, etc. Samanta et al.13 have employed the genetic algorithm for condition monitoring of machines. The concept of mutual information has been applied by Kappaganthu and Nataraj14 and the authors concluded that the fault detection accuracy improved significantly with the use of feature ranking methods. Sharma et al.15 have examined various feature ranking techniques for the analysis of bearing faults and summarized that performance of analysis can be improved in the presence of ranked features.

Catastrophic failure of the bearing and the associated system can be reduced significantly with known defect severity. Hong and Liang16 and Wang et al.17 used the Lempel-Ziv complexity and continuous wavelet transform (CWT) based model to quantify the defect severity. The authors conclude that the Lempel-Ziv measure, as non-dimensional index, can be used for fault severity estimation. Jiang et al.18 have extracted residual signals and statistical features from the conducted experiments to quantify the defect severity. The multi-frequency band energies (MFBEs) are also extracted from the acquired signals and summarize that varying trend of residual signals can be a useful tool for defect severity estimation. Yaqub et al.19 presented a defect severity estimation model based on...
on wavelet packet decomposition and support vector machine (SVM). The authors also extracted various statistical features for the severity estimation. Moshou et al.\(^\text{20}\) have extracted statistical features for the defect severity estimation in rolling element bearing. The authors have quantified the defect severity by graphical representation of self-organizing maps (SOMs).

This paper presents a new methodology for defect severity classification and defect severity estimation of rolling element bearings. Four feature ranking techniques are used to select the most appropriate features. The selected features are further used as an input to two machine learning techniques, support vector machine (SVM), and artificial neural network (ANN) for classification and estimation purposes.

2. FEATURE RANKING TECHNIQUES

A number of features are extracted from raw signals to interpret them in meaningful results. However, not all the features are equally important for a specific purpose. Thus, optimal feature selection is the important task in fault diagnosis and severity estimation. The objective of feature ranking techniques is to rank the features based on information and physical spacing. In this study four feature ranking techniques; Chi-square, gain ratio (GR), ReliefF and principal component analysis (PCA), are employed to select the most appropriate features from the extracted features. The selected features are fed as input to the machine learning techniques for defect classification and defect severity estimation analysis. Feature ranking techniques used in this study are described as follows:

2.1. Chi-square

Chi-square is a very commonly used feature selection method. It evaluates the importance of a feature with respect to the class by calculating the value of Chi-squared statistic. Mathematically,

\[
\chi^2 = \sum_{i=1}^{2} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}. \tag{1}
\]

The necessary steps for the analysis using Chi-square technique are summarized as follows:\(^\text{21}\)

1. Calculate the Chi-square value for every pair of adjacent intervals in a signal using Eq. (1).
2. It considers a high significance level for all numeric attributes for discretization.
3. A significance level for each of the attribute \((a = 1, \ldots, n)\) is calculated and merged.
4. The consistency checking is performed after each attribute’s merging.
5. Consistency checking is conducted to ensure that the discretized data set accurately represent the original one.
6. If the inconsistency rate is not exceeded, significance level \((\alpha)\) is determined for attribute \(a\)’s next round of merging; otherwise attribute \(a\) will not be involved in further merging.
7. The process is continued until no attribute’s value is merged to only one value. When the discretization ends, feature ranking is accomplished.

Chi-square discretize the relevant attributes and remove irrelevant attributes. It automatically selects the Chi-square value, determine the interval of numeric attribute as well as select features according to the characteristics of the data. It ensures that the fidelity of the training data can remain after Chi-square is applied. Chi-square feature ranking technique is a useful and reliable tool for discretization and feature selection of numeric attribute.

2.2. Gain Ratio

Gain ratio is based on the principle of information gain. In GR, features are selected in an incremental manner based on the iteration and the iteration ends when a predefined number of features remain.\(^\text{22}\) Higher GR value indicates the higher applicability of features in a feature set, as well as improves the information gain by taking the inherent information of a split into account and is expressed as;

\[
\text{Gain ratio} = \frac{\text{Information gain}}{\text{Splitted information}} \tag{2}
\]

where \(\text{Information gain} = \text{Unsplitted information} - \text{Splitted information}\). Gain ratio is an entropy based feature selection technique and calculates the usefulness of a feature by evaluating the performance of feature randomly in its presence. In GR, features are ranked based on maximizing the feature’s information gain with minimizing the number of its value. The GR values lies between the range \((0, 1)\), where higher GR value of a feature indicates its higher ranking in a feature set.\(^\text{23}\)

2.3. ReliefF

ReliefF evaluates the worth of an attribute by frequently considering an instance and by taking the value of given attribute for the nearest instance of the same and different class. Basically, it is defined for the two-class problem, but can also be used for multiple class problems.\(^\text{24}\) For two class problem ReliefF is;

\[
RF(Z_i) = \frac{1}{2} \sum_{i=1}^{N} p(Z_{t, i} - Z_{dc(x_i)}) - p(Z_{t, i} - Z_{sc(x_i)}) \tag{3}
\]

where, \(Z_{dc(x_i)}\) and \(Z_{sc(x_i)}\) indicates the value of \(i\)th feature of nearest points to \(x_i\) with different and same class label, respectively.

ReliefF is a supervised feature ranking technique. It is employed in data preprocessing as a feature subset selection method. During the features evaluation process, a weight is assigned to each feature based on the ability of the feature to distinguish among the classes and selects those features whose weight exceed a predefined threshold as a relevant feature. The weight computation is executed based on the probability of nearest neighbors from two different classes having different values for a feature and the probability of two nearest neighbors of the same class having the same value of the feature.
The higher the difference between two probabilities represents the more importance of the feature. 23

ReliefF feature ranking technique is more robust and can deal with the noisy and incomplete data. However, its larger computational complexity can reduce the efficiency.

2.4. Principal Component Analysis

Principal component analysis is one of the major linear unsupervised dimensionality reduction techniques. It tries to set the data point from a higher dimensional space to a lower dimensional space with keeping all of the important information intact. 25 It considers the eigenvector to evaluate the influence, to the feature extraction result of each feature element. In PCA, the eigenvector corresponding to a large eigenvalue is able to capture more information of samples. 26

3. MACHINE LEARNING TECHNIQUES

A variety of machine learning techniques such as support vector machine, 3,4 artificial neural network, 27,28 fuzzy logic, 29,30 genetic algorithm, 31,32 and others have been successfully employed in many engineering applications. Among them, support vector machine (SVM) and artificial neural network (ANN) are most widely used artificial intelligence (AI) techniques due to their proven outstanding performance on rolling element bearings applications. 4,33 These two supervised soft computing techniques are considered in this study.

3.1. Support Vector Machine

Support vector machine is a supervised machine learning method based on structural risk minimization principle derived in statistical learning theory. SVM is extensively used for classification and regression problems due to its high generalization performance, robustness, ability to model non-linear relationships, and potential to handle very large feature space. 34,35 For a two-class problem SVM can be formulated as following optimization problem;

Minimize \( \frac{1}{2} ||W||^2 + C \sum_{i=1}^{n} \xi_i \)  \hspace{1cm} (4)

Subject to \[
\begin{align*}
    y_i(W^T x_i + q) &\geq 1 - \xi_i, \\
    \xi_i &\geq 0, \quad i = 1, 2, \ldots, n
\end{align*}
\]  \hspace{1cm} (5)

where \( x_i, y_i \) is the data set and \( q \) is a real constant.

The sequential minimal optimization (SMO) is an improved faster training algorithm, used for solving the dual problem arising from the derivation of the SVM.

3.2. Artificial Neural Network

Artificial neural network is a group of especially interconnected artificial nodes, called neurons. ANN is an adaptive system that changes its structure according to the information flows through the network. Having various architectures of ANN, multilayer feed forward back propagation algorithm is widely used for rotary machine elements.

4. EXPERIMENTAL SETUP

The bearing vibration data used for analysis in this study are collected from Case Western Reserve University Bearing Data Centre website. 36 Figure 1 shows the brief outlines of the experimental test rig. The test rig has a 2HP three phase induction motor, an encoder, and a dynamometer. The drive-end side of the motor consists the test bearing and is loaded by the dynamometer. Accelerometer, having magnetic base, is mounted on the housing of the test bearing and used for acquiring the vibration signals. Healthy bearing data are considered as the baseline data in the analysis. The drive-end side test bearing parameters used in this study are listed in Table 1. The schematic representation of various bearing components defects, i.e. inner race defect, outer race defect, and ball defect, are shown in Fig. 2.

A single neuron consists of synapses, summing function, and an activation function. Mathematically a neuron can be represented as:

\[ K = Z \left( \sum_{i=1}^{J} w_i v_i + b \right). \]  \hspace{1cm} (6)

5. FEATURE EXTRACTION AND SELECTION

A wide set of statistical features is extracted from the vibration signals. The extracted features are described as follows:
5.1. Mean

Mean is referred as the average value of the signal.

\[
\text{Mean} = \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}.
\] (7)

5.2. Root Mean Square (RMS)

Root mean square is the square root of the average of the squared values of the signal.

\[
\text{RMS} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} x_i^2}.
\] (8)

5.3. Standard Deviation (SDEV)

Standard deviation is a measure of energy contain in the signal.

\[
\text{SDEV} = \sqrt{\frac{n \sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2}{n(n-1)}}.
\] (9)

5.4. Kurtosis

Kurtosis is used to describe the distribution of observed data around the mean and is defined as the degree to which a statistical frequency curve is peaked.

\[
\text{Kurtosis} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^4}{(n-1)(\text{SDEV})^4}.
\] (10)

5.5. Skewness

Skewness measures the symmetry of a distribution around its mean. Skewness can be negative or positive.

\[
\text{Skewness} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^3}{(n-1)(\text{SDEV})^3}.
\] (11)

5.6. Crest Factor

Crest factor is the ratio of peak value to RMS value of the signal and indicates the shape of the waveform.

\[
\text{Crest factor} = \frac{\max |x_i|}{\sqrt{\frac{1}{n} \sum_{i=1}^{n} x_i^2}}.
\] (12)

5.7. Minimum Value (MIN)

Minimum value represents the minimum value of the signal.
5.8. Maximum Value (MAX)

Maximum value represents the maximum value of the signal.

5.9. Covariance (COV)

Covariance is a measure that represents the strength of the correlation between two random variables in a signal.

5.10. Shape Indicator

Shape indicator is defined as the ratio of the RMS value to the mean value of the signal.

\[ \text{Shape indicator} = \frac{\text{RMS}}{\text{Mean}}. \]  

These statistical features are initially used to form a feature vector. To improve the defect classification and defect severity estimation efficiency, these extracted features are fed as input to various feature ranking techniques as discussed; thereafter, the features are shortlisted and selected as per their ranking. The selected features are then fed as input to machine learning techniques, i.e. SVM and ANN. The overview of the methodology for defect severity classification and defect severity estimation is shown in Fig. 3.

6. RESULTS AND DISCUSSION

In this study, defects in all bearing components, i.e. inner race, outer race and rolling element, with various defect severity levels, i.e. 0.1778 mm, 0.3556 mm and 0.5334 mm, and with healthy bearing, are considered for the fault diagnosis and defect severity estimation. Various statistical features are extracted from the considered bearing conditions. Further, features are selected as per their ranking using four feature ranking techniques. All the extracted features are supplied to four feature ranking techniques for their ranking. As suggested by Wang et al., \((\log_2 f_n)\) number of features may be used for classification with various learning algorithms, where \(f_n\) is the number of features. Table 2 summarizes the ranking of features corresponding to various feature ranking methods. A comparative study between SVM and ANN with all feature ranking techniques is carried out for defect classification and defect severity estimation.

6.1. Defect Classification

As a part of analysis, first the classification among all the considered cases is carried out, which includes the following forty bearing conditions: four corresponding to healthy bearing, twelve corresponding to inner race defects, twelve corresponding to outer race defect, and twelve corresponding to ball defect, having localized defects of 0.1778 mm, 0.3556 mm, and 0.5334 mm in inner race, outer race, and ball and each corresponding to four speeds, i.e. 1797 rpm, 1772 rpm, 1750 rpm, and 1730 rpm. A sample training/testing vector used in the investigation is shown in Table 3 (where, HY = healthy bearing, ID = bearing having inner race defect, OD = bearing having outer race defect and BD = bearing having ball race defect).

The results for the two machine learning techniques, i.e. SVM and ANN, using 10-fold cross validation are shown. In 10-fold cross validation, data is randomly divided into ten equal sized training and testing folds. During iterations, nine of the 10-folds are used for training and remaining one fold is used for testing the dataset and finally it provides a single value after averaging all the iterations. 10-fold cross validation is preferred due to its capability of eliminating any biasness while dividing data into training and testing set. The detailed accuracy for SVM and ANN using the following four feature ranking techniques: Chi-square, GR, ReliefF, and PCA and these are shown in Table 4. It represents 100% classification accuracy and 0% incorrectly classified instances for each of SVM and ANN. Results also indicate the value of Kappa statistics for each of SVM and ANN with all the feature ranking techniques as 1 (or 100%), which indicates the perfect categorization of the data with the highest accuracy. Kappa statistics is an important measure which is used to predict the agreement between actual and predicted classes.38

6.2. Defect Severity Estimation

In the previous section, classification between defective inner race, outer race and rolling elements have been carried out. The classification accuracies of both SVM and ANN with all the feature ranking techniques are obtained as 100%. In this section, defect severities in bearing components are estimated. The estimation is carried out on three different defect severities, i.e. 0.1778 mm, 0.3556 mm, and 0.5334 mm of inner race, outer race, and rolling elements at four different speeds, i.e. 1797 rpm, 1772 rpm, 1750 rpm, and 1730 rpm. A sample training/testing vector used for defect severity estimation purpose is shown in Table 5 (where IR = inner race, OR = outer race, and Ball = rolling element).

The detailed accuracies of defect severity estimation of SVM and ANN for inner race defects using various feature ranking techniques, i.e. Chi-square, GR, ReliefF, and PCA, are listed in Table 6. The correlation coefficients show a good agreement between the actual class and the predicted class, as its value is observed as 1 for all the ranking techniques for both of SVM and ANN. The maximum percentage error is reported as 0.2812% for ANN with PCA ranking technique. It indicates highly correlated results having very few errors.

The results of SVM and ANN for defect severity estimation of outer race defects with various feature ranking techniques are listed in Table 7. The results show the superior relationship between the actual class and the predicted class than that for inner race. For both of the artificial intelligence techniques, the correlation coefficient is observed as 1. Also, the maximum percentage error is reported as 0% for both of SVM and ANN. It shows perfectly correlated results for both SVM and ANN.

Table 8 indicates the results of defect severity estimation of SVM and ANN for rolling element with various feature ranking techniques. Results show that prediction capability of SVM is better than that of ANN. The correlation coefficient shows a perfect synchronization between the actual and predicted class for SVM while ANN has fewer prediction capabilities in this case. The maximum percentage error for SVM is observed as 0% and for ANN it is found as 3.3746%. The
maximum and minimum values of correlation coefficients for
ANN are also noticed as 0.9997 and 0.999, respectively, which
are very close to 1 and show good agreement between the ac-
tual and predicted class.

7. CONCLUSIONS

The present study deals with defect severity classification
and estimation in various rolling element bearing components.
Defects having three fault severities in inner race, outer race,
and rolling elements are considered for the analysis. A wide set
of statistical features is extracted from vibration signals. Four
feature ranking techniques are used to rank the extracted fea-
tures and the performance of two machine learning techniques,
support vector machine and artificial neural network, are eval-
uated. The following conclusions are drawn from the present
study:

- Both SVM and ANN show good performance for defect
  severity classification and estimation, but ANN performs
  a bit underneath for estimating the defect severity with
  principal component analysis feature ranking technique.
The results obtained from SVM are superior due to its
inherent capability of generalization.

- Results indicate that the two features of standard devia-
tion and root mean square are proven to be the best two
  indicators irrespective of feature ranking method.

- The classification and quantitative assessment of fault
  severity of rolling element bearings can be improved sig-
nificantly with feature ranking techniques. Results show
  that Chi-square method outperforms other techniques in
terms of correlation coefficient as well as the minimum
  Maximum error (%).

- Proposed methodology can be effectively used for dimen-
sionality reduction of features without compromising the
  performance and it would be beneficial in real practices
to analyze the defect severities accurately.

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Western Reserve University for their efforts to make bearing
data set available and permission to use data set.

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014-1107-1
### Table 4. Detailed accuracy of SVM and ANN for defect classification using different feature ranking techniques.

<table>
<thead>
<tr>
<th>Feature ranking technique</th>
<th>SVM Correctly classified instances</th>
<th>SVM Incorrectly classified instances</th>
<th>ANN Correctly classified instances</th>
<th>ANN Incorrectly classified instances</th>
<th>Kappa statistics</th>
<th>Error (%)</th>
<th>Classification accuracy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-square</td>
<td>40</td>
<td>0</td>
<td>40</td>
<td>0</td>
<td>0.005</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>Gain Ratio</td>
<td>40</td>
<td>0</td>
<td>40</td>
<td>0</td>
<td>0.009</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>ReliefF</td>
<td>40</td>
<td>0</td>
<td>40</td>
<td>0</td>
<td>0.144</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>PCA</td>
<td>40</td>
<td>0</td>
<td>40</td>
<td>0</td>
<td>0.154</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>120</strong></td>
<td><strong>0</strong></td>
<td><strong>120</strong></td>
<td><strong>0</strong></td>
<td><strong>0.288</strong></td>
<td><strong>0</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

### Table 5. Sample training/testing vector for SVM and ANN for fault severity estimation.

<table>
<thead>
<tr>
<th>Features</th>
<th>Mean</th>
<th>RMS</th>
<th>SDEV</th>
<th>Kurtosis</th>
<th>Skewness</th>
<th>Crest factor</th>
<th>MIN</th>
<th>MAX</th>
<th>COV</th>
<th>Shape indicator</th>
<th>Speed (rpm)</th>
<th>Condition</th>
<th>Defect severity (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.004</td>
<td>0.314</td>
<td>0.314</td>
<td>5.291</td>
<td>-0.01133</td>
<td>5.329</td>
<td>-1.536</td>
<td>1.672</td>
<td>0.098</td>
<td>1.399</td>
<td>1721</td>
<td>IR</td>
<td>0.1778</td>
</tr>
<tr>
<td></td>
<td>0.004</td>
<td>0.163</td>
<td>0.163</td>
<td>21.686</td>
<td>0.02355</td>
<td>11.532</td>
<td>-1.88</td>
<td>1.854</td>
<td>0.026</td>
<td>1.652</td>
<td>1752</td>
<td>IR</td>
<td>0.3556</td>
</tr>
<tr>
<td></td>
<td>0.003</td>
<td>0.449</td>
<td>0.449</td>
<td>8.345</td>
<td>0.303</td>
<td>8.056</td>
<td>-3.087</td>
<td>3.614</td>
<td>0.201</td>
<td>1.482</td>
<td>1728</td>
<td>IR</td>
<td>0.5334</td>
</tr>
<tr>
<td></td>
<td>0.005</td>
<td>0.58</td>
<td>0.58</td>
<td>7.964</td>
<td>-0.003</td>
<td>5.576</td>
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<td>3.236</td>
<td>0.337</td>
<td>1.634</td>
<td>1725</td>
<td>OR</td>
<td>0.1778</td>
</tr>
<tr>
<td></td>
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<td>0.097</td>
<td>0.097</td>
<td>3.024</td>
<td>0.0002</td>
<td>4.932</td>
<td>-0.409</td>
<td>0.478</td>
<td>0.009</td>
<td>1.255</td>
<td>1749</td>
<td>OR</td>
<td>0.3556</td>
</tr>
<tr>
<td></td>
<td>0.004</td>
<td>0.559</td>
<td>0.559</td>
<td>23.542</td>
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<td>11.902</td>
<td>-6.654</td>
<td>6.653</td>
<td>0.313</td>
<td>2.054</td>
<td>1721</td>
<td>OR</td>
<td>0.5334</td>
</tr>
<tr>
<td></td>
<td>0.004</td>
<td>0.154</td>
<td>0.154</td>
<td>2.84</td>
<td>0.02</td>
<td>4.69</td>
<td>-0.72</td>
<td>0.672</td>
<td>0.024</td>
<td>1.245</td>
<td>1722</td>
<td>Ball</td>
<td>0.1778</td>
</tr>
<tr>
<td></td>
<td>0.005</td>
<td>0.144</td>
<td>0.144</td>
<td>9.753</td>
<td>0.145</td>
<td>12.819</td>
<td>-1.386</td>
<td>1.839</td>
<td>0.02</td>
<td>1.422</td>
<td>1749</td>
<td>Ball</td>
<td>0.3556</td>
</tr>
<tr>
<td></td>
<td>0.005</td>
<td>0.118</td>
<td>0.118</td>
<td>3.101</td>
<td>0.025</td>
<td>4.886</td>
<td>-0.493</td>
<td>0.577</td>
<td>0.014</td>
<td>1.259</td>
<td>1729</td>
<td>Ball</td>
<td>0.5334</td>
</tr>
</tbody>
</table>

### Table 6. Detailed accuracy of SVM and ANN for defect classification using different feature ranking techniques.

<table>
<thead>
<tr>
<th>Feature ranking technique</th>
<th>Correlation coefficient</th>
<th>Maximum error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVM</td>
<td>ANN</td>
<td>SVM</td>
</tr>
<tr>
<td>Chi-square</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Gain Ratio</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>ReliefF</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>PCA</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 7. Detailed accuracy of SVM and ANN for defect classification using different feature ranking techniques.

<table>
<thead>
<tr>
<th>Feature ranking technique</th>
<th>Correlation coefficient</th>
<th>Maximum error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVM</td>
<td>ANN</td>
<td>SVM</td>
</tr>
<tr>
<td>Chi-square</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Gain Ratio</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>ReliefF</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>PCA</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 8. Detailed accuracy of SVM and ANN for defect classification using different feature ranking techniques.

<table>
<thead>
<tr>
<th>Feature ranking technique</th>
<th>Correlation coefficient</th>
<th>Maximum error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVM</td>
<td>ANN</td>
<td>SVM</td>
</tr>
<tr>
<td>Chi-square</td>
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<td>Gain Ratio</td>
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<tr>
<td>ReliefF</td>
<td>1</td>
<td>0.9997</td>
</tr>
<tr>
<td>PCA</td>
<td>1</td>
<td>0.999</td>
</tr>
</tbody>
</table>


Vibration Response Prediction on Rubber Mounts with a Hybrid Approach

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Accurate prediction of the vibration response at a point on a complex structure, where the operational behavior cannot be measured directly, is an important engineering problem for design optimization, component selection and condition monitoring. Identifying the exciting forces acting on the structure is a major step in the vibration response prediction (VRP). At the point where direct measurement is impossible or impractical due to physical constraints, a common approach is to identify the exciting forces based on multiplication of an inverted frequency response function (FRF) matrix and a vector of vibration responses measured at the points where the exciting forces are transmitted through. However, in some cases measuring FRFs are almost impossible. In other cases, where measuring is possible, they may be prone to significant errors. Furthermore, the inverted FRF matrix may be ill-conditioned due to the one or few modes that dominate the dynamics of the structure.

In order to improve the force identification step and reduce the experimental challenges, previous studies focused on either conditioning methods or numerical models. However, conditioning methods result in additional measurements, and using only numerical models causes reduced accuracy due to incongruities between the simulation model and the real system. Considering these problems, a hybrid VRP methodology that incorporates the numerical modeling and experimental measurement results is proposed in this study. Creating an accurate numerical model and properly selecting the force identification points are the main requirements of the proposed methodology. A structure coupled with rubber mounts is used to demonstrate the proposed methodology. The numerical model includes hyperelastic and viscoelastic modeling of the rubber to represent the system behavior accurately. The selection of force identification points is based on a metric that is composed of the average condition number of the FRF matrix across the whole frequency of interest. The results show that the proposed hybrid methodology is superior to other alternative methods where predictions are solely based on numerical results or experimental measurements.

1. INTRODUCTION

Mechanical systems are usually composed of various subsystems coupled by several links, such as rubber mounts. Any excitation acting on the system is divided into several internal forces, which propagate throughout the mounts. The structural vibration response at a point of interest, as a result of the exciting forces, is usually of great importance in terms of design optimization, component selection and condition monitoring. For the locations in complex structures where the operational behavior cannot be measured directly, a methodology is required for accurate prediction of the response. A major step of the vibration response prediction (VRP) is to identify the exciting forces acting on the structure. The most evident solution is to measure the forces directly. However, this may not be possible due to the complexity of the structure and the challenges of load cell applications. Consequently, indirect methods have been widely studied in the literature.1–20 The following studies are worthwhile to mention here. Verheij1 introduced the dynamic stiffness method which seems the most straightforward approach, especially for rubber linked structures. However, accurate complex dynamic stiffness data of the rubber mounts is rarely available, and even if present, it is only valid for a given load condition. Another approach, called the transmissibility method, can be implemented to predict the vibration response at a point of interest.2–5 In this method, the forces are replaced by the measured responses at the force identification points, and the propagation paths are represented by the transmissibility. This approach is much simpler and faster, but unconsidered potential cross-coupling between the paths can lead to incorrect predictions.

In the early 1980s, the matrix inversion method was developed.6,7 The inverse method basically involves multiplication of a vector of vibration responses with an inverted matrix constituted by the frequency response functions (FRFs). The main drawback of this method is the need of the FRF measurements. However, measuring the FRFs is sometimes not possible, especially for complex structures. Even when it is possible, it is very time consuming and prone to significant errors based on excitation, environment, sensor, structure, and unconsidered sources. The possible sources of the above measurement errors are shown in Fig. (1).21–27

Modal behavior of the structure also influences the accuracy of the matrix inversion method. The FRF matrix includes information about the vibration modes contributing to the response, and the amplitude of the modal contribution depends on the location of the FRF measurements. Accordingly, the condition number of the FRF matrix, which is simply the ratio of the largest singular value to the smallest, varies significantly across the structure at particular frequencies. In the condition that one or few modes dominate the responses of specific points that constitute the FRF matrix, the rows or columns of the FRF matrix may become linearly dependent resulting in high condition numbers. High condition number refers to smaller singular values. In this case, the solution may not be unique, and the FRF matrix can be defined as “almost singular” or “ill-conditioned.”13,14 By selecting the points where the forces are identified properly, the condition number can be reduced and ill-conditioning can be improved. Therefore, con-
dation number of the FRF matrix can be perceived as a good quality indicator of the force identification process.

The ill-conditioned FRF matrix and measurement errors cause erroneous results in the force identification process and consequently, in the response prediction. Thus, eliminating these problems is of great importance in order to improve the accuracy of the prediction results. For this purpose, conditioning methods have been applied in the literature. It has been shown that over-determining the FRF matrix by including additional measurements can improve the inverse problem. However, in this case, the matrix becomes rectangular and a least-squares error method is required for the inversion. In certain cases, the over-determination does not help to reduce the prediction errors, and thus, regularization techniques, such as Tikhonov, singular value decomposition, L-curve etc., are required to make the inverse problem a unique and well-conditioned problem. However, these methods are limited to a certain range of condition numbers and sensitive to the selection of the regularization parameters.

As discussed above, previous studies focused on either reducing the experimental measurement efforts and/or improving the conditioning of the FRF matrix. In order to overcome the experimental challenges as well as measurement errors, numerical simulations have been widely used. However, the vibration responses at the force transmission locations required for the inverse method cannot be computed accurately in a numerical model. This is usually due to the fact that the excitation on the structure cannot be modeled precisely.

Conditioning methods result in additional measurements, and using a numerical model causes reduced accuracy due to incongruities between the simulation model and the real system. Considering these challenges, a hybrid VRP methodology that incorporates numerical modeling and experimental measurement results is proposed in this study. In the proposed hybrid methodology, the experimental errors and difficulties of FRF measurements as well as time consumption are eliminated since FRFs are calculated numerically. Moreover, inverse problem is improved by reducing the condition number of the FRF matrix by choosing the right points for the FRF measurements. An accurate numerical model and selection of the points where the forces are identified are critical steps of the proposed methodology. If the representative dynamics of the structure are not included in the measured FRFs due to a wrong selection of the measurement locations, the forces are not identified correctly, and it affects the accuracy of the response prediction process. Accordingly, force identification points are selected based on the “combined condition number” metric within the scope of the hybrid VRP. The proposed hybrid methodology is applied to a structure coupled through rubber mounts. All of the above studies are also validated by applying an experimental VRP study.

This paper is organized as follows: Section 2 focuses on the vibration response prediction methodology based on the direct matrix inversion technique. In Section 3, hybrid VRP method and theoretical considerations such as accuracy of the numerical model and the importance of selection of the force identification points is introduced. Section 4, presents VRP case studies along with the hyperelastic and viscoelastic modeling of the rubber mounts. Finally, the paper is concluded with a summary and conclusion in Section 4.3.

2. VIBRATION RESPONSE PREDICTION METHODOLOGY

The vibration response of any point depends on the corresponding transfer function and the force acting on that point if there is only one transmission path. However, if more than one source or path is present, the cross-coupling terms should be taken into consideration. Cross-coupling means that the response at a particular point depends not only on the force acting on that point but also on the other internal forces. Thus, cross-coupling effects are considered by means of including all FRFs between the transmission paths. Accordingly, vibration response in terms of acceleration can be calculated by multiplying the internal forces and the corresponding FRFs. Assuming that the system is linear and time-invariant, partial contributions from each force are summed up, and the vibration response at the point of interest, \( \ddot{X}_k(\omega) \), can be predicted, as follows:

\[
\ddot{X}_k(\omega) = \sum_{i=1}^{n} \ddot{X}_{ki}(\omega) = \sum_{i=1}^{n} F_i(\omega)H_{ki}(\omega);
\]

where \( n \) is the number of transmission paths.

The major step of the VRP study is to identify the operating forces by implementing indirect methods as discussed above. In the direct matrix inversion method, vibration response measurements are performed at the connection points of the transmission paths, and the FRFs are measured between these points in order to consider the cross-couplings. A square FRF matrix, \( n \times n \), is created since the number of forces and responses are equal to each other. Thus, internal forces can be identified in a matrix notation by applying ordinary matrix inversion as shown in Eq. (2).

\[
\{F_j(\omega)\} = [H_{ij}(\omega)]^{-1}\left\{\ddot{X}_i(\omega)\right\};
\]

where \( i = j \) which denotes the number of paths and forces, respectively.

3. HYBRID VIBRATION RESPONSE PREDICTION METHODOLOGY

In this study, a hybrid VRP methodology was proposed to reduce the measurement effort and errors as well as the ill-
conditioning of the inverse problem. The hybrid VRP methodology differs from the classical VRP methods in the construction of the FRF matrix. The acceleration (FRF) matrix, \([H_{ij}(\omega)]\) of Eq. (2), is formed by the numerical FRFs, whereas the acceleration vector, \(\{\ddot{X}_i(\omega)\}\) is measured through the experimental studies. For obtaining reliable predictions from the hybrid VRP, an accurate numerical model of the structure needed to be created and validated. Numerical models of the rubber-linked structures are more critical compared to that of rigid structures since rubber may expose the structure to non-linearity. In this approach, two important steps needed to be completed before performing the hybrid VRP:

a) Numerical FRFs on the structure needed to be validated using measured FRFs.

b) Force identification points needed to be selected properly such that representative dynamics of the structure is included in the measured FRFs. Condition number of the FRF matrix was used as a metric to select the measurement points, and it was related to the spatial position of the measurement points depending on the modal behavior of the structure.

A flow diagram of the hybrid VRP is shown in Fig. (2). The detailed process was as follows;

a) Create a finite element (FE) model of the structure and include all the components.

b) Determine the validation points where the numerical and experimental FRFs are compared. If there are discrepancies between the FRFs over the frequency of interest, the FE model should be updated.

c) After the validation process, select \(m\) candidate points to determine the best force identification points.

d) Create a combination with each set of two candidate points.

e) Calculate the coupled FRFs between the points for each combination.

f) Construct the FRF matrix, \([H_{ij}(\omega)]\) as in Eq. (2), \((2 \times 2)\) for each set of two measurement points \((i, j)\).

g) Calculate the combined condition number, \(C_{ij}\) as follows:

\[
C_{ij} = \frac{1}{n} \left( \sum_{\omega} K_2 \{H_{ij}(\omega)\} \right)
\]

(3)

where \(n\) is the number of discrete frequencies and \(K_2\) stands for the 2-norm condition number.

h) Select the combination having the minimum combined condition number.

i) Measure the vibration responses at the selected points.

j) Calculate the FRFs between the selected and target points from the numerical model.

k) Perform hybrid VRP based on the direct matrix inversion method with the set of experimental measurements and numerical FRFs by implementing Eq. (1) and (2).

4. VRP CASE STUDIES

In order to implement the proposed hybrid VRP methodology, an experimental set-up was built as shown in Fig. (3). The set-up consisted of two plates coupled with two rubber mounts.

The schematic representation of the experimental set-up with the force and target locations, \(T_1, T_2,\) and \(T_3\), where the responses are predicted, is illustrated in Fig. (4). A modal shaker was fixed to the structure and used to generate the excitations representing the operating source, \(F\). Although there was one operational force acting on the structure, vibrational energy flowed through the rubber links resulting in internal forces. Note that moments and rotations were ignored in the calculations since it is very difficult to measure those quantities.

4.1. Rubber Components Modeling

Rubber mounts are widely used as vibration isolators as they have elastic and viscous properties such as high inherent damping, deflection capacity, and energy storage. They can also be characterized as compact, easily available, cost effective, and maintenance free. The dynamic properties of the rubber components were of primary concern in designing rubber isolators to reduce transmissibility. In order to determine these properties accurately, hyperelastic and viscoelastic constitutive models were applied. The hyperelastic material model captured the material’s nonlinear elasticity with no-time dependence, whereas the viscoelastic model described the material response, which contains an elastic and viscous part depending on time, frequency, and temperature.

The rubber mounts used in the case study were made of natural rubber. In order to evaluate the dynamic properties of the rubber, tensile and relaxation tests were conducted on the specimens created according to DIN 53504-S1 with a thickness of 2 mm, as shown in Fig. (5)a and Fig. (5)b.

4.1.1. Hyperelastic Constitutive Model

A hyperelastic material is still an elastic material, which means that the material returns to its original shape once the force is released. The difference is that for a hyperelastic material, the stress-strain relationship derives from a strain energy density function, \(W\), as follows:

\[
\sigma = \frac{\partial W}{\partial \varepsilon}. 
\]

(4)

The strain energy density function can be defined in terms of strain invariants and stretch ratios. Deviatoric and volumetric terms of the strain energy density function for incompressible materials can be written as follows:

\[
W = W_d(T_1,T_2) + W_v(J);
\]

(5)

where \(J\) is the ratio of the final volume to the initial volume and \(T_1, T_2\) are the strain invariants. Since the rubber was assumed to be incompressible, meaning that the Poisson’s ratio is very close to 0.5, no volumetric test was performed, and the volumetric term was neglected.

Several mathematical models for \(W\) were proposed for the analytical and numerical prediction of stress-strain behavior of elastomer materials where the most prominent ones are Neo-Hookean, Mooney-Rivlin and Yeoh models. All of these
models are semi-empirical models and require experimental parameters from shear, uniaxial, or biaxial tests. In this study, Neo-Hookean, Mooney-Rivlin and Yeoh models were considered and the corresponding mathematical models are shown in Table 1.

To study the hyperelastic properties, specimens were subjected to quasi static tensile loading with constant strain rate of 25 mm/min. Three cycles of the experiment were conducted to minimize experimental errors. Figure (6) presents the stress-strain responses obtained from the quasi static test, and the estimated coefficients of hyperelastic models for the rubber component were tabulated in Table 2.
4.1.2. Viscoelastic Rheological Model

As discussed before, the hyperelastic model represents the nonlinear elastic response with no time dependence. In order to model time dependency of the rubber behavior, a viscoelastic rheological model was used by means of a relaxation test. In a relaxation test, the strain is held constant and the stress anticipates relaxation behavior and is usually applied in the case of small deformations.

One of the basic rheological viscoelastic models is the Maxwell model, which includes both the elastic and viscous property of the material. The Maxwell model consists of a linear ideally viscous, Newtonian dashpot and linear elastic Hookean spring in series. In the advanced form of this model, which is called the generalized Maxwell model, the relaxation does not take place at a single time but during a series of times. Thus, it can have many spring-dashpot Maxwell elements as shown in Fig. (7). This model can anticipate relaxation behavior and is usually applied in the case of small deformations.

Prony series is one of the best functions for modeling the linear viscoelasticity and the generalized Maxwell model. The resulting stress vs. time data from a tensile relaxation test can be fitted with Prony series as shown in Eq. (6).

\[
E(t) = E_\infty + \sum_{i=1}^{N} E_i \exp \left( -\frac{t}{\tau_i} \right); \tag{6}
\]

where \( E_\infty \) is the long-term modulus, \( \tau_i \) is the relaxation time and \( N \) is a finite integer. The instantaneous modulus is given by \( E(t = 0) \).

The relaxation behavior of the rubber specimens was examined through relaxation tests. Figure (8) shows the time history of the stress decrement at a constant strain. The relaxation curve initially revealed a very fast stress relaxation and then, slowed down and continued in an asymptotic sense as confirmed in the references. The generalized Maxwell model was used for the rubber components used in this study, and the Prony coefficients were determined by applying curve fitting to the relaxation test data. These coefficients were used for modeling the viscoelastic properties of the rubber in the FE model, which is presented in the following section.

4.2. Numerical Model of the Set-up and Validation

As discussed in previous sections, an accurate numerical model was required in order to implement hybrid VRP study. For this purpose, the measured properties of the rubber, presented in previous section, was used for modeling the rubber mounts, and the numerical model of the set-up was generated by the finite element method (FEM) tool, ANSYS, as shown in Fig. (9).

A system with rubber components can display nonlinearity due to the hyperelastic properties of the rubber. The equation of motion of a general nonlinear system subjected to a time dependent excitation is as stated in Eq. (7).

\[
[M]\{x(t)\} + [C]\{\dot{x}(t)\} + i[D]\{x(t)\} + [K]\{x(t)\} + [G]\{\dot{x}(t),x(t)\} = \{f(t)\}; \tag{7}
\]

where \( M, C, D, \) and \( K \) are mass, viscous damping, hysteretic damping, and stiffness matrices, respectively. The nonlinear component of the system is represented by the nonlinear vector, \( G \), which is a function of all displacements and velocities in a general case. Due to the nonlinear material properties of the model, linear harmonic analysis could not be implemented in FEM solvers. Thus, transient analysis was used for a more accurate solution. The responses obtained in the time domain were transformed to the frequency domain using post processing and the Fast Fourier Transform (FFT) algorithm in MATLAB, as described in Fig. (10). In the algorithm, time response data was divided into smaller blocks in order to compute multiple FFTs instead of computing a single FFT for the whole data set. Overlapping, windowing, and averaging was used in order to obtain a good representation of the data set and improve the accuracy of FFT.

<table>
<thead>
<tr>
<th>Hyperelastic Model</th>
<th>( C_{10} )</th>
<th>( C_{20} )</th>
<th>( C_{50} )</th>
<th>( C_{61} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neo-Hookean</td>
<td>0.6933</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Mooney-Rivlin</td>
<td>0.3724</td>
<td>-</td>
<td>0.4397</td>
<td></td>
</tr>
<tr>
<td>Yeoh</td>
<td>0.9658</td>
<td>0.06213</td>
<td>0.3263</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7. Generalized Maxwell model.

Figure 8. Normalized relaxation test results for the rubber specimen.

Figure 9. Numerical FEM model of the experimental set-up.
The first step of the hybrid VRP was the model validation, which was performed by comparing the numerical FRFs with the measured ones. A representative FRF is shown in Fig. (11), and it can be seen that the numerical FRF agrees reasonably well with the experimentally measured FRF. Other selected points were also verified to assure that the numerical model could be accepted as valid.

### 4.3. Selection of the Force Identification Points

Another consideration for the hybrid VRP was the condition number of the accelerance matrix. Thus, numerical simulations were carried out to select the points where the transmitted force would be identified. The points were selected based on the "combined condition number" metric as discussed in Section 3. As shown in Fig. (12), a total of 17 candidate points were selected and the accelerance matrix, \([H_{ij}(\omega)]\), was created for each set of 2 candidate points. A total of 136 combinations were considered. Combined condition numbers for each combination were shown in Fig. (13). According to Fig. (13), out of 136 combinations, the best and worst combinations are determined as 5-8 and 13-16, respectively. The condition numbers of the accelerance matrices formed by these combinations are given in Fig. (14) as a function of frequency. As shown in the figure, the condition numbers were reduced considerably when the accelerance matrix was created by the best combination.

### 4.4. Experimental and Hybrid ORP Results

After selecting the measurement points, vibration responses of each target on the structure were predicted by implementing Eq. (1) and are shown in Fig. (4). The responses of T1 and T2 points predicted by the hybrid VRP with the best and worst combinations were compared with the experimentally measured ones in Figs. (15)a and (15)b. All of these results were also compared with the responses predicted by the experimental VRP where all required operational responses and FRFs were obtained experimentally for validation purposes. The condition number of the numerical accelerance matrix was considerably lower than that of the experimental accelerance matrix and is shown in Fig. (16). Due to the reduction in the condition numbers, it clearly shows that the hybrid VRP method with the best combination improves the results significantly. Note that, a similar behavior was exhibited by the other measurement location, T3. It can be seen that the combined condition number is an efficient tool for selecting the force identification points. However, closer inspection reveals some discrepancies at specific frequencies for both methods. These discrepancies may have occurred due to the unconsidered in-plane lateral forces, rotations, and moments.

### 5. CONCLUSIONS

A hybrid vibration response prediction methodology was proposed for rubber linked structures with numerical and experimental case studies. FRF measurements were obtained numerically and vibration responses were measured experimentally in the proposed method. A validated numerical model of the structure which incorporates hyperelastic and viscoelastic behavior of rubber mounts, was used for the hybrid VRP method. It was demonstrated that the selection of the force identification points significantly improves the vibration response predictions compared to the experimental VRP method.
identification points plays an important role in determining the condition number of the accelerance matrix. The force identification point selection is based on a metric that is composed of the average condition number of the FRF matrix across the whole frequency of interest. The results show that the proposed hybrid methodology is superior to other alternative methods where predictions are solely based on the numerical results or experimental measurements. Consequently, the hybrid VRP methodology can be considered a reliable and efficient tool to predict the vibration responses, especially for complex structures.

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REFERENCES


Vibration Analysis of Elastic Beams With Unconstrained Partial Viscoelastic Layer

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Material damping treatments, such as adding viscoelastic layers (VEL) to engineered structures, are commonly used for vibration attenuation. Study of the VEL geometry, shape, location (in case of partially adding), and arrangement (in composites), are of engineering interest to optimize the damping effect versus the weight and cost. In order to show the possibility of higher damping characteristics for shorter VEL, this paper aims for a two-step vibration analysis of an elastic cantilever beam with an unconstrained partial VEL. The governing equations are developed based on Euler-Bernoulli beam theory and Kelvin-Voigt viscoelastic model. In order to answer how the VEL length and thickness affect the modal parameters and dynamic response, both free and forced vibration problems are solved analytically, and the results are manipulated to achieve a much more applicable size range with higher damping characteristic. A non-uniform trend and inconsistent behaviour in both frequency and amplitude changes is observed versus increasing the VEL length, which addresses the necessity of an optimization challenge and gives a good insight to take into account the concept of VEL critical length.

NOMENCLATURE

- $E_e, E_v$: Elastic and viscoelastic Young’s Modulus.
- $H$: Frequency response Function (FRF).
- $I_e, I_v$: Elastic and viscoelastic moment of inertia.
- $k_e, k_{ev}$: Elastic and composite (elastic-viscoelastic) bending stiffness.
- $K(x)$: Complex bending stiffness function.
- $K_e, K_{ev}$: Elastic and composite (elastic-viscoelastic) complex bending stiffness.
- $l_e$: Length of the elastic beam.
- $m_e, m_v$: Elastic and viscoelastic mass.
- $p$: Distributed force.
- $p_0$: Magnitude of harmonic excitation.
- $q$: Time dependent variable.
- $r$: Curvature radius.
- $W$: Spatial or time independent variable.
- $w$: Transverse displacement in $z$ direction.
- $x_v$: Length of the viscoelastic layer.
- $\Gamma$: Modal participation factor.
- $\Omega$: Excitation frequency.
- $\varepsilon$: Strain.
- $\phi$: Phase angle.
- $\eta$: Viscoelastic loss factor.
- $\theta$: Argument of the complex bending stiffness.
- $\lambda_1, \lambda_2$: Roots of characteristic equation.
- $\rho$: Mass density.
- $\sigma$: Stress.
- $\omega$: Natural frequency.
- $\omega_0(x)$: Stiffness to mass function.
- $\omega_e, \omega_{ev}$: Elastic and composite (elastic-viscoelastic) stiffness to mass ratio.
- $\psi$: Quotient of the excitation frequency to the function of stiffness to mass.
- $\xi$: Damping ratio.

ACRONYMS

- ACLD: Active constrained layer damping.
- FEM: Finite element method.
- FRF: Frequency response function.
- PCLD: Passive constrained layer damping.
- VDM: Viscoelastic damping material.
- VEL: Viscoelastic layer.

1. INTRODUCTION

Material damping is one of the most inexpensive, achievable, safe, and efficient ways to mitigate the vibration of structures, especially in aircrafts, rotating machines, vehicles, and so on. Seeking the modal parameters, i.e., natural frequency, mode shape, and damping ratio, is reckoned as a main analysing object and priority issue to assess the structural properties. On the other hand, a study on the dynamic response for various types of excitation such as transient, harmonic, and random plays a significant role to deal with real case of loadings, so that can be deemed as an essential step and mandatory procedure of structural design. In recent years, utilizing VEL (e.g., glass type composites, polyurethane foams, polymers, rubbers, etc.) on account of dissipating abilities are widespread to access the vibration attenuation and absorbing the impact loads. At first glance, the principal mission aims to reduce acoustic noise in walls, dissipation in massive structures, such as towers accompanied by some friction and hysteresis effects, or foundation of rotary machines.\(^1\) Viscoelastic (VE) materials, similar to the discrete viscous dampers, have an inherent dissipation factor which can implicitly change by time, temperature, and elastic properties.\(^2,3\) Based on the VE properties mentioned above, some of the laminates are basically composed of elastic and VELs to improve the strength and damping properties simultaneously. Therefore, one may take into account the optimization of the geometry and location of the VEL added to the elastic host structures. Moreover, plenty of structures which can be considered as beam structures with a
clamped edge, (e.g., aircraft wing, tower crane jib, power line pile) may be exposed to long-term oscillation, while there is no possibility of attaching the VEL over the entire span of the beam. Hence, this justifies the idea of utilizing partial VEL in this type of structure.

In this article, of those conditions mentioned above, the effects of the length and thickness of attached VELs on free and forced vibrations of cantilever beam is investigated, to evaluate whether adding a larger unconstrained VEL improves the beam’s damping characteristics or not. The word “unconstrained” accentuates taking advantage of bending effects rather than shear deformation of the VEL as a core between two elastic layers (constraints) and is very common in the literature.

During the past few decades, a lot of research has been carried out on dynamic analysis of structures with added VEL, and several numerical and experimental approaches have been developed. Zhou et al. reviewed various research methods and the theory calculation models that are employed in engineering to study the static and dynamic vibration characteristics of viscoelastic damping materials (VDM). Rearranging and optimizing the VELs within sandwich panels or composites is one of the interesting subjects which has been marked by some researchers. Some researchers have been focused on achieving higher energy dissipation, tuning, and vibration control for various compressional, bending, and shear vibrations, by means of analytical, numerical, and experimental methods. Vibration of structures considering VEL damping effects under stochastic excitation have been surveyed in literature. Some authors have regarded either material or geometrical nonlinearities in the presence of VELs or VE damping. Of the early studies on vibration of VE composites, is kerwin’s investigation for an elastic laminate with VE core using both theoretical and experimental method. Douglas and Yang investigated the effect of compressional damping in vibration of sandwich cantilevers composed of two elastic constraints bounding a VE core. Ardaño suggested a model containing a set of distributed VE hung cantilevers under a mass-spring system. Yamaguchi studied vibration of elastic beams with two built in edges, connected to a VED beam from mid span by a spring-damper. Inman employed the method of separation of variables and used the kernel relaxation function to analyse a single VE beam. The laplace transformation process, based on orthogonality assumptions, was introduced afterwards to express the coupling between equations due to damping.

Adhikari proposed an approach for the model-order reduction of linear multiple degree of freedom VE systems via equivalent second order systems. Karim and Chen investigated the effects of surface damping between VELs and a simply supported elastic beam. Considering nonlinearities due to the tension, the transverse vibration of VE sandwich beam was studied by Haiwei et al. Lázaro et al. surveyed vibration of VE structures with proportional damping assumptions and extracted complex eigenvalues by utilizing the fixed point iteration method. Lázaro and Pérez studied the dynamics of frame structures with VEL and utilized fractional derivative to express constitutive relations. Eigenvalues were extracted and frequency response plotted for the cases with, and without, VEL. Eigenvalues of an Euler-Bernoulli beam supported by VE solids was studied by Cha et al. and their sensitivities were approximated by using Taylor series expansion. Hu and Wang investigated dynamics of viscoelastically damped structures by eliminating higher modes in frequency response, considered contribution of the lower modes, and the first two terms of the Neumann expansion of the unavailable modes. An iterative method, by means of normal modes, was used to calculate harmonic response of VE structures. Seismic behaviour and vibrational tests of steel frame structures with added VE dampers were experimentally investigated by Chang, Won Min, and Bilbao, and their co-workers. Avcar and Saploğlu studied free vibration of steel beams with different boundary conditions and various geometrical characteristics, using Newton-Raphson and artificial neural network techniques. Effects of shear rotary inertia and non-homogeneous Young’s modulus on natural frequencies of simply supported beam was also investigated by Avcar. An analytical model of the transient time response of an impacted cantilever beam with partial constrained layer damping (PCLD) was developed by Granger and Ross, and the effects of PCLD parameters on the initial transient response was studied. Following the recent work and case study, Blais et al. studied the suppression of non-causal effects, due to time aliasing, occurring when continuous frequency spectra are discretized.

Kumar and Singh examined the effect of parametric variation of active constrained layer damping (ACLD) on the vibration control of the beams. Optimum active or passive constrained layer damping (ACLD/PCLD) patches was placed by means of closed and open loop FEM model. El Hachemi et al. proposed an intuitive computational multi-scale homogenization procedure and tool for the estimation of the effective static and mechanical properties of complex viscoelastic composite material and structures. The proposed solution consisted of numerically computing the complex effective properties (storage and loss moduli) as a function of frequency. Hågsberg investigated the free vibration characteristics of structures with viscoelastic treatment by the complex-valued natural frequencies, and modified the assumed single mode by a correction term representing the influence of residual vibration modes.

The present work, investigates the effect of the VEL geometric parameters on the damping characteristic of a cantilever beam in free and forced vibration. Modal parameters of the structure, as well as harmonic resonant response, are studied to explain their behaviour over the changes of VEL parameters. The geometry of the problem is divided into two segments of composite elastic-VEL. The governing differential equations of motion are determined based on Euler-Bernoulli beam theory for composite elastic-VE segment, and then by applying a singular step function, it is generalized for all the domain, both the composite and elastic parts.

In Section 2.1, a separation of variable technique is used to solve the Eigen problem and to extract the modal parameters. To verify and compare the analytical natural frequencies and modes, and also to find an acceptable size of the structure in which Euler beam assumptions (analytical solution) are valid, an FEM simulation (3D) is also carried out by means of commercial software.

In Section 2.2, a harmonic analysis is exploited by assuming a harmonic excitation at the tip of the structure to obtain the response and comparing for various size of VEL length and thickness.
2. GOVERNING EQUATIONS

As shown in Fig. (1), the geometry of the problem in which a partial unconstrained VEL is attached to an elastic cantilever beam (as the host or base structure), and loaded by a concentrated force at the free edge.

The elastic beam and the VEL attachment are assumed to be thin, so that the Euler-Bernoullii beam theory can be used. It is assumed that the elastic beam and VEL are completely tightened together therefore, there is no contact between them and shear forces, and delamination effects are neglected. No structural damping is considered except VEL loss effects. Considering the Kelvin-Voigt model for VEL,\(^2\) the stress-strain relationship can be expressed by the following relation:

\[
\sigma = E\varepsilon + \eta E\varepsilon/dt;
\]

where \(\sigma\) and \(\varepsilon\) are the stress and strain, \(E\) is the Young modulus, and \(\eta\) is the damping coefficient. In the case of bending of the VE beam, Hooke’s law states

\[
\sigma_x = E_v^* \varepsilon_x;
\]

where \(E_v^*\) denotes the complex modulus of VE material as:

\[
E_v^* = E_v \left(1 + \frac{\partial}{\partial l}\right);
\]

Using the force and moment equilibrium equations, and strain-curvature relation, one can express the general equation of motion along the structure by:

\[
\left(E_v I_v + E_{v'} I_{v'} (x_v - x_v^0)^0\right) \frac{d^4 W}{dx^4} = \\
\left(\eta E_v I_v (x_v - x_v^0)^0\right) \frac{d^4 W}{dx^4} = \\
\left(m_v + m_v (x_v - x_v^0)^0\right) \frac{d^2 W}{dx^2} = p(x, t);
\]

where \(w\) represents the transverse deflection and subscript indices "\(v\)" and "\(v'\)" denote VEL and elastic properties, respectively. \(M\) and \(I\) are mass per unit length and moment of inertia, and \(p\) is the distributed load. The operator \(< x_v - x >^0\) is a step function, the value of which switches between 0 for \(x_v < x \leq l_v\), and 1 for \(0 < x \leq x_v\), whereby the distribution of VE bending stiffness over the longitudinal direction can be piece-wisely figured.

2.1. Free Vibration

Solution of Eq. (4) in the absence of external loads results in an eigenvalue problem and the extraction of natural frequencies and mode shapes. Applying the method of separation of variables (i.e., \(w(x, t) = W(x) \cdot T(t)\)) to Eq. (4), one can obtain the following two differential equations:

\[
\frac{d^2 W}{dt^2} + 2\zeta \omega \frac{dW}{dt} + \omega^2 W = 0;
\]

\[
\frac{d^4 W}{dx^4} - \left(\frac{\omega}{\omega_0(x)}\right)^2 W = 0;
\]

where \(\omega\) is the natural frequency, and \(\zeta\) and \(\omega_0\) are the damping ratio and stiffness to the mass (inertia) ratio, respectively. Dividing the domain into elastic and composite (elastic-VEL) parts, the solution of Eq. (8), can be written as:

\[
W_{|0<x\leq x_v} = W_{ev} = C_{11} \cos \lambda_1 x + C_{12} \sin \lambda_1 x + C_{13} \cosh \lambda_1 x + C_{14} \sinh \lambda_1 x; \quad \text{(7a)}
\]

\[
W_{|x_v<x\leq l_v} = W_e = C_{21} \cos \lambda_2 x + C_{22} \sin \lambda_2 x + C_{23} \cosh \lambda_2 x + C_{24} \sinh \lambda_2 x; \quad \text{(7b)}
\]

where subscript ‘\(ev\)’ and ‘\(e\)’ stand for elastic-VE and elastic parts, respectively, \(\lambda_1\) and \(\lambda_2\) are the roots of the characteristic equation of Eq. (6) which correspond to \(\omega_0\) of each segment. \(C_{ij}\) are the coefficients that can be determined using boundary condition. In order to find these coefficients, one should apply the four boundary conditions of the cantilever beam as:

\[
W_{ev} |_{x=0} = \left. \frac{dW_{ev}}{dx} \right|_{x=0} = \left. \frac{d^2W_{ev}}{dx^2} \right|_{x=0} = \left. \frac{d^3W_{ev}}{dx^3} \right|_{x=0} = 0. \quad \text{(8)}
\]

Besides, four continuity conditions are necessary to apply in transition region of jointing elastic to composite segment as \((x = x_v)\):

\[
W_e = W_{ev}; \quad \text{(9a)}
\]

\[
\left. \frac{dW_{ev}}{dx} \right|_{x=x_v} = \left. \frac{dW_e}{dx} \right|_{x=x_v}; \quad \text{(9b)}
\]

\[
\left. \frac{k_{ev}}{dx^2} \right|_{x=x_v} = \left. \frac{k_e}{dx^2} \right|_{x=x_v}; \quad \text{(9c)}
\]

\[
\left. \frac{k_{ev}}{dx^3} \right|_{x=x_v} = \left. \frac{k_e}{dx^3} \right|_{x=x_v}; \quad \text{(9d)}
\]

where \(k_e\) and \(k_{ev}\) denote the elastic and composite bending stiffness, respectively. Expressing Eq. (6) in terms of Eq. (7), and applying the conditions in Eqs. (8) and (9), leads to an algebraic set of equations shown by Eq. (10).

The determinant of the left-hand side matrix in the recent relation, must be zero for the existence of the non-trivial solution, which leads to a characteristic equation in terms of \(\omega\). In fact, all the components of matrix are known by the natural frequency \(\omega\). (i.e., each component is a trigonometric and hyperbolic functions of \(\omega\)). Hence, it is sufficient to enter the so-called known inputs and use an arbitrary determinant technique to obtain characteristic equation in terms of \(\omega\), and then finding zeros or intersections, with the \(x\) direction vector. In the present work, simple programming has been carried out in Maple-13 to obtain characteristic equation and zeros.
2.2. Forced Vibration

Considering harmonic force \(p_0 e^{j\Omega t}\) at the tip of cantilever and dividing response to two segments (e.g., composite and only elastic region), one can take the response as the following form:

\[
W(x, t) = W(x, \Omega)e^{j(\Omega t - \phi)} = (W_{ev}(0 < x < x_e, \Omega) + W_{ce}(x_e < x < l_e, \Omega))e^{j(\Omega t - \phi)};
\]  

where \(\Omega\) denotes excitation frequency, \(\phi\) is the phase difference due to damping (ineffective in computations), \(W\) is the response amplitude in terms of frequency and position, and \(j\) denotes imaginary part. Applying the above solution to Eq. (4), results in the following equation:

\[
- \left( m_e + m_v \langle x_v - x \rangle \right) \Omega^2 W + (E_e I_e + E_v I_v) \langle x_v - x \rangle + j \Omega \Omega_1 E_e I_e \langle x_v - x \rangle^0 W_{xxx} = 0.
\]  

(12)

It must be noted that the external force acts as the boundary load. Solution of the recent equation can be obtained by defining complex roots \(\lambda\) as:

\[
\lambda_n = \sqrt[\pm j n + \phi]p_{0}, \quad n = 0, 1, 2, 3;
\]  

where \(\psi\) can be defined by

\[
\psi(x) = \Omega^2 / [K(x)] m(x);
\]  

(14)

where \(K\) is complex bending stiffness with argument \(\theta\) (see Ref.39). Assuming complex roots of Eq. (12), and no structural damping in the elastic segment, leads \(\theta\) to be zero in this region and the solution can be expressed by the following complex form:

\[
W_{ev} = a_1 \cos \left( \psi_{ev} \sin \left( \frac{\theta}{4} \right) x \right) \cosh \left( \psi_{ev} \cos \left( \frac{\theta}{4} \right) x \right)
+ a_2 \cos \left( \psi_{ev} \cos \left( \frac{\theta}{4} \right) x \right) \sinh \left( \psi_{ev} \sin \left( \frac{\theta}{4} \right) x \right)
+ a_3 \sin \left( \psi_{ev} \sin \left( \frac{\theta}{4} \right) x \right) \cosh \left( \psi_{ev} \cos \left( \frac{\theta}{4} \right) x \right)
+ a_4 \sin \left( \psi_{ev} \cos \left( \frac{\theta}{4} \right) x \right) \sinh \left( \psi_{ev} \sin \left( \frac{\theta}{4} \right) x \right);
\]  

(15a)

\[
W_{ce} = a_5 \cos \psi_{ce} x + a_6 \sin \psi_{ce} x + a_7 \cosh \psi_{ce} x + a_8 \sinh \psi_{ce} x;
\]  

(15b)

where \(\psi_{ce}\) and \(\psi_{ev}\) correspond to the elastic and composite segments in Eq. (14), respectively. In a similar manner to the previous section, after applying the boundary and continuity conditions, constants \(a_1\) to \(a_8\) can be obtained from the following algebraic set of equations, presented in Eq. (16), where \(p_0\) is the magnitude of concentrated harmonic load at the tip of elastic beam, \(K_{ce}\) and \(K_e\) are complex and real bending stiffness corresponding to the composite and elastic segments, respectively. Rest of the symbols are defined by:

\[
c = \cos; \quad s = \sin; \quad ch = \cosh; \quad sh = \sinh; \quad \psi_{ev} = \psi_{ev} \cos \left( \frac{\theta}{4} \right); \quad \psi_{ce} = \psi_{ce} \sin \left( \frac{\theta}{4} \right).
\]  

Solving the above complex system results in the frequency response function (FRF) and phase of the structure.
3. RESULTS AND DISCUSSIONS

To verify the analytical formulations presented in previous section, several numerical case studies were examined. The analytical solution was carried out using MAPLE software, and compared with the finite element results using commercial software. An aluminium cantilever beam having 40 mm × 40 mm cross section and 2000 mm length, is considered. The beam has a Young’s modulus of 70 GPa, Poisson’s ratio of 0.33 and density of 2700 kg/m³. The VEL material is made of G11/Glass (see Ref.28, 29) with modulus of elasticity, density, and poison’s ratio of 181 GPa, 1500 kg/m³ and 0.3, respectively. It has the same cross sectional area as the aluminum beam with a variable length. The finite element model consisted of 100 solid elements for the beam and another 100 solid elements for the attached VEL. The analytical and FEM solutions has been carried out by Maple-13 and ANSYS-APDL12, respectively. The corresponding first three natural frequencies of the beam with attached VEL are presented in Table 1, for some particular characteristics and geometries. As seen from the table, the results obtained by the analytical method are in very good agreement with those obtained by FEM.

![Figure 2](image_url)

Figure 2. Variation of the first natural frequency versus the VEL size for \( t_x = 0.04 \text{ m, } l_x = 2 \text{ m}. \)

The analytical and FEM results are also presented in Fig. (2), with different length and thickness ratios. Since the structure has been modelled using 3-D solid elements, increasing the VEL thickness, causes some discrepancies compared to 1-D analytical results. Based on finite strip (wide beam) and shear deformation (thick beam) theories, restrictions of appli-
cable size for which the exact solution is valid can be appropriated for the length and thickness of VELs with errors less than 1 percent.

As demonstrated in Fig. (2), there is an acceptable alignment in two diagrams for relatively long and thin partial VEL. Comparisons of the first three mode shapes between FEM and the analytical solutions are shown in Fig. (3). Only the transverse modes of the 3D modelling in ANSYS are considered here. The results illustrated in Fig. (4) point out the frequency ratio diagram versus the VEL size variation. Hence, increasing the VEL length up to the half of the elastic beam span, causes an increase in fundamental frequency (dashed line) where beyond this region it starts to decrease. Since the fundamental frequency of a cantilever beam is proportional to the inverse of the square length, a frequency decline is expected due to increasing the length. But, in the discussed structure (elastic-VEL composite), fundamental frequency growing due to length increasing (for a length ratio less than 0.6) is related to the contribution of VEL and the amount of its participation to the response. On the other hand, the effect of increasing the total thickness of a structure (composite segment) is dominant in comparison with increasing the VEL length. This is more observable in higher thickness ratios, as shown in the diagram. The diagram for the second mode (dashed-point line) is clustered to one valley and two peaks on both sides, and maximum frequency occurs almost at the quarter and three quarters of length ratios.

In general, adding partial VEL to an elastic beam, causes an increase in the thickness and length of the composite segment; the former will result in an increase in natural frequencies, while the latter will cause the reduction of natural frequencies. Therefore, it is either a thickness or length increase that dictates how the frequency must change, so that reconciliation makes maximum frequency for length ratio about 0.6, and for any arbitrary geometrical size of the elastic part and VEL thickness, where shown in Fig. (4) and Fig. (5). According to the stiffness to mass ratios \( \omega_{ev}, \omega_{e} \), corresponding to the elastic and composite segments respectively, one can consider this as a scale or evaluation criterion of natural frequencies in structures, so that multiplying each by participation factors:

\[
\Gamma_{ev} = \left( \int_{0}^{l_e} \frac{W_{IV}^{eV}}{W_{ev}} dx \right)^{\frac{1}{2}} ;
\]

\[
\Gamma_{e} = \left( \int_{0}^{l_e} \frac{W_{IV}^{eV}}{W_{e}} dx \right)^{\frac{1}{2}} ;
\]

results in exactly the natural frequencies (i.e., \( \omega = (\Gamma_{ev}\omega_{ev})_{|x=x_{ev}/l_e} = (\Gamma_{e}\omega_{e})_{|x=x_{ev}/l_e} \)). Increasing the length ratio causes a decrease in \( \omega_{ev} \) and an increase \( \omega_{e} \), respectively, and that is the corresponding participation factor which specifies quantity of the natural frequency as a function of geometry. As a result, for the length ratio about 0.6, this is the aforesaid factor which determines destiny of the maximum frequency in this region.

The magnitude of FRF at the tip of the structure is shown in Fig. (6), for various loss factors. In higher frequency range, utilizing VEL even with a poor loss factor, makes a significant decrease in the response amplitude. For the first resonant frequency of 14.22 Hz and a loss factor of \( \eta = 0.047 \), damping ratio equals to the critical value of \( \zeta = 1 \) and therefore, for frequencies or loss factors a bit more than these, over damping occurs.

Impedance phase angles concerning the first three modes are presented in Fig. (7). As seen from the figure, there is a \( 2\pi \) difference between responses with higher and less than first resonant mode. The figure also shows that, the next \( 2\pi \) phase differences do not occur exactly at the \( 2^{\text{nd}} \) and \( 3^{\text{rd}} \) resonant modes (e.g., 62.80 Hz and 181.00 Hz), because, as mentioned, the critical damping appears for the cases with lower loss factors. However, varying the dimension of the VEL makes no instrumental sense to compare and evaluate the results to deduct which size of VEL represents higher damping performance. Hence, it will be helpful utilizing the FRF peak at resonant mode of each size (rather than expressing in terms of frequency) and then comparing to each other.

Figure (8), shows the variation of the ratio of the FRF peak at the first resonant mode \( H(\omega_1) \), to static level \( H(0) \), for various length ratios at several thickness ratios.

For any VEL size, minimum ratio happens when the length ratio is rising to about 0.6; i.e., for VEL with a length about half (or a little more than half) of the size of the elastic beam. The diagrams for various thicknesses in this region \( (x_e/l_e < 0.6) \) are descending and, in spite of intersecting each other at some particular length ratios, totally increasing the thickness has a positive effect in vibration amplitude reduction.

As a matter of fact, a thicker structure naturally has a lower transverse deflection, therefore, intersections can be depicted such that the static response or denominator of the ratio, is always descending versus increasing the thickness or length ratio. On the other hand, although the numerator (first resonant response) decreases as the length ratio increases, for the length ratio less than 0.6, the static stiffness has a dominant effect on decaying the amplitude than the dynamic damping. Thus, the numerator does not decrease as much as the denominator in this region. Nevertheless, it does not mean the smaller thickness of VEL is better in decaying the resonant amplitude.

Contrarily, for a length ratio more than about half \( (x_e/l_e > 0.6), \) meaning longer VEL up to the elastic length, the amplitude begins to increase and there is no intersection for various thicknesses in this region. It simply means using a larger VEL does not necessarily guarantee a further amplitude decay, though, it contributes to decrease the total static deflection and the strength of structure.

4. CONCLUSIONS

The modal parameters of an elastic beam with a partial unconstrained VEL were analytically obtained based on the Kelvin-Voigt VEL model and the Euler-Bernoulli beam theory. Applicable size and dimension range for which analytical solutions and assumptions have been valid, were detected by comparing to the FEM results.

An irregular variation was observed in natural frequencies versus VEL size. It was shown that the natural frequencies are proportional to the stiffness to mass ratio, as well as the participation factor related to the contribution of the VEL length in mode shapes.
The fundamental frequency increases via enlarging the VEL to about 60 percent of the elastic beam span, whereas by lengthening more than this, the frequency gradually begins to decrease. As a result, adding a larger VEL does not necessarily increase the fundamental frequency.

Moreover, frequency response, phase angle, and first resonant FRF ratio, were analytically extracted. It was demonstrated that the amplitude of the structure response always decreases by increasing the loss factor as well as VEL thickness. The first resonant peak meets its minimum for VEL length of about 0.6 of the elastic beam span, whereas for longer VEL, it shows a growing amplitude because of the higher participation of longer VEL in this region, where decreases the natural frequency and damping ratio.
REFERENCES


Efficient Model for Acoustic Attenuators using the Method of Fundamental Solutions

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In this work, the three-dimensional formulation of the Method of Fundamental Solutions (MFS) is applied to model acoustic problems in the frequency domain. This formulation is developed by making use of adequate Green’s function defined by the image-source technique, reducing the discretization necessary for the definition of the numerical model. The proposed approach is applied to study the sound attenuation provided by an acoustic attenuating device, consisting of a closed acoustic space located between inlet and outlet tubes. Absorbent properties of the lining materials of the acoustic device, defined using laboratory measurements of their absorption coefficients, are incorporated into the model. The proposed model is verified against reference numerical models based on a boundary integral equation formulation. An experimental validation is also performed, demonstrating that the model adequately simulates the sound propagation under experimental conditions. Numerical applications are then presented to demonstrate the behaviour of the system under different conditions.

1. INTRODUCTION

The sound field inside closed acoustic spaces is dependent on several factors, such as the excitation frequency of the sound source, its amplitude, its geometry and sound absorption properties. Therefore, several numerical approaches have been developed to deal with this type of problem. Some of these approaches have been applied to the study of attenuating devices that are currently used, for example, in duct systems or in vehicles to reduce the sound emissions and increase user comfort. Indeed, the sound propagation through attenuators is of special practical interest, since these devices are of great importance to reduce the sound that reaches the inhabited spaces within buildings, minimizing the impact of the mechanical noise produced by ventilators and by airflow. Their study has been the subject of research using approaches varying from simplified theoretical formulas based on surface and transfer impedance concepts to much more complex numerical models based on finite elements or boundary elements techniques.

Many analytical and numerical methods were widely used to predict the acoustic attenuation performance of expansion chamber mufflers with the effects of mean flow and higher-order modes (see, for example, the works of Ih and Lee" and Ji et al." for further details on these effects in duct systems). Expansion chambers were also analyzed by Selamet and Ji" using both analytical, numerical, and experimental techniques for assessing the importance of extended inlets/outlets in the case of circular expansion chambers. Later, Xu et al." analyzed these devices in the presence of internal lining using different absorptive materials, making use of numerical techniques and validating the method experimentally. Kim and Ih" also evaluated the performance of lined plenum chambers with rectangular shape, making use of a Rayleigh-Ritz approach. In both works, it was concluded that the thickness and the type of lining play a fundamental role in the final performance of the chamber.

In what concerns silencers or attenuators, usually inserted into HVAC (abbreviation for Heating, Ventilation, Air Conditioning) networks to enable the reduction of noise that reaches climatized spaces, while allowing the adequate air flow through the piping system, many researchers have developed modelling strategies, using different approaches. Some examples are the publications from Wu et al." proposing boundary element formulations for the analysis of packed silencers, from Lee" which analyzed in detail the performance of hybrid and dissipative mufflers using the Boundary Element Method (BEM), and from Denia et al." who analyzed the performance of silencers using Finite Element Methods (FEM) and an analytical approach. Many works have also focused on the development of the BEM and its variants for these types of problems, including more intricate acoustic models" and proposing highly efficient numerical approaches to tackle large scale problems. Recently, Hua et al." demonstrated how transmission and insertion loss for multi-inlet mufflers can be determined using impedance matrix and superposition approaches, and Yang et al." combined the BEM and the point collocation approach to calculate the transmission loss of silencers in the absence of mean flow and temperature gradient.

The development, mostly in the last 20 years, of Meshless Methods has also widened the range of options to simulate sound propagation in that type of configuration. The MFS is one these numerical methods found in the literature. As with the BEM, the MFS is also based on the use of fundamental solutions, which are, themselves, solution of the governing equation of the problem, but not requiring the numerical and...
analytical integrations that need to be performed in boundary integral equation method (hence the singular or hyper-singular integrals are not evaluated). Works such as those by Godinho et al., Antnio et al. or Godinho et al. have shown that, despite its simplicity, the MFS is accurate and constitutes a good tool for simulation of physical systems.

The MFS has attracted great interest of scientists and researchers, due to its simple mathematical formulation that only requires prior knowledge of the fundamental solutions. The linear superposition of these fundamental solutions were employed to approximate the solution of the problem, assuming the virtual sources to be located outside the domain to avoid singularities in the analyzed space. Many researchers have applied the MFS to problems of acoustics, such as Kondapalli et al., which were among the first to apply the MFS to the Helmholtz equation in the analysis of acoustic scattering in fluids and solids. Fairweather et al. have also described and analyzed the MFS for the numerical solution to the acoustic scattering problem in fluid and solid media. Alves and Valtchev compared the plane wave method and MFS for acoustics waves scattering and concluded that the first method can be seen as an asymptotic case of the MFS.

One of the main difficulties in the application of the MFS was the determination of the position of the pseudo-boundary, on which the singularities in terms of which the approximation is expressed, were placed. Therefore, Karageorghis has proposed a simple practical algorithm for determining an estimate of the pseudo-boundary which yields the most accurate MFS approximation when the method is applied to certain boundary value problems. Later, Godinho et al. analyzed the behaviour of the MFS numerical frequency domain approach with regard to stability, accuracy, and efficiency, applying strategies for improving stability and accuracy of the method such as the use of different distributions of collocation points and virtual sources or a singular value decomposition (SVD) solver. These strategies allow for the conclusion that the concentration of the sound field inside closed acoustic spaces in the frequency domain.

In the present paper, the authors make use of a numerical approach in the frequency domain, based on the three-dimensional formulation of the MFS to calculate the attenuation of the sound field within an acoustic attenuating device. Here, the effect of airflow is not considered. The proposed model considers the symmetry of the problem, modelling thus only a quarter of the numerical model. Moreover, the inner boundary (symmetry planes) of the proposed model require no discretization due to the use of a Green’s function which incorporates the appropriate boundary conditions related to symmetry of the problem. The absorbing properties of a possible lining material are incorporated in the numerical model, by means of impedance (Robin) boundary conditions. To validate the numerical implementation of the MFS model, the results are compared with the results obtained in laboratory tests. Finally, a study detailing the behaviour of the acoustic silencer through a set of simulations of its performance considering different absorbing properties of the materials is presented. In this study, the possibility of using lining materials previously characterized by means of laboratory measurements of their absorption coefficients is also discussed.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

2.1. Sound Propagation in 3D Closed Space

The propagation of sound within a three-dimensional space can be mathematically represented in the frequency domain by the Helmholtz differential equation:

$$\nabla^2 p(x) + k^2 p(x) = 0;$$  \hspace{1cm} (1)

where \( p(x) \) is acoustic pressure, \( k = \omega/c \) is the wave number, with \( \omega \) being the angular frequency and \( c \) the sound propagation velocity within the acoustic medium.

Assuming a source point placed within this propagation domain, at point \( x_0 \) with coordinates \((x_0, y_0, z_0)\), it is possible to establish the fundamental solution for the sound pressure \( G \) and particle velocity \( H \) at a point \( x \) with coordinates \((x, y, z)\), which can be written respectively as:

$$G(x, x_0, k) = \frac{e^{-ikr}}{4\pi r};$$  \hspace{1cm} (2)

$$H(x, x_0, k, n) = \frac{1}{\rho c} \frac{e^{\pm ikr}}{4\pi r^2} \frac{\partial}{\partial n};$$  \hspace{1cm} (3)

where \( r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} \) represents the direction along which the particle velocity is to be calculated, \( \rho \) is the density of the acoustic medium and \( i = \sqrt{-1} \).

2.2. Formulation of the Method of Fundamental Solutions

Using the Method of Fundamental Solutions, the frequency domain response inside a 3D enclosure is computed as a linear combination of fundamental solutions for a set of virtual sources. These sources are placed outside the domain of interest, to avoid singularities. Thus, at point \( x \) within the propagation domain, the sound pressure field can be written as:

$$p(x, k) = \sum_{j=1}^{NV_S} A_j G(x, x_j, k);$$  \hspace{1cm} (4)

where \( x_j \) represents the coordinates of the virtual sources \((x_j, y_j, z_j)\), \( A_j \) are the unknown amplitudes of the virtual sources which are computed by imposing the appropriate boundary conditions at a set of \( NCP \) points (collocation points) placed along the surfaces, \( NV_S \) is the number of virtual sources and \( G(x, x_j, k) \) is the fundamental solution at point \( x \) for a virtual source placed at \( x_j \). The particle velocity field can be expressed in a similar way. In this work an equal number of collocation points and virtual sources is assumed, which allows obtaining a \( NV_S \times NV_S \) system. This system is built by prescribing at each collocation point \( x \), placed in the boundaries of the closed space, Neumann and Robin conditions. By applying this procedure one obtains:

$$\sum_{j=1}^{NV_S} A_j H(x, x_j, k, n) = v_n(x, k);$$  \hspace{1cm} (5)

$$\sum_{j=1}^{NV_S} A_j G(x, x_j, k) = \tilde{Z} \sum_{j=1}^{NV_S} A_j H(x, x_j, k, n);$$  \hspace{1cm} (6)
where $\vec{v}_n(x,k)$ is the normal component of the particle velocity and $\vec{Z}$ is the surface impedance.

If the problem involves more than one sub-domain, then the MFS equation (Eq. (4)) must be written for each sub-domain and continuity conditions must be enforced at interfaces between sub-domains. These continuity conditions can be written as:

$$
\begin{align*}
  p^+ &= p^- \quad \text{in } S_c; \\
  \vec{v}_n^+ &= -\vec{v}_n^- \quad \text{in } S_c;
\end{align*}
$$

where $S_c$ is a common interface between the sub-domains, $p$ is the acoustic pressure, and $\vec{v}_n$ is the particle velocity along the normal direction pointing outwards of each sub-domain (i.e. with opposing directions between sub-domains).

In this work, the proposed model has symmetry in relation to the two planes $xy$ and $xz$, as shown in Fig. 1a. Due to this symmetry, it is possible to reduce the discretization to a quarter of the model if the following Green’s function is assumed, written as:

$$
G(x, x_0, k) = \frac{e^{-ikr_1}}{4\pi r_1} + \frac{e^{-ikr_2}}{4\pi r_2} + \frac{e^{-ikr_3}}{4\pi r_3} + \frac{e^{-ikr_4}}{4\pi r_4};
$$

where:

$$
\begin{align*}
  r_1 &= \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}; \\
  r_2 &= \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z+z_0)^2}; \\
  r_3 &= \sqrt{(x-x_0)^2 + (y+y_0)^2 + (z-z_0)^2}; \\
  r_4 &= \sqrt{(x-x_0)^2 + (y+y_0)^2 + (z+z_0)^2}.
\end{align*}
$$

To model the proposed problem, we considered that the problem to be solved incorporated three sub-domains: $\Omega_1$ and $\Omega_2$ corresponding to the entrance (inlet) and exit (outlet) of the silencer; $\Omega_3$ corresponding to the inner domain of the silencer. The inlet and outlet surfaces of the silencer are designated as $S_1$ and $S_2$, respectively, while the inner surface of the silencer is designated as $S_3$; the common interface (where continuity of pressure and velocity is imposed) between sub-domains $\Omega_1$ and $\Omega_3$ is designated as $S_{C1}$ and the common interface between sub-domains $\Omega_2$ and $\Omega_3$ is designated as $S_{C2}$. As illustrated in Fig. 1b.

In this numerical model, the excitation source is introduced by means of ascribing a boundary condition of $v_n = 10^{-3}$ m/s along the direction of the inward normal, at the inlet tube; on the opposite face (outlet tube), an anechoic termination is considered by $p/v_n = \rho c$.

For this configuration, two variants were considered. The first one considers totally rigid internal surfaces for the silencer, and thus the corresponding boundary conditions can be written as:

$$
\begin{align*}
  v_n^{\Omega_1} &= 0 \text{ m/s in } S_1; \\
  v_n^{\Omega_2} &= 0 \text{ m/s in } S_2; \\
  v_n^{\Omega_3} &= 0 \text{ m/s in } S_3.
\end{align*}
$$

In the second, the internal surfaces of the silencer are considered to be lined with an absorbive material, and thus the corresponding boundary condition can be written as:

$$
\begin{align*}
  p^{\Omega_1}/v_n^{\Omega_1} &= \bar{Z} \text{ in } S_1; \\
  p^{\Omega_2}/v_n^{\Omega_2} &= \bar{Z} \text{ in } S_2; \\
  p^{\Omega_3}/v_n^{\Omega_3} &= \bar{Z} \text{ in } S_3.
\end{align*}
$$

where $\bar{Z}$ is the impedance of the material calculated by an absorption coefficient. This coefficient can be evaluated experimentally, appealing, for example, by impedance tube method.

In both analyses, the equilibrium and continuity condition of the two interfaces between sub-domains of the numerical model can be written as:

$$
\begin{align*}
  p^{\Omega_1} &= p^{\Omega_3} \text{ in } S_{C1}; \\
  v_n^{\Omega_1} &= -v_n^{\Omega_3} \text{ in } S_{C1}; \\
  p^{\Omega_2} &= p^{\Omega_3} \text{ in } S_{C2}; \\
  v_n^{\Omega_2} &= -v_n^{\Omega_3} \text{ in } S_{C2}.
\end{align*}
$$

Considering that $NVS_1$ virtual sources are distributed around the domain $\Omega_1$, $NVS_2$ virtual sources are placed around the domain $\Omega_2$, and that $NVS_3$ virtual sources are placed around the domain $\Omega_3$, the acoustic pressure field inside each domain can be written as:

$$
\begin{align*}
  p(x,k)_{\Omega_1} &= \sum_{j=1}^{NVS_1} A_j G(x, x_{1,j}, k) \quad \text{for } x \text{ in } \Omega_1; \\
  p(x,k)_{\Omega_2} &= \sum_{j=1}^{NVS_2} B_j G(x, x_{2,j}, k) \quad \text{for } x \text{ in } \Omega_2; \\
  p(x,k)_{\Omega_3} &= \sum_{j=1}^{NVS_3} C_j G(x, x_{3,j}, k) \quad \text{for } x \text{ in } \Omega_3.
\end{align*}
$$

where $A_j$, $B_j$, and $C_j$ are the amplitudes of the virtual sources placed around the domains $\Omega_1$, $\Omega_2$, and $\Omega_3$.

Assuming that the number of collocation points exclusively in $S_1$, $S_2$ and $S_3$ is $NPC_{S1}$, $NPC_{S2}$ and $NPC_{S3}$, respectively, and that the number of collocation points on the coupling interfaces $S_{C1}$ and $S_{C2}$ is $NPC_{BC1}$ and $NPC_{BC2}$, respectively, after imposing the boundary conditions, a $(NPC_{S1} + NPC_{S2} + NPC_{S3} + 2NPC_{BC1} + 2NPC_{BC2}) \times (NFV_1 + NFV_2 + NFV_3)$ system of equations may be established, allowing the calculation of the $A_j$, $B_j$, and $C_j$ unknowns.
3. MODEL VERIFICATION

Previous works by the authors Godinho et al. have shown that the accuracy of the response computed using a 3D MFS model depended on the position of the virtual sources used to simulate the pressure field. The proposal for defining this distance as a function of the average spacing between neighboring collocation points was previously found to be a good strategy; following previous results in Godinho et al., positioning the virtual sources at a distance that is between 4 and 8 times this spacing was found to be a good compromise between accuracy and stability in 3D configurations. Given this framework, a preliminary study will be performed here, verifying the behaviour of the proposed method against a reference solution based on the boundary integral equation for a geometric configuration of the acoustic silencer. With the purpose of confirming the adequacy of the position of the virtual sources, and to ensure that meaningful results will be obtained, 4 times the average spacing between collocation points will be employed. Thus, it is considered a parallelepiped space with internal dimensions of 0.60 m × 0.40 m × 0.25 m, filled with air, with density of 1.22 kg/m³, and sound propagation velocity of 340 m/s; connected to this space, an inlet and an outlet tube are considered. Assume that all the surfaces of this space are totally rigid, with null normal particle velocities, except in the entrance of the device, where the boundary condition is \( v = 1 \) m/s, and in the exit of the device, where the prescribed boundary condition is \( v = 0 \) m/s (rigid termination).

A schematic representation of the numerical models can be found in Fig. 2. For the MFS model was used a maximum of 554 collocation points, while that for the BEM model a maximum of 880 boundary elements was used.

The proposed MFS model has been used to compute the acoustic pressure inside this system, dividing the propagation medium in three separate domains, and coupling them by imposing full continuity of pressures and particle velocities at the interfaces. In this model, it is assumed a collocation point distribution based on Chebyshev points, closing the extreme points located at the boundary of the surfaces. The basic expression that allows defining these points can be written for the one-dimensional case and, considering \( npts \) points to be distributed along a line with unit length, as:

\[
x_j = 0.5 + 0.5 \cos \left( \frac{j \pi x}{npts + 1} \right), \quad \text{with } j = 1, ..., npts.
\]

In order to illustrate the responses obtained in the verification, Fig. 3 displays the comparison between MFS and BEM results of the sound reduction in the tested configuration. The calculations were performed by assuming frequencies up to 2000 Hz and a 10 Hz frequency step. Analysis of the numerical results clearly confirmed that there was an excellent agreement between the two solutions, meaning that the proposed MFS model is accurate in the calculation of the pressure field within the proposed geometry.

In the results shown in Fig. 3, a significantly smaller number of collocation points were employed for the MFS model; this reduction clearly decreased the computational requirements in terms of memory and time while still ensuring good accuracy. Additionally, it must be stressed that no integrations need to be performed, and thus the MFS model can be very efficient from the computational point of view. This behaviour further justified the use of the MFS in these types of problems.

4. EXPERIMENTAL VALIDATION

In order to assess the practical applicability of the MFS model in the simulation of sound propagation, it is important to compare the results it provides with those that can be measured experimentally. For this purpose, a simple experimental setup was developed, and subjected to the incidence of sound waves. The sound waves are introduced in this system through an inlet tube with a circular cross-section (with a diameter of 0.05 m) centred on one of the faces of the enclosure. The use of an inlet tube with an increased length (0.90 m) allows for adequate control of the excitation introduced into the system, ensuring that this excitation can be adequately represented as a plane wave. On the face opposite to the source, an exit tube was installed, formed by a short PVC tube (0.30 m), with a diameter of 0.05 m, with an anechoic termination materialized by a block of mineral wool 0.04 m thick, with a density of 70 kg/m³. The sound pressure level was then measured at the entrance (within the impedance tube) and in the exit tubes at opposite faces of the model by means of two microphones.

The signal is acquired using a 2 channel “Symphonie” acquisition system, from 01dB, connected to two GRAS Sound & Vibration 40AF microphones. These are free-field microphones subject to an incidence at an angle of 90°, and thus a corrective term should be introduced at each of them, obtained from the characteristic curve provided by the producer (GRAS Sound & Vibration). However, for this specific microphone...
model, the correction factor is approximately null up to 3 kHz, and thus it does not influence the measured sound pressure levels within the analyzed frequency range. The response of this system when the impedance tube generates a broad band noise is registered during 6 s, and this test is repeated 5 times. Average SPLs are calculated at each microphone for frequencies up to 2000 Hz. An average SPL reduction is then calculated as the differences between the average SPLs are registered at the entrance and exit tubes. A schematic representation of this model, including its dimensions, can be found in Fig. 4.

This geometry was also modelled to make use of the MFS model considering the symmetry of the problem in relation to the planes $xy$ and $xz$, modelling only 1/4 of the numerical model. In addition, the inner boundary (symmetry planes) of the problem did not need to be discretized, due to use of the Green’s function (see Eq. (8)) that considers the boundary conditions of the symmetry of the problem. 600 collocation points are used to model the proposed configuration.

The sound reduction (SR) is calculated by the difference between the registers of the SPLs in the entrance and exit tubes of the acoustic silencer, written as:

$$SR = SPL_1 - SPL_2, \text{ (dB)};$$

where $SPL_1$ is the sound pressure level in the entrance of the acoustic silencer and $SPL_2$ is the sound pressure level in the exit of the acoustic silencer.

Figure 5 illustrates the comparison between experimental and numerical results computed for the defined configuration. Observing this figure, it is possible to conclude that, in general, there is a good agreement between the measured results and those estimated using the numerical model. In fact, the two curves have very similar trends, with the position of peaks and valleys matching well between both. Some discrepancies occur in what concerns the involved amplitudes, a situation which can be justified by small differences between the experimental apparatus and the modelled geometry.

5. NUMERICAL APPLICATIONS

In order to analyze the mentioned device in greater detail, a set of simulations was performed to analyse its behaviour. In these simulations, the SR was evaluated considering the average of the sound pressures in several receivers localized in the entrance and exit of the acoustic silencer, written as:

$$SR = 20 \log \left( \frac{\sum_{i=1}^{nrec_1} |(p_1)_i|/nrec_1}{2.10^{-5} \text{Pa}} \right) - 20 \log \left( \frac{\sum_{i=1}^{nrec_2} |(p_2)_i|/nrec_2}{2.10^{-5} \text{Pa}} \right) \text{ (dB)}; \quad (16)$$

where $p_1$ is the sound pressure in the entrance, $p_2$ is the sound pressure in the exit of the silencer; $nrec_1$ and $nrec_2$ are the number of receivers localized in the entrance and exit of the device, respectively. In this analysis, two sets of 16 receivers were considered in the entrance and in the exit of the silencer, equally spaced of 0.01 m along the $x$-direction (see Fig. 6).

Two different boundary conditions of the acoustic domain of the silencer were considered: the first corresponding to Neumann condition, which consists in considering null velocities in all internal walls of the device; and the second corresponding to Robin conditions able to simulate an absorption coefficient.
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Figure 7. Numerical curves of the sound reduction for a device with a cross section of 0.4 m × 0.4 m, with different lengths: (a) device with rigid walls; (b) device with absorbing walls ($\bar{\alpha} = 0.2$).

Figure 8. Numerical curves of the sound reduction for varying cross-section dimensions, with $L_1 = 0.6$ m for a device with absorbing walls ($\bar{\alpha} = 0.2$).

Figure 9. Numerical curves obtained in a device with dimensions 0.6 m × 0.4 m × 0.25 m, for the sound reduction for different value of the absorption coefficient ($\bar{\alpha}$).

A generic model is here considered, as described in Fig. 6, consisting of a parallellepipided silencer device (an expansion chamber) connected to an inlet and an outlet tube with cross-section of 0.15 m × 0.15 m. The criterion described in the previous sections to define the collocation point distribution in the numerical model, was also used in this chapter.

Figure 7 illustrates the results of the SR (in dB) as a function of the frequency, in the range [10; 2000 Hz] with a frequency step of 10 Hz, for different lengths of the expansion chamber device when its cross-section is 0.4 m × 0.4 m. Two types of model were analyzed, namely with totally reflective internal walls (Fig. 7a) and with its walls lined with an absorbing material with $\bar{\alpha} = 0.2$ (Fig. 7b). For both cases, the sound reduction curves exhibit an oscillatory behaviour, associated with the multiple internal reflections that occur within the acoustic space. More oscillations are visible when the larger values of $L_1$ are adopted (namely 1.2 m), given the lower frequency of internal resonance that occurs in that case. At higher frequencies, a sequence of peaks and dips occurs, and originates a very complex behaviour, alternating attenuation with amplification phenomena. When the internal surfaces are lined, a smoother trend is observed in the sound reduction and those peaks and dips are greatly attenuated. For both cases, and for all values of $L_1$, it can be seen that the attenuation peaks around 500 Hz, with similar values for all configurations. This behaviour seems to be more associated with the cross-section of the device than with its length. Indeed, this can be explained by the fact that the attenuating effect of these devices are based on the contrast between impedances of the inlet/device/outlet.

A more realistic situation was also modelled, considering the sound absorption coefficient as a function of the frequency, following the pattern determined in the laboratory for mineral wool. As usual for porous and fibrous materials, the evaluated sound absorption coefficient varies with the frequency and
When the internal walls of the silencer are completely rigid, sound levels recorded on the receivers are completely rigid), sound levels recorded on the receivers reach higher amplitudes for the higher frequencies of 1500 Hz and 2000 Hz, the attenuation is significantly more pronounced within the attenuating device with reductions around 15 to 30 dB (see Figs. 12c2 and 12d2) within this space. Since the mineral wool exhibits higher absorption at higher frequencies, lining the inside of the device is very effective at those higher frequencies.

6. CONCLUSIONS

A numerical frequency domain approach, based on the three-dimensional formulation of the MFS was used to calculate the attenuation of the sound field within of an acoustic attenuator device in the absence of mean flow. Due to the symmetry of the acoustic problem, only a quarter of the model was discretized. Moreover, the discretization of the inner boundary (symmetry planes) of the model is not required due to the use of the Green’s functions that incorporate the boundary conditions related to symmetry of the problem. The numerical model also accounts for possible absorbing properties of the material lining the interior of the device, eventually defined by laboratory measurement of its sound absorption coefficients. The numerical results were validated with the results obtained in the laboratory test. In what concerns the proposed model, it was concluded that it can be quite an interesting tool for providing accurate results with increased computational efficiency when compared to traditional models such as those based on the boundary element model.

The parametric study conducted by evaluating the effect of the dimensions of the device and of the material lining its interior in the sound reduction it provides, revealed interesting behaviours. As expected, it was observed that both increasing the dimensions (in terms of cross-section) and the absorption of the lining material further increases the attenuating capabilities of the silencer device.

7. ACKNOWLEDGMENTS

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Figure 12. SPL obtained over a grid of receivers localized on the plane xy for the excitation frequencies of: (a) 100 Hz, (b) 500 Hz, (c) 1500 Hz, and (d) 2000 Hz.


Modal Density of Honeycomb Sandwich Composite Cylindrical Shells Considering Transverse Shear Deformation

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Honeycomb sandwich shells with composite face sheets are of extensive use in the spacecraft industry. Information on the number of resonant modes present in a frequency band is required to study their response behaviour under acoustic excitation. Modal densities of thin composite cylindrical shells have been reported while transverse shear deformations have not been considered. But in honeycomb sandwich panels, transverse shear deformations are significant, especially at higher order modes of vibration. In this work expressions for natural frequency and modal density of composite cylinders incorporating transverse shear deformation are derived. The modal densities of a typical cylinder that are obtained using the derived expression are compared with the results obtained using the finite element method and they are similar. Effects of transverse shear and orthotropic nature of the face sheets on the modal densities are investigated. It is shown that computing the modal density of honeycomb sandwich cylinders without considering the transverse shear deformation can lead to significant errors at high frequencies. Expressions of modal densities for special cases are also derived from the general expression.

1. INTRODUCTION

Honeycomb sandwich plates/shells are widely used in satellite structures due to their high stiffness to weight ratio. Broad band acoustic excitation is one of the critical loading conditions for satellite structures. Statistical Energy Analysis (SEA) is a promising tool to study the response behaviour of structures subjected to such high frequency dynamic loads. Modal density is an important parameter encountered in the calculation of response using SEA. Expressions for modal densities of several structural forms are derived and are in use. Xie et al. presented mode counts of several simple structural systems with various basic boundary conditions. They confirmed that at high frequencies the modal density is independent of boundary conditions.

In several larger spacecrafts, the primary structure consists of a central cylinder which is a honeycomb sandwich construction with composite face sheets. Several studies have been reported on modal densities of cylindrical shells. Modal densities of honeycomb sandwich cylinders have also been reported. Wilkinson derived an expression for modal densities of sandwich cylinders incorporating shear deformation of the core and Erickson modified the expression considering rotary inertia. Ferguson and Clarkson obtained an expression for estimating modal density of paraboloidal structural element. Elliot presented expressions for the modal densities of thin as well as honeycomb sandwich cylindrical shells in the form of integrals which were evaluated numerically. Finnvedan presented a finite element based approach to estimate the modal density of shells and hence no expressions are derived and it was applied to isotropic shells. An alternate methodology to evaluate the modal density of circular cylindrical shells is presented by Farshidianfar et al. but it is applicable to only isotropic shells. Ramachandran and Narayanan studied the effect of stiffeners on the modal density of isotropic cylindrical shells. All the
above works are on the isotropic shells or honeycomb sandwich shells with isotropic face sheets. An expression for the modal densities of composite cylindrical shells was derived earlier which can be used for estimating the modal densities of thin composite cylindrical shells. But the modal density determined using this expression does not consider the transverse shear flexibility. In honeycomb sandwich structures the shear modulus of the core is considerably low and can have a significant effect on the modal density. Therefore, an expression for modal density of cylindrical shells considering the shear flexibility is essential, especially for determining the modal densities of honeycomb sandwich composite cylinders. In this work, modal densities of honeycomb sandwich cylindrical shells with composite face sheets are obtained. An expression for modal density of composite cylindrical shells considering transverse shear deformation is derived. An expression for natural frequency is first derived. Expression for modal density incorporating transverse shear. Therefore, an expression for modal density of honeycomb sandwich composite cylindrical sheets with isotropic face sheets.

In this work, Donnell’s shell theory incorporating first order shear deformation theory is then derived by adopting wave space integration technique. The expression derived here are compared with the results obtained using the finite element method. Influence of transverse shear deformation on the modal densities is investigated. The displacements along the longitudinal direction is \( u_x \), along the tangential direction (linear displacement) is \( u_\theta \) and along the radial direction is \( u_r \).

In this work, Donnell’s shell theory incorporating first order shear deformation along with Airy’s stress function is used.

2. DIFFERENTIAL EQUATIONS OF MOTION
Consider a cylinder having a radius \( a \), length \( L \) and mass per unit area of \( \rho_m \). The co-ordinate axes are denoted by \( x \) for longitudinal, \( \theta \) for tangential and \( r \) for radial as shown in Fig. 1. The displacement along the longitudinal direction is \( u_x \), along the tangential direction (linear displacement) is \( u_\theta \) and along the radial direction is \( u_r \).

In this work, Donnell’s shell theory incorporating first order shear deformation along with Airy’s stress function is used.

2.1. Force and Moment Equilibrium Equations
For laminated cylindrical shells under free vibration, neglecting rotary inertia,

\[
\begin{align*}
\frac{\partial N_{xx}}{\partial x} + \frac{1}{a} \frac{\partial N_{x\theta}}{\partial \theta} &= \rho_m \frac{\partial^2 u_x}{\partial t^2}; \\
\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{a} \frac{\partial N_{\theta\theta}}{\partial \theta} &= Q_{r\theta} = \rho_m \frac{\partial^2 u_\theta}{\partial t^2}; \\
\frac{\partial Q_{xx}}{\partial x} + \frac{1}{a} \frac{\partial Q_{x\theta}}{\partial \theta} &= -\frac{N_{\theta\theta}}{a} = \rho_m \frac{\partial^2 u_r}{\partial t^2}; \\
\frac{\partial M_{xx}}{\partial x} + \frac{1}{a} \frac{\partial M_{x\theta}}{\partial \theta} &= Q_{r\theta} = 0; \\
\frac{\partial M_{x\theta}}{\partial x} + \frac{1}{a} \frac{\partial M_{\theta\theta}}{\partial \theta} &= Q_{r\theta} = 0;
\end{align*}
\]

which is a set of coupled equations with 5 displacement components \((u_x^0, u_\theta^0, u_r, \psi_x, \psi_\theta)\). In the above equations \( N_{xx}, N_{x\theta}, N_{\theta\theta}, M_{xx}, M_{x\theta}, \) and \( M_{\theta\theta} \) are the force and moment resultants (per unit length) and \( Q_{r\theta} \) are the shear forces per unit length. In the above equations \( u_x^0 \) and \( u_\theta^0 \) are the mid-surface displacements and \( u_r \) is the radial displacement.

Combining Eqs. (3), (4), and (5) we get

\[
\frac{\partial^2 M_{xx}}{\partial x^2} + \frac{2}{a} \frac{\partial^2 M_{x\theta}}{\partial x \partial \theta} + \frac{1}{a^2} \frac{\partial^2 M_{\theta\theta}}{\partial \theta^2} - \frac{N_{\theta\theta}}{a} = \rho_m \frac{\partial^2 u_r}{\partial t^2}. \tag{6}
\]

Combining only the normal loading and neglecting the inertia in the in-plane directions, Eq. (1) becomes

\[
\frac{\partial N_{xx}}{\partial x} + \frac{1}{a} \frac{\partial N_{x\theta}}{\partial \theta} = 0. \tag{7}
\]

Further, since the shear term \( \frac{\partial u_r}{\partial \theta} \) is small relative to the other terms of Eq. (2), Eq. (2) reduces to

\[
\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{a} \frac{\partial N_{\theta\theta}}{\partial \theta} = 0. \tag{8}
\]

Equations (6), (7), and (8) form the force and moment equilibrium equations. It is to be noted that these differential equations do not change whether the transverse shear deformation is considered or not.

2.2. Strain Displacement Relations
The strains are related to displacements as

\[
\begin{align*}
\varepsilon_{xx} &= \frac{\partial u_x}{\partial x}; \\
\varepsilon_{x\theta} &= \frac{1}{a} \frac{\partial u_\theta}{\partial \theta} + \frac{u_x}{a}; \\
\gamma_{x\theta} &= \frac{\partial u_\theta}{\partial x} + \frac{1}{a} \frac{\partial u_x}{\partial \theta}.
\end{align*}
\]

Equations (9), (10), and (11) do not change whether the transverse shear deformation is considered or not. Also, the normal strains acting on the plane parallel to the middle surface are negligible compared to the other strain components. In other words, no stretching is present in the radial/normal direction, i.e. \( \varepsilon_r = 0 \). The radial displacement is independent of thickness.

2.3. First Order Shear Deformation Theory
Denoting the rotations of the transverse plane as \( \psi_x \) and \( \psi_\theta \), the displacement field in a thin shell as well as in a thick shell with first order shear deformation theory is

\[
\begin{align*}
u_x &= u_x^0 + z \psi_x; \\
u_\theta &= u_\theta^0 + z \psi_\theta;
\end{align*}
\]

where \( u_x^0 \) and \( u_\theta^0 \) are the mid-surface displacements. In a thin shell, \( \psi_x = \frac{1}{a} \frac{\partial u_r}{\partial \theta} \) and \( \psi_\theta = \frac{1}{a} \frac{\partial u_r}{\partial \theta} \).
Donnell’s first order shear deformation theory is used in the present formulation. Therefore, the transverse planes that are normal to the un-deformed mid-surface remain straight but not normal to the mid-surface after deformation. The rotations of the transverse planes are

\[
\psi_x = \gamma_{xx} - \frac{\partial u_x}{\partial x};
\]

\[
\psi_\theta = \gamma_{\theta r} - \frac{1}{a} \frac{\partial u_r}{\partial \theta}.
\]

In the first order shear deformation theory the curvatures are given by

\[
\kappa_{xx} = \frac{\partial \psi_x}{\partial x} = \frac{\partial \gamma_{xx}}{\partial x} - \frac{\partial^2 u_x}{\partial x^2};
\]

\[
\kappa_{\theta \theta} = \frac{\partial \psi_\theta}{\partial \theta} = \frac{1}{a} \frac{\partial \gamma_{\theta r}}{\partial \theta} - \frac{1}{a^2} \frac{\partial^2 u_r}{\partial \theta^2};
\]

\[
\kappa_{\theta r} = \frac{\partial \psi_x}{\partial x} + \frac{1}{a} \frac{\partial \psi_\theta}{\partial \theta} = \frac{2}{a} \frac{\partial^2 u_x}{\partial x \partial \theta} + \frac{1}{a} \frac{\partial \gamma_{xx}}{\partial \theta}.
\]

Substituting the expressions for the rotations of the transverse plane [Eqs. (14–15)] and the displacement field [Eqs. (12–13)] in Eqs. (9–11), the strains become

\[
\varepsilon_{xx} = \varepsilon_{0xx} + 2z \kappa_{xx};
\]

\[
\varepsilon_{\theta \theta} = \varepsilon_{0\theta \theta} + 2z \kappa_{\theta \theta};
\]

\[
\gamma_{\theta r} = \gamma_{0\theta r} + 2z \kappa_{\theta r},
\]

where \( \varepsilon_{0xx}, \varepsilon_{0\theta \theta} \) and \( \gamma_{0\theta r} \) are the mid-surface strains.

The transverse shear strains will be constant across the section

\[
\gamma_{xx} = \gamma^0_{xx},
\]

\[
\gamma_{\theta r} = \gamma^0_{\theta r},
\]

where \( \gamma^0_{xx} \) and \( \gamma^0_{\theta r} \) are the mid-surface shear strains

\[
\gamma^0_{xx} = \psi_x + \frac{\partial u_x}{\partial x};
\]

\[
\gamma^0_{\theta r} = \psi_\theta + \frac{1}{a} \frac{\partial u_r}{\partial \theta},
\]

It should be noted that the term \( \frac{1}{a} \) is neglected in the expression for \( \gamma_{\theta r} \) as per Donnell’s theory, whereas other theories (Love’s theory, Loo’s theory, Sander’s theory) include this term in the formulation. In this work Donnell’s theory is used.

### 2.4. Force and Moment Resultants

The force and moment resultants are defined as in the case of thin shells as

\[
[N_{xx}]_{i\theta \theta} = \sum_{k=1}^{n_k} \int_{r_{k-1}} r_k \left[ \sigma_{xx} \right]_k dz;
\]

\[
[N_{\theta \theta}]_{i\theta \theta} = \sum_{k=1}^{n_k} \int_{r_{k-1}} r_k \left[ \sigma_{\theta \theta} \right]_k dz;
\]

\[
[N_{\theta r}]_{i\theta \theta} = \sum_{k=1}^{n_k} \int_{r_{k-1}} r_k \left[ \sigma_{\theta r} \right]_k dz;
\]

where \( R_i \) is the radii of curvature in the respective direction and \( k \) refers to the number of layers in the laminate. For cylindrical shell, \( R_i = \infty \) and \( R_0 = a \).

It is assumed that the term \( 1 + \frac{z}{R_i} \) is very close to unity. The stresses are related to the strains through elastic stiffness coefficients denoted by \( Q_{ij} \), details of which are not given here for brevity but explained in Josephine Kelvina Florence and Ranji work. The strains are related to the mid-surface strains and curvatures through Eqs. (19–21). Combining all the above relations the force and moment resultants become

\[
[N_{xx}]_{i\theta \theta} = \left[ \begin{array}{cccc}
A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16}
\end{array} \right] \left[ \begin{array}{c}
\varepsilon_{0xx}
\varepsilon_{0\theta \theta}
\gamma_{0\theta r}
\end{array} \right] + \left[ \begin{array}{c}
\frac{\partial \psi_x}{\partial x}
\frac{\partial \psi_\theta}{\partial \theta}
\end{array} \right];
\]

where \( A_{ij} = \sum_{k=1}^{n_k} (Q_{ij})_k (h_k - h_{k-1}) \), also called as extensional stiffness terms; \( B_{ij} = \frac{1}{2} \sum_{k=1}^{n_k} (Q_{ij})_k (h^2_k - h^2_{k-1}) \), also called as coupling stiffness terms; \( D_{ij} = \frac{1}{4} \sum_{k=1}^{n_k} (Q_{ij})_k (h^3_k - h^3_{k-1}) \), also called as bending stiffness terms; and \( Q_{ij} \) are coefficients of elastic stiffness.

In the present work the laminate considered is symmetric and balanced, therefore \( B_{ij} = 0; A_{16} = 0; A_{20} = 0 \). Also, assume that \( D_{16} \) and \( D_{20} \) are negligible, the above relations become

\[
[N_{xx}]_{i\theta \theta} = \left[ \begin{array}{cccc}
A_{11} & A_{12} & 0 & 0 & 0 & 0
\end{array} \right] \left[ \begin{array}{c}
\varepsilon_{0xx}
\varepsilon_{0\theta \theta}
\gamma_{0\theta r}
\end{array} \right] + \left[ \begin{array}{c}
\frac{\partial \psi_x}{\partial x}
\frac{\partial \psi_\theta}{\partial \theta}
\end{array} \right].
\]

The above set of equations is the same as those for thin composite cylindrical shells, but the expressions for the curvatures include the transverse shear effects.

### 2.5. Governing Differential Equations of Motion

Using Airy stress function, the stress resultants can be defined as

\[
N_{xx} = 1 \frac{\partial^2 \phi}{\partial x^2};
\]

\[
N_{\theta \theta} = 1 \frac{\partial^2 \phi}{\partial \theta^2};
\]

\[
N_{\theta r} = 1 \frac{\partial^2 \phi}{\partial x \partial \theta}.
\]

The function \( \phi \) was first introduced by Airy and is in general known as Airy’s stress function. Using this function, the first two equations of motion are satisfied completely. In other words, two independent in-plane displacements are eliminated and the unknowns reduce to two which are \( \phi \) and \( u_r \). For solving the unknowns, we make use of the third equation of motion and an additional equation generated using compatibility condition.

Substituting Eq. (29), Eqs. (16–18) and Eq. (31) in Eq. (6),
we get
\[
D_{11} \frac{\partial^3 \gamma_{xx}}{\partial x^3} + \frac{D_{22}}{a^3} \frac{\partial^2 \gamma_{xy}}{\partial y^2} + \left( \frac{D_{12} + 2D_{66}}{a} \right) \left( \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} + \frac{1}{a} \frac{\partial \gamma_{xx}}{\partial x} \right) = \frac{D_1}{a} \frac{\partial^4 u_r}{\partial x^4} + \frac{D_2}{a^2} \frac{\partial^4 u_r}{\partial x^2 \partial y^2} + \frac{1}{\rho_n} \frac{\partial^2 u_r}{\partial t^2} + \frac{D_{23}}{a^3} \frac{\partial^4 u_r}{\partial x^2 \partial y^2}.
\]
(33)

Equation (33) represents the first governing differential equation which is in terms of \( \gamma_{xx}, \gamma_{xy}, u_r \) and \( \Theta \).

The terms involving \( \gamma_{xx}, \gamma_{xy} \) need to be eliminated to have a closed form expression for the modal density. For this, we make use of force equilibrium consideration which is given below for ready reference
\[
\frac{\partial Q_{xx}}{\partial x} + \frac{1}{a} \frac{\partial Q_{xy}}{\partial y} - \frac{N_{xy}}{a} \frac{\partial}{\partial x} \gamma_{xx} = 0.
\]
(34)

For a honeycomb sandwich construction, the average shear angle \( \gamma_{xy} = \frac{Q_{xy}}{a} \) where \( N_x = G_x h (1 + b^2) \) where \( G_x \) is the core shear modulus, \( h \) is the thickness of core and \( t \) is the thickness of face sheet. Assuming the core to be isotropic \( G_x = G_y = G \) and \( N_x = N_y = N \), called shear rigidity of the shell, the shear angle can be expressed as
\[
\gamma_{xy} = \frac{Q_{xy}}{a}.
\]
(35)

Substituting Eq. (35) into Eq. (34), the force equilibrium equation becomes
\[
\frac{\partial^2 \gamma_{xx}}{\partial x^2} + \frac{1}{a} \frac{\partial^2 \gamma_{xy}}{\partial y \partial x} = \frac{N_{xy}}{a} \frac{\partial^2 \Theta}{\partial x^2} = -g_r.
\]
(36)

From Eq. (36), upon suitable algebraic operations, we get
\[
D_{11} \frac{\partial^2 \gamma_{xx}}{\partial x^2} + \frac{D_{22}}{a} \frac{\partial^2 \gamma_{xy}}{\partial y \partial x} = -\frac{1}{N} \left( \frac{D_{11}}{a} \frac{\partial^2 q_r}{\partial x^2} + \frac{D_{22}}{a^2} \frac{\partial^2 q_r}{\partial y^2} \right) + \frac{1}{N} \left( \frac{D_{11}}{a} \frac{\partial^2 \Theta}{\partial x^2} + \frac{D_{22}}{a^2} \frac{\partial^2 \Theta}{\partial y^2} \right) - \frac{D_1}{a^2} \frac{\partial^2 \gamma_{xy}}{\partial x^2 \partial y}.
\]
(37)

Substitution of Eq. (37) into Eq. (33) gives
\[
- \frac{1}{N} \left( \frac{D_{11}}{a^2} \frac{\partial^2 q_r}{\partial x^2} + \frac{D_{22}}{a^2} \frac{\partial^2 q_r}{\partial y^2} \right) + \frac{1}{N} \left( \frac{D_{11}}{a} \frac{\partial^2 \Theta}{\partial x^2} + \frac{D_{22}}{a^2} \frac{\partial^2 \Theta}{\partial y^2} \right) + \left( \frac{D_{12} + 2D_{66} - D_{11}}{a} \right) \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{D_1}{a} \frac{\partial^4 u_r}{\partial x^4} + \frac{D_2}{a^2} \frac{\partial^4 u_r}{\partial x^2 \partial y^2} + \frac{1}{\rho_n} \frac{\partial^2 u_r}{\partial t^2} + \frac{D_{23}}{a^3} \frac{\partial^4 u_r}{\partial x^2 \partial y^2}.
\]
(38)

It is not possible to eliminate \( \gamma_{xx} \) and \( \gamma_{xy} \) completely. If the terms having \( \gamma_{xx} \) and \( \gamma_{xy} \) are neglected, the differential equa-
tion becomes
\[
- \frac{1}{N} \left( \frac{D_{11}}{a^2} \frac{\partial^2 q_r}{\partial x^2} + \frac{D_{22}}{a^2} \frac{\partial^2 q_r}{\partial y^2} \right) + \frac{1}{N} \left( \frac{D_{11}}{a} \frac{\partial^2 \Theta}{\partial x^2} + \frac{D_{22}}{a^2} \frac{\partial^2 \Theta}{\partial y^2} \right) = \frac{D_1}{a} \frac{\partial^4 u_r}{\partial x^4} + \frac{1}{a^2} \frac{\partial^4 u_r}{\partial x^2 \partial y^2} + \frac{2(D_{12} + 2D_{66})}{a^2} \frac{\partial^4 u_r}{\partial x^2 \partial y^2} + \frac{D_{22}}{a^2} \frac{\partial^4 u_r}{\partial y^2} + \frac{1}{a} \frac{\partial^2 \Theta}{\partial x^2} + \frac{\rho_n}{a^2} \frac{\partial^2 u_r}{\partial t^2}.
\]
(39)

As \( q_r = -\rho_n \frac{\partial^2 u_r}{\partial t^2} \), the above differential equation becomes
\[
D_{11} \frac{\partial^4 u_r}{\partial x^4} + \frac{2(D_{12} + 2D_{66})}{a^2} \frac{\partial^4 u_r}{\partial x^2 \partial y^2} + \frac{D_{22}}{a^2} \frac{\partial^4 u_r}{\partial y^2} + \frac{1}{a^2} \frac{\partial^2 \Theta}{\partial x^2} + \frac{\rho_n}{a^2} \frac{\partial^2 u_r}{\partial t^2} - \frac{D_1}{a^2} \frac{\partial^2 \Theta}{\partial x^2} = 0.
\]
(40)

Equation (40) is the first governing differential equation in terms of Airy’s stress function and normal displacements. In this formulation, all the terms representing the shear effects are not included. However, if one considers the differential equation of a thin composite cylinder (which is given below for reference),
\[
D_{11} \frac{\partial^4 u_r}{\partial x^4} + \frac{2(D_{12} + 2D_{66})}{a^2} \frac{\partial^4 u_r}{\partial x^2 \partial y^2} + \frac{D_{22}}{a^2} \frac{\partial^4 u_r}{\partial y^2} + \frac{1}{a^2} \frac{\partial^2 \Theta}{\partial x^2} + \frac{\rho_n}{a^2} \frac{\partial^2 u_r}{\partial t^2} = 0;
\]
(41)

one can infer that most of the significant terms are included. It should also be noted that if the terms having \( \gamma_{xx} \) and \( \gamma_{xy} \) are not neglected, it will not be possible to incorporate shear effects, which will result in significant error. Since these terms are third derivatives of the shear angle it is expected that these are negligible.

To obtain the second differential equation, compatibility condition is enforced. This is carried out by eliminating the displacements from the strain displacement relationships and is given by Eq. (42) as
\[
\frac{\kappa_{xx}}{a} \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial x \partial y} + \frac{1}{a} \frac{\partial^2 \phi}{\partial x \partial y} + \frac{1}{a^2} \frac{\partial^2 \phi}{\partial y^2} = \frac{1}{a} \frac{\partial^2 \gamma_{xx}}{\partial x \partial y} = 0.
\]
(42)

Using Eq. (29) and the definition of Airy stress function as given in Eqs. (31–32), the mid-surface strains can be written as
\[
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{xy}
\end{bmatrix} = \frac{1}{A_{11} A_{22} - A_{12}^2} \begin{bmatrix}
A_{22} & -A_{12} & 0 \\
-A_{12} & A_{11} & 0 \\
0 & 0 & A_{44}
\end{bmatrix} \begin{bmatrix}
\frac{\partial^2 \phi}{\partial x^2} \\
\frac{\partial^2 \phi}{\partial x \partial y} \\
\frac{\partial^2 \phi}{\partial y^2}
\end{bmatrix}.
\]
(43)

Using Eq. (31) and Eq. (43) in Eq. (42), we get
\[
A_{12}^2 - A_{11} A_{22} \frac{\partial^2 u_r}{\partial x^2} + A_{11} A_{22} \frac{\partial^2 \phi}{\partial x^2} + A_{11} A_{22} \frac{\partial^2 \phi}{\partial x \partial y} + 2 A_{11} A_{22} \frac{\partial^2 \phi}{\partial x \partial y} + \frac{A_{22}}{A_{44}} \frac{\partial^2 \phi}{\partial y^2} = 0.
\]
(44)

This is the second equation in terms of \( \phi \) and \( u_r \).

Hence, Eqs. (40) and (44) are the governing differential equations in terms of Airy’s stress function and normal displacements.
2.6. Assumptions

Though the assumptions involved in arriving at the differential equations are presented whenever they are applied they are summarized here to have an overall idea. Some of these assumptions are part of any two-dimensional structural analysis but given here for completeness.

1. The shell is cylindrical.
2. The shell is shallow; the term \(1 + \frac{x}{R}\) is very close to one. The radius of curvature is very large compared to the in-plane displacements.
3. Material is linearly elastic.
4. The laminate is symmetric, therefore \(B_{ij} = 0\).
5. The laminate is balanced, therefore \(A_{16}, A_{26} = 0\).
6. \(D_{16}\) and \(D_{26}\) are negligible.
7. Plane stress condition exists.
8. \(\varepsilon_r = 0\), i.e., the displacement \(u_r\) is independent of \(z\).
9. The transverse shear strains in the face sheets are neglected.
10. Mass distribution is uniform, i.e. mass per unit area is constant.
11. Rotary inertia is neglected.
12. In a honeycomb sandwich construction the core is homogeneous; the wavelength is far greater than the cell size.
13. The transverse planes that are normal to the undeformed axial direction of the shells considering transverse shear deformation have the expression for natural frequency. The parameter \(\delta\) represents an arbitrary angle indicating that there is no preference in circumferential direction.
14. Upon substitution of Eqs. (45) and (46) in the governing differential equations we get

\[
M_{11} M_{12} M_{21} M_{22} \times \left[ \begin{array}{c} u_{mn} \\ \theta_{mn} \end{array} \right] = 0; \quad (47)
\]

for \(m\) half waves in the axial direction and \(n\) full waves in the circumferential direction. The parameter \(\delta\) is restricted to normal loading.

To determine the modal density and mode count, an expression for natural frequency of orthotropic cylindrical shells considering transverse shear deformation of first order.

If transverse shear effects are neglected, i.e. when \(N\) tends to \(\infty\), the expression for natural frequency becomes Eq. (49), which is the same as that reported for thin composite cylinders.\(^{11}\)

To assess the influence of transverse shear on the natural frequencies of typical honeycomb sandwich composite cylinder used in spacecraft the natural modes are obtained considering the transverse shear [Eq. (48)] as well as neglecting the transverse shear [Eq. (49)]. The length of the cylinder is 3000 mm and the mean radius is 600 mm.

Material properties of the sandwich cylinder are given below.

**Face sheet**

Material: 2 layers of bidirectional CFRP.
Each layer has the following properties:
- Young’s modulus = 1.47 \(\times\) 10\(^{11}\) N/m\(^2\)
- Shear modulus = 4 \(\times\) 10\(^8\) N/m\(^2\)
- Poisson’s ratio = 0.03
- Thickness = 0.08 mm
- Density = 1660 kg/m\(^3\)

**Core**

Material: Aluminium honeycomb
- Thickness = 12 mm
- Density = 32 kg/m\(^3\)
- Shear modulus = 1.4 \(\times\) 10\(^8\) N/m\(^2\)

The cross-section of the cylinder has the following elastic properties: \(A_{11} = 4.71 \times 10^7\) N/m, \(A_{22} = 4.71 \times 10^7\) N/m, \(A_{12} = 1.41 \times 10^6\) N/m, \(A_{06} = 1.28 \times 10^6\) N/m, \(D_{11} = 1.74 \times 10^7\) Nm, \(D_{22} = 1.74 \times 10^7\) Nm, \(D_{12} = 52.2\) Nm, and \(D_{06} = 47.3\) Nm. Shear rigidity of the section is 17.25 \(\times\) 10\(^6\) N m. Mass per unit area of the cylinder is 0.92 kg/m\(^2\) (includes 2 face sheets, core and adhesive).

As the transverse shear effects are expected to be significant for higher order modes the results are given in Table 1 for the higher order modes. The results show the need for the inclusion of transverse shear deformation even for these cylinders.

4. MODE COUNT AND MODAL DENSITY

Modal density is the average number of modes per unit frequency. Modal density can be determined from the constant
By defining the wave numbers as
\[ \omega_{mn}^2 = \frac{1}{\rho_m} \left[ 1 + \left( \frac{D_{11} m \pi}{n \pi} \right)^2 + \frac{D_{22}}{\rho_m} \left( \frac{n}{a} \right)^2 \right] \left( D_{11} \left( \frac{m \pi}{L} \right)^4 + 2(D_{12} + 2D_{66}) \left( \frac{m \pi}{L} \right)^2 \left( \frac{n}{a} \right)^2 + D_{22} \left( \frac{n}{a} \right)^4 + \right. \\
\left. \frac{A_{11}A_{22} - A_{12}^2}{\rho_m} \sqrt{\frac{m \pi}{n \pi}} \left( \frac{m \pi}{L} \right)^2 \left( \frac{n}{a} \right)^2 \right) \times \left\{ \frac{1}{\rho_m} \left( \frac{m \pi}{n \pi} \right)^2 + \frac{1}{\rho_m} \left( D_{11} \left( \frac{m \pi}{L} \right)^4 + D_{22} \left( \frac{n}{a} \right)^4 \right) \right\} \right); \tag{48} \]
\[ \omega_{mn}^2 = \frac{1}{\rho_m} \left( D_{11} \left( \frac{m \pi}{L} \right)^4 + 2(D_{12} + 2D_{66}) \left( \frac{m \pi}{L} \right)^2 \left( \frac{n}{a} \right)^2 + D_{22} \left( \frac{n}{a} \right)^4 + \right. \\
\left. \frac{A_{11}A_{22} - A_{12}^2}{A_{66}} \sqrt{\frac{m \pi}{n \pi}} \left( \frac{m \pi}{L} \right)^2 \left( \frac{n}{a} \right)^2 \right) \times \left\{ \frac{1}{\rho_m} \left( \frac{m \pi}{n \pi} \right)^2 + \frac{1}{\rho_m} \left( D_{11} \left( \frac{m \pi}{L} \right)^4 + D_{22} \left( \frac{n}{a} \right)^4 \right) \right\} \right); \tag{49} \]

\begin{table}[h]
\centering
\caption{Comparison of natural frequencies of a cylinder with and without shear deformation.}
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{m},\text{axial half wave number} & \textbf{n}, \text{circumferential} & \textbf{Frequency neglecting shear deformation (Hz)} & \textbf{Frequency considering shear deformation (Hz)} \\
\hline
9 & 2 & 1019 & 1002 \\
10 & 3 & 1008 & 970 \\
6 & 7 & 1018 & 941 \\
7 & 7 & 1062 & 975 \\
10 & 12 & 2944 & 2394 \\
20 & 11 & 4004 & 3008 \\
15 & 17 & 5967 & 4161 \\
16 & 18 & 6700 & 4528 \\
30 & 15 & 8420 & 5204 \\
28 & 20 & 10144 & 5872 \\
\hline
\end{tabular}
\end{table}

4.1. Expression for Mode Count

Mode count, denoted by \( N(\omega) \), is the number of modes below the frequency \( \omega \). The derivative of the expression for the number of modes below the frequency \( \omega \), with respect to the frequency gives modal density.

4.2. Expression for Modal Density

Modal density, denoted by \( n(\omega) \), is related to mode count by the relation \( n(\omega) = \frac{dN(\omega)}{d\omega} \). It is to be noted that modal density as a function of frequency is expressed as \( n(f) = 2\pi n(\omega) \).

Differentiating the expression for mode count given by Eq. (53) with respect to \( \omega \), we get
\[ n(\omega) = \frac{\rho_m \omega A L}{N \pi} \int_0^{\frac{\pi}{2}} \left\{ \frac{f_2}{f_1} + \sqrt{\frac{f_2^2}{f_1^2} + \frac{4N^2}{f_1^2(\rho_m \omega^2 - f_3)}} \right\} d\theta; \tag{54} \]
\[ n(f) = \frac{2Af \rho_m}{N} \int_0^{\frac{\pi}{2}} \left\{ \frac{f_2}{f_1} + \sqrt{\frac{f_2^2}{f_1^2} + \frac{4N^2}{f_1^2(\rho_m \omega^2 - f_3)}} \right\} d\theta; \tag{55} \]
5. COMPARISON WITH THE RESULTS OF FINITE ELEMENT METHOD

Modal densities for a typical honeycomb sandwich cylinder with composite face sheets are obtained using the expression derived and they are compared with those obtained using the finite element method. The length of the cylinder considered is 2260 mm, mean radius is 452 mm, which results in a surface area of 6.42 m².

Material properties of the above sandwich cylinder are given below.

<table>
<thead>
<tr>
<th>Material: 4 layers (0/-35/0/35) of CFRP [1 layer of Bidirectional CFRP + 3 layers of Unidirectional CFRP]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness = 0.365 mm</td>
</tr>
<tr>
<td>Young’s modulus of Unidirectional lamina = 2.15 × 10¹¹ N/m² (along the fibre direction) = 6.6 × 10⁹ N/m² (along the transverse direction)</td>
</tr>
<tr>
<td>Shear modulus of Unidirectional lamina = 3.9 × 10⁹ N/m²</td>
</tr>
<tr>
<td>Major Poisson’s ratio = 0.23</td>
</tr>
<tr>
<td>Density = 1600 kg/m³</td>
</tr>
<tr>
<td>Properties of the Bidirectional lamina are the same as those described in Section 3.</td>
</tr>
<tr>
<td>Core Material: Aluminium honeycomb</td>
</tr>
<tr>
<td>Thickness = 12 mm</td>
</tr>
<tr>
<td>Shear modulus = 1.4 × 10⁸ N/m²</td>
</tr>
<tr>
<td>Density = 32 kg/m³</td>
</tr>
</tbody>
</table>

The cross-section of the cylinder has the following elastic properties: \( A_{11} = 1.03 \times 10^8 \) N/m, \( A_{22} = 0.363 \times 10^8 \) N/m, \( A_{12} = 0.186 \times 10^8 \) N/m, \( A_{66} = 1.99 \times 10^7 \) N/m, \( D_{11} = 3.95 \times 10^6 \) Nm, \( D_{22} = 1.43 \times 10^6 \) Nm, \( D_{12} = 0.705 \times 10^3 \) Nm, and \( D_{66} = 0.753 \times 10^3 \) Nm. The other elastic properties are \( D_{16} = 0.03 \times 10^6 \) Nm, \( D_{26} = 0.015 \times 10^3 \) Nm. Shear rigidity of the section is 17.83 × 10³ N/m. Mass per unit area of the cylinder is 4.7 kg/m² (includes 2 face sheets, core and adhesive).

Modal densities computed using the expression given by Eq. (55) are given in Table 2 and Fig. 2. Finite element model is developed for the cylindrical geometry using 4-noded quadrilateral element. It is a shell element with 5 degrees-of-freedom for each node. The face sheets and the core together are considered as a laminate. The elements are assigned with the properties of the laminate. The elastic properties of the laminate are determined from the properties of each layer. To represent transverse shear deformation, Mindlin’s theory is used and the shear effects are included in the finite element model in terms of shear correction factor.

Natural frequencies of the normal modes of this cylinder are determined using NASTRAN solver. All the modes up to 3000 Hz are extracted. Modal density for each one third octave band is then computed as the ratio of the number of modes in that band to the bandwidth. The results are given in Fig. 2 and a very good agreement is seen validating the expression derived.

6. EXPRESSION FOR MODAL DENSITY IN SPECIAL CASES

Modal density of a composite cylindrical shell considering transverse shear deformation can be obtained using Eq. (55). It is essential to verify whether the derived expression under limiting cases converges to the existing expressions. Also, the expression can be in a simple form under certain conditions. These are discussed here.

6.1. Negligible Transverse Shear Deformations

When the transverse shear effects are negligible, expression for modal density of sandwich cylinder should converge to that of thin composite cylinders.\(^\text{11}\) This condition can be achieved by taking \( N \to \infty \). Taking \( N \) inside the braces of general expression, we get

\[
n(f) = 2Af \rho_m \int_0^{\pi/2} \left\{ \frac{f_2}{Nf_1} + \sqrt{\left(\frac{f_2}{Nf_1}\right)^2 + \frac{4}{f_1(\rho_m \omega^2 - f_3)}} \right\} \, d\theta. \tag{56}
\]

As \( N \) tends to \( \infty \),

\[
n(f) = 2Af \rho_m \int_0^{\pi/2} \left( \frac{4}{f_1(\rho_m \omega^2 - f_3)} \right) \, d\theta. \tag{57}
\]

On simplification

\[
n(f) = 2Af \rho_m \left( \frac{\sqrt{\rho_m}}{\sqrt{f_1}} \frac{d\theta}{\sqrt{\omega^2 - f_3/\rho_m}} \right) \tag{58}
\]

Upon substitution of \( f_1 \) and \( f_3 \), we get Eq. (59), which is the same as the expression reported for modal density of thin composite cylindrical shells.\(^\text{11}\) When the shear effects are neglected, the expression for modal density derived here converges to that of thin composite cylindrical shells.
\[ n(f) = \frac{A}{\pi \rho_m} \int_{0}^{\frac{\pi}{2}} \left\{ 1 - \frac{c^4(A_{11} A_{22} - A_{11}^2)}{4\pi^2 f^2 \rho_m a^2 (A_{11} c^4 + A_{22} s^4 + \frac{A_{11} A_{22} - A_{11}^2}{A_{22}} c^2 s^2)} \right\}^{\frac{A_{22}}{2}} \left\{ D_{11} c^4 + 2(D_{12} + 2D_{06}) c^2 s^2 + D_{22} s^4 \right\}^{-\frac{A_{22}}{2}} \, d\theta \]

(59)

6.2. Isotropic Shells

For a cylinder with isotropic material, \( A_{11} = A_{22}, D_{11} = D_{22} = D \). The functions of \( \theta \) become
\[ f_1 = D_{11} c^4 + 2(D_{12} + 2D_{06}) c^2 s^2 + D_{22} s^4 = D(1 - 2c^2 s^2 + \gamma^2 c^2 s^2); \]
\[ f_2 = D_{11} c^4 + D_{22} s^4 = D_1 = D; \]
\[ f_3 = \frac{A_{11} c^2 (1 - \mu^2)}{\alpha^2 (1 - 2c^2 s^2 + \gamma^2 c^2 s^2)}; \]
where \( \frac{A_{11}}{A_{11}} = \alpha; \frac{A_{22}}{A_{11}} = \beta; \frac{2(D_{12} + 2D_{06})}{D_1} = \gamma. \) For an isotropic cylinder, \( \alpha = \mu; \beta = \frac{1 - \mu}{2}; \gamma = 2. \)

The product \( \cos^2 \theta \sin^2 \theta \) is approximately constant except for values of \( \theta \) near 0 and \( \pi/2. \) Since there is no preference for a wave number component, in other words equal probability of occurrence for the wave components, an average value of a wave number component, in other words equal probability \( \theta \) for values of

\[ \text{The face sheet has a thickness of } 1.1 \text{ mm, mean radius of } 600 \text{ mm resulting in an area of } 11.3 \text{ m}^2. \]

The honeycomb core has a density of 32 kg/m\(^3\) and the thickness of the core is 12 mm. The face sheet has a thickness of 0.3 mm made of Aluminium. Young’s modulus of Aluminium is considered as 7.2 \times 10^{10} \text{ N/m}^2. The mass per unit area of the cylinder is 0.0151 kg/m\(^2\).

The modal densities of the cylinder whose properties are given above are computed using Eq. (61) and compared with the results obtained using the expression derived by Wilkinson in Fig. 3. The results show that the modal densities deter-

\[ \text{Figure 3. Modal densities of sandwich cylinders with isotropic face sheets.} \]

6.3. Modal Density at High Frequencies

At very high frequencies, the structural wavelengths are very much lower compared to the circumference so that a cylinder tends to behave like a flat plate.\(^6\) The expression at high frequencies can be obtained by setting \( R \) to \( \infty \) as
\[ n(f) = \frac{2A f \rho_m}{N} \int_{0}^{\frac{\pi}{2}} \left\{ 1 + \frac{4N^2}{D(\rho_m \omega^2 - \frac{A_{11} c^2 (1 - \mu^2)}{\alpha^2})} \right\}^{-\frac{A_{22}}{2}} \frac{2N^2}{\rho_m \omega^2 f_1 \sqrt{\left( \frac{f_2}{f_1} \right)^2 + \frac{4N^2}{\rho_m \omega^2 f_1}}} \, d\theta. \]

(62)

where \( f_1 = D_{11} c^4 + 2(D_{12} + 2D_{06}) c^2 s^2 + D_{22} s^4; \)
\[ f_2 = D_{11} c^4 + D_{22} s^4. \]

It is interesting to compare the modal densities obtained using Eq. (62) with those using the expression for the composite flat panels, considering transverse shear deformation, reported by Renji which is given as Eq. (63)
\[ n(f) = \frac{2abpf}{N} \int_{0}^{\frac{\pi}{2}} \left\{ \frac{f_2}{f_1} \left( \rho \omega f_2^2 + \frac{4\rho \omega^2 N^2}{\sqrt{D_{11} D_{22}}} \right) \right\}^{-\frac{A_{22}}{2}} \left( \rho \omega f_2^2 + \frac{2N^2 f_1}{\sqrt{D_{11} D_{22}}} \right) \, d\theta. \]

(63)

where \( f_1 = 1 - \gamma^2 \sin^2 \theta \) with \( \gamma = 2 \left( 1 - \frac{D_{12} + 2D_{06}}{D_{11} D_{22}} \right); \)
\[ f_2 = \left( \frac{D_{11}}{D_{22}} \right)^{\frac{1}{2}} c^2 + \left( \frac{D_{11}}{D_{22}} \right)^{\frac{1}{2}} s^2. \]

It should be noted that the two equations are similar in nature. The forms, in which the wave numbers \( (K_1 \text{ and } K_2) \) and the functions of \( \theta (f_1 \text{ and } f_2) \) are defined, are different in both the expressions. In the case of shells, wave numbers are defined as \( K_1 = r \cos \theta; K_2 = r \sin \theta, \) whereas they are defined...
as $D_{11}K_1 = r \cos \theta$; $D_{22} \frac{1}{r} K_2 = r \sin \theta$ for plates. As an example, both the expressions give a modal density of 0.5 modes/Hz at 10000 Hz for the cylinder described in Section 3.

6.4. Modal Density for Cylinders Having Equal Properties in Both the Directions

For cylinders having equal properties in the two material-property directions, i.e., $A_{11} = A_{22}$, $D_{11} = D_{22}$, the expression for modal density can be written in a simple form as

$$n(f) = \frac{2AF_{pm}}{N} \int_0^{\pi} \left( \frac{f_2}{f_1} + \sqrt{\left( \frac{f_2}{f_1} \right)^2 + \frac{4N^2}{f_1(p_m \omega^2 - f_3)}} - \frac{2N^2}{f_1(p_m \omega^2 - f_3)} \sqrt{\left( \frac{f_2}{f_1} \right)^2 + \frac{4N^2}{f_1(p_m \omega^2 - f_3)}} \right) d\theta. \quad (64)$$

The functions of $\theta$ become

$$f_1 = D_{11}(1 - 2c^2s^2 + \gamma c^2s^2);$$
$$f_2 = D_{11};$$
$$f_3 = \frac{A_{11};c^2(1-\alpha^2)}{a^2\left[1-2c^2s^2 + \frac{c^2(1-\alpha^2)}{c^2s^2}\right]};$$

with $A_{11} = \alpha; \frac{A_{11}}{A_{11}} = \beta; \frac{2(D_{12} + 2D_{16})}{D_{11}} = \gamma.$

7. INFLUENCE OF TRANSVERSE SHEAR DEFORMATION AND ORTHOTROPY

7.1. Transverse Shear

To understand the effect of the transverse shear deformation, modal densities are obtained (in one-third octave bands) for a honeycomb sandwich composite cylinder, whose properties are given in Section 5, and the results are presented in Table 3 and Fig. 4.

![Modal densities of honeycomb sandwich composite cylinder.](image_url)

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Modal density (modes/Hz)</th>
<th>With transverse shear</th>
<th>Without transverse shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.132</td>
<td>0.139</td>
<td></td>
</tr>
<tr>
<td>630</td>
<td>0.161</td>
<td>0.137</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>0.210</td>
<td>0.211</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>0.263</td>
<td>0.272</td>
<td></td>
</tr>
<tr>
<td>1250</td>
<td>0.246</td>
<td>0.271</td>
<td></td>
</tr>
<tr>
<td>1600</td>
<td>0.261</td>
<td>0.270</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.290</td>
<td>0.330</td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.332</td>
<td>0.377</td>
<td></td>
</tr>
<tr>
<td>3150</td>
<td>0.390</td>
<td>0.147</td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>0.473</td>
<td>0.144</td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>0.575</td>
<td>0.145</td>
<td></td>
</tr>
<tr>
<td>6300</td>
<td>0.712</td>
<td>0.145</td>
<td></td>
</tr>
<tr>
<td>8000</td>
<td>0.895</td>
<td>0.145</td>
<td></td>
</tr>
</tbody>
</table>

Modal density of a thin cylinder increases with frequency, reaches a maximum and then converges to the modal density of flat plates. The transverse shear deformation increases modal density. The influence is negligible at low frequencies but very significant at high frequencies. At high frequencies due to the presence of transverse shear deformation modal densities increase with frequency whereas it remains constant if the transverse shear deformation is neglected. Figure 4 clearly indicates that in the absence of the present expression, modal density calculated by neglecting transverse shear can result in significant error at higher frequencies.

Modal densities, as a function of frequency, for different values of $N$ are shown in Fig. 5. Results show that modal density increases sharply with increase in shear flexibility and the effect is larger at high frequencies. When the shear rigidity is very high the modal densities of the cylinder are identical to those of thin cylinder.

7.2. Orthotropic Nature of the Cylinder

Modal density is also influenced by the in-plane shear property, apart from other factors, which is represented by the parameters $\beta$ and $\gamma$. As both the parameters have similar effects on the modal density, results for various values of $\gamma$ only are presented.

A typical cylinder whose properties are same in both the directions, given in Section 3, is considered. Modal densities for various values of $\gamma$ are shown in Fig. 6. The results show that the parameter $\gamma$ has a significant role and it has similar influence at all frequencies. Modal densities increase with decrease in the values of $\gamma$. In other words, modal densities increase with decrease in in-plane shear modulus and Poisson’s ratio. For an isotropic material $\gamma = 2$ and it is equal to 0.17 for the cylinder considered.

7.3. Use of Expression for Isotropic Cylinders

In the absence of the expression derived in this work one could use the available expression reported in the literature which is for isotropic materials. The term $D_{11}$ of the given cylinder can be equated to $D$ of the equivalent isotropic cylinder and the thickness can be worked out. Wilkinson’s expression can now be used to obtain the modal densities. The cylinder whose details are given in Section 3 is considered. The isotropic cylinder is a honeycomb sandwich cylinder with the same core as the given cylinder but face sheets made of Aluminium alloy having Young’s modulus $7.2 \times 10^10$ N/m$^2$. The thickness of the face sheet of the isotropic cylinder is 0.3 mm. The results are given in Fig. 7.
The results show that the estimated modal densities are very much lower if isotropic models are used signifying the need for the expression derived here.

8. CONCLUSIONS

Expressions for estimating the natural frequency, mode count and modal density of composite cylindrical shells incorporating transverse shear deformation are derived. Modal densities of typical cylinders of spacecraft are obtained. The results are in accordance with the number of modes determined using the finite element method. Transverse shear deformation increases modal density and the impact is very significant at higher frequencies. Effect of the orthotropic nature of the face sheets is to increase the modal density further but its impact is present at all frequencies. It is shown that in the absence of the expression derived here the modal densities computed will be in significant error.

REFERENCES

The Influence of Elastic Boundary on Modal Parameters of Thin Cylindrical Shell

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This research combines theory with experiment to investigate the influence of an elastic boundary on modal parameters of a thin cylindrical shell (TCS). First, artificial stiffness method and finite element method (FEM) are employed to calculate natural frequencies and modal shapes of TCS under condition such that vibration characteristics of elastically supported shell can be roughly mastered. Then, the following measurements and identification techniques are used to get precise frequency, damping, and shape results: non-contact laser Doppler vibrometer and vibration shaker with excitation level being precisely controlled are used in the test system; “pre-experiment” is adopted to determine the required tightening torque as well as to verify whether or not the tested shell is under constraint boundary; and small-segment FFT processing technique is employed to accurately measure nature frequency, and laser rotating scanning technique is used to get shape results with high efficiency. Finally, based on the accurate measured data, the influences on natural frequencies, modal shapes, and damping ratios of TCS under elastic boundary are analysed and discussed. It can be found that an elastic boundary can significantly affect frequency and damping results, clearly reducing high order damping and decreasing natural frequencies of most modes. However, high order natural frequencies and mode shapes are still the same as the ones under the constraint condition, and the changing trend of natural frequencies with mode shapes is constant when the order of axial mode is $m = 1$, which agrees well with the results calculated by artificial stiffness method and FEM.

1. INTRODUCTION

Thin cylindrical shell (TCS) has long been an important structural component due to its high stiffness to weight and strength to weight ratios, which is widely used in engineering fields, such as aircraft casings, pipes and ducts, rotary drums in granulators and aircraft engines. Modal parameters of TCS are mainly composed of natural frequencies, modal shapes, and damping ratio, and these parameters are the basis of further study on vibration characteristics of TCS, which are of great importance to theoretical modelling, response prediction, vibration reduction optimization, vibration mechanism research, structural damage identification etc.

In engineering practice, in order to reduce the vibration stress and suppress vibration fatigue of TCS, some vibration reduction techniques are used artificially. For example, friction damping snubber, metal rubber, elastic rubber and other material are gradually used to substitute traditional constraint or fixed boundary for elastic boundary, which can bear proper extrusion deformation to absorb and consume structural vibration energy. At present, the advantages of elastic boundary are recognized by an increasing number of engineers and researchers, and it has been approved to be an effective application in vibration control of the shell. There are also shell structures running without under complete restraint boundary, or this may be called elastic boundary in a wider sense, especially when it is excited by the complex external loads, such as high-speed aerodynamic load and centrifugal load. Besides, TCS are often installed or connected by welding, riveting, and bolting, which can inevitably lead to loose or elastically restrained condition; thus, it is hard to ensure whether the shell is working under complete restraint boundary, and elastically supported shell structures are common in engineering field.

Currently, great efforts have been made to study vibration character of TCS under elastic boundary by scholars and researchers, and many encouraging research results have been obtained. For example, Forsberg studied the influence of boundary conditions on the modal characteristics of thin cylindrical shells. Total 16 possible sets of homogeneous boundary conditions were specified independently at each end of the shell, and these sets of conditions were discussed in detail. It has been found that even for long cylinders (length to radius ratio of 40 or more) the minimum natural frequency may differ...
by more than 50% depending upon whether the displacement of any point on the middle surface is in axial direction \( u = 0 \) or the longitudinal stress resultant is \( N_x = 0 \) at both ends. Koga studied the effects of boundary conditions on the free vibrations of TCS and a simple formula for the natural frequency was derived as an asymptotic solution for the eigenvalue problem of the breathing vibrations, whose accuracy was sequentially examined by a comparison with numerical solutions and experimental results. The results showed that the formula was accurate enough for engineering and it was applicable under any possible combinations of the boundary conditions for the simply supported, clamped, and free ends of the shell. Ama- bili and Dalpiaz studied the vibrations of circular cylindrical shells with non-axisymmetric mass distribution on elastic bed, and Rayleigh-Ritz method and Fourier series method were used to obtain natural frequencies and modal shapes of the shell. Loveday and Rogers analysed the free vibration of TCS with elastic boundary conditions by application of the exact solution of the Flugge shell theory equations of motion. The elastic boundaries were represented by distributed linear springs, and elastic boundary conditions could be simulated by varying the eight spring constants. Liang et al. studied stiffness optimization of TCS under elasticity boundary condition. The explicit formula of initial parameter solution of variable thickness shell was derived by transfer matrix method, and the optimization process was transformed into a constraint nonlinear solving process; thus, the objective function can be successfully obtained by the stepped reduction method. Zhou et al. used wave propagation method to solve the equations of motion of TCS under elastic-support boundary condition, and the elastic-support boundary condition was specified in terms of eight independent sets of distributed springs which have arbitrary stiffness values. Besides, the effects on natural frequencies of the restraining springs were also studied for a range of stiffness values and different geometrical characteristics of the shells, and it was found that the restraining stiffness can drastically affect frequency parameters of TCS. Sun studied free vibration and dynamic response of rotating TCS and employed three methods to analyse its natural characteristics under the different boundary condition, including Fourier series expansion method and wave propagation approach for TCS with classical boundary conditions, and Rayleigh-Ritz method with artificial spring for TCS with elastically constrained condition. Wu et al. studied vibration of a joined cylindrical-spherical shell under elastic boundary by a domain decomposition method. The elastic-support boundary was regarded as a combination of distributed linear springs and can be treated as a special interface as well as the interface between two adjacent shell segments, and the theoretical results were compared with those derived by ANSYS to confirm the reliability and accuracy. Chen et al. studied vibration characteristics of cylindrical shell under complicated boundary conditions, such as elastic and non-uniform boundary conditions, and the exact solution was obtained by improved Fourier series method, but the calculated results were only compared with the finite element results rather than experimental results.

However, most of researches done by the above scholars and researchers are mainly based on theory or simulation; experimental studies on the influence of elastic boundary on modal parameters of TCS are still scarce. Besides, due to the complexity of the damping mechanism, it is difficult to obtain the reliable damping parameters of the shell-only-based theoretical model, let alone analysing the related influence on damping parameters. Therefore, it is necessary to combine theory with experiment to investigate the influence of elastic boundary on modal parameters of TCS.

This research has investigated the influence of elastic boundary on modal parameters of TCS. First, in Section 2 artificial stiffness method (ASM) and finite element method (FEM) are employed to calculate natural frequencies and modal shapes of TCS under conditions such that vibration characteristics of elastically supported shell can be roughly mastered. Then, experiment system is set up to accurately measure modal parameters of TCS, and the corresponding test procedures and identification techniques suitable for elastically supported shell are proposed in Section 3. Finally, in Section 4, based on the accurate measured data, the influence on natural frequencies, modal shapes, and damping ratios of TCS under elastic boundary are analysed and discussed in detail. This research can provide dynamic modelling service for TCS under complex boundary condition, provide experimental data for effective selection of boundary parameters in the theoretical model, and also provide an important reference for diagnosis of vibration fault of elastically supported shell.

### 2. THEORETICAL ANALYSIS OF NATURE FREQUENCY AND MODAL SHAPE OF TCS UNDER ELASTIC BOUNDARY

#### 2.1. Research Object and Simulation of Elastic Boundary

The TCS studied in this research is shown in Fig. 1 and its dimension and material parameters are listed in Table 1 and Table 2, respectively. There is an extension edge with 150 mm external radius and 3 mm thickness on this shell which is machined to be clamped by a clamping-ring with eight M8 bolts, so that it can be of certain that the shell be restricted under fixed-free boundary (or called under constraint boundary with one end free). Then, we can simulate different elastic boundary conditions by filling rubber ring of different thickness into the position between the clamping-ring and extension edge and tightening M8 bolts with certain tightening torque, as seen in Fig. 1. For example, we can fill 1 mm, 2 mm, and 3 mm rubber rings to simulate the various types of elastic boundaries, and the related material parameters of ZN33 elastomer used in this research are listed in Table 3.

#### 2.2. Artificial Stiffness Method

In practice, elastic boundary conditions commonly exist in the connection positions of TCS and complete constraint

### Table 1. Dimension parameters of TCS

<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>Thickness (mm)</th>
<th>Internal radius (mm)</th>
<th>External radius (mm)</th>
<th>Extension edge radius (mm)</th>
<th>Extension edge thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>95</td>
<td>2</td>
<td>142</td>
<td>144</td>
<td>150</td>
<td>3</td>
</tr>
</tbody>
</table>

### Table 2. Material parameters of TCS.

<table>
<thead>
<tr>
<th>Name</th>
<th>Elastic modulus (Gpa)</th>
<th>Poisson ratio</th>
<th>Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structural steel</td>
<td>212</td>
<td>0.3</td>
<td>7850</td>
</tr>
</tbody>
</table>
under elastic boundary based on ASM. Figure 2 gives the dynamic model of TCS under elastic boundary, whose length, radius, and thickness is $L$, $R$, and $H$. First, the assumptions that the TCS studied in this research can meet isotropic and homogeneity hypothesis and the influences of shear stress as well as boundary damping are ignored. Then, the curvilinear coordinates system $x - y$ is used in this model by taking the generatrix and parallel direction of TCS as two main directions, any point on the middle surface of the shell can be described by $(x, \theta)$, where $x$ is the generatrix length from the vertex to the point $(x, \theta)$ and $\theta$ is the corresponding rotation angle. Besides, $u$, $v$ and $w$ is the displacement of any point on the middle surface in the $x$, $\theta$, $z$ direction, and $k^u$, $k^v$, $k^w$ is the support stiffness in $u$, $v$ and $w$ direction while $k^\theta$ is the rotation stiffness.

In above method, boundary damping is ignored while the influence of the artificial spring is taken into consideration, and different support stiffnesses of artificial spring are used to simulate different elastic boundaries by changing the magnitude of $k^u$, $k^v$, $k^w$, $k^\theta$. According to the linear shell theory and small deformation theory, the strain equation in the middle surface of the shell can be described as:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_x \theta \end{bmatrix}_{(0)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}; \quad (1)$$

where subscript $(0)$ refers to the middle surface. Based on Donnell-Mushtari shell theory, the curvature coefficient $\kappa_x, \kappa_\theta$ and twist coefficient $\chi_x \theta$ at the middle surface are:

$$\begin{bmatrix} \kappa_x \\ \kappa_\theta \\ \chi_x \theta \end{bmatrix} \approx \begin{bmatrix} 0 & 0 & \frac{-1}{2} \frac{\partial^2}{\partial R \partial \theta} \\ 0 & \frac{1}{R} \frac{\partial}{\partial \theta} & \frac{-1}{2} \frac{\partial^2}{\partial R \partial \theta} \\ 0 & 0 & \frac{1}{R} \frac{\partial}{\partial \theta} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}. \quad (2)$$

Because the deformation of the shell is closely related with the displacement of middle surface, the strain at any point in the shell can be written in terms of strain, curvature, and twist coefficients in the middle surface and it can be described as:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_x \theta \end{bmatrix} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_x \theta \end{bmatrix}_{(0)} + z \begin{bmatrix} \kappa_x \\ \kappa_\theta \\ \chi_x \theta \end{bmatrix}; \quad (3)$$

where $z$ is the distance from any point in the shell to the middle surface.

The physical equation of the shell can be expressed as:

$$\sigma_x = \frac{E}{1-\mu^2}(\varepsilon_x + \mu \varepsilon_\theta); \quad (5)$$

$$\sigma_\theta = \frac{E}{1-\mu^2}(\varepsilon_\theta + \mu \varepsilon_x);$$

$$\tau_{x\theta} = G\varepsilon_{x\theta};$$

where $E$ is Young’s modulus and $\mu$ is Poisson’s ratio.

The expression of kinetic energy of TCS can be described as:

$$T = \frac{\rho H L}{2} \int_0^{2\pi} \int_0^1 (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) R d\theta d\xi; \quad (6)$$

where $\xi = xL, (\cdot) = \partial/\partial \xi$ is the partial derivative of time.

Taking bending and shear deformation of shell into consideration, the potential energy of the shell can be expressed as:

$$U_z = \frac{H L}{2} \int_0^{2\pi} \int_0^1 (\sigma_x \varepsilon_x + \sigma_\theta \varepsilon_\theta + \tau_{x\theta} \gamma_{x\theta}) R d\theta d\xi. \quad (7)$$

Simplifying Eq. (7), we can deduce that:

$$U_z = \frac{L H}{2} \int_0^{2\pi} \int_0^1 \left( \frac{E}{1-\mu^2} (\varepsilon_x^2 + \varepsilon_\theta^2) + \frac{2G}{1-\mu^2} \varepsilon_{x\theta} + G(\gamma_{x\theta})^2 \right) R d\theta d\xi. \quad (8)$$
where $G = \frac{E}{2(1 + \nu)}$ is the shear modulus.

Because the studied TCS is under free-clamped boundary condition, we can employ ASM to place artificial spring on two sides of the shell, as shown in Fig. 2. It should be noted that in this method, when the support stiffness $k_{0i} = k_{0j} = k_{0k} = 0$, and $k_1 = k_2 = k_3 = k_4 = 0$, it represents the free-clamped boundary condition, and the subscript 0 denotes the fixed end while the subscript 1 denotes the free end of the shell. When support stiffness $k_0 = k_0 = k_0 = k_0 = 10^4 \sim 10^8$, and $k_1 = k_2 = k_3 = k_4 = 0$, it represents the elastic-free boundary condition. Therefore, by changing the magnitude of $k_1, k_2, k_3, k_4$, we can simulate different elastic boundaries, and the related elastic potential energy of artificial spring under such boundary can be described as:

$$U_{sp} = \frac{1}{2} \int_0^{2\pi} \left\{ k_{0u}[u(0, \theta, t)]^2 + k_{0v}[v(0, \theta, t)]^2 \right. $$

$$+ k_{0w}[w(0, \theta, t)]^2 k_{ef} \frac{1}{L_2} \left[ \frac{\partial w(0, \theta, t)}{\partial \xi} \right]^2 \right\} R d\theta \tag{9}$$

$$+ \frac{1}{2} \int_0^{2\pi} \left\{ k_{1u}[u(1, \theta, t)]^2 + k_{1v}[v(1, \theta, t)]^2 \right. $$

$$+ k_{1w}[w(1, \theta, t)]^2 k_{ef} \frac{1}{L_2} \left[ \frac{\partial w(1, \theta, t)}{\partial \xi} \right]^2 \right\} R d\theta.$$

Next, we use orthogonal polynomials to assume shape functions of the shell, and the Rayleigh-Ritz method is employed to calculate the natural characteristics of TCS under elastic boundary. According to Rayleigh-Ritz method, we can assume that the vibration displacement of the shell has the following forms:

$$\left\{ \begin{array}{l}
    u = e^{-j\omega t}\cos n\theta U(\xi) \\
    v = e^{-j\omega t}\sin n\theta V(\xi) \\
    w = e^{-j\omega t}\cos n\theta W(\xi)
\end{array} \right. \tag{10}$$

Here, $\omega$ is the natural frequency, $n$ is the circumferential waves of the TCS, and $U(\xi), V(\xi), W(\xi)$ are modal shape functions in $x$, $\theta$ and $z$ direction, respectively.

Then transforming $U(\xi)$, $V(\xi)$, and $W(\xi)$ into polynomial forms, we have the following expression:

$$U(\xi) = \sum_{m=1}^{NT} a_m \varphi^u(\xi)$$

$$V(\xi) = \sum_{m=1}^{NT} b_m \varphi^v(\xi) \tag{11}$$

$$W(\xi) = \sum_{m=1}^{NT} c_m \varphi^w(\xi)$$

Here, $a_m$, $b_m$, and $c_m$ are shape coefficients, $\varphi^u(\xi), \varphi^v(\xi)$, and $\varphi^w(\xi)$ are orthogonal polynomials, which should satisfy the geometrical boundary conditions.

Construst the following equation:

$$\psi^2_2(\xi) = (\xi - B_2) \psi^2(\xi);$$

$$\psi^k_p(\xi) = (\xi - B_2) \psi^k_p(\xi) = C_k \psi^{k-2}_k(\xi), k;$$

where $B_2 = \frac{\psi^2_{,\xi}}{\psi^2_{,\xi}(\xi=0)^{2\xi}}$, $C_k = \frac{\psi^{k-2}_{,\xi}}{\psi^{k-2}_{,\xi}(\xi=0)^{2\xi}}$

Define the 2-norm of $\psi_k(\xi)$ as the following:

$$\|\psi_k(\xi)\|_2 = \sqrt{\int_0^1 [\psi^k_2(\xi)]^2 d\xi; (p = u, v, w) \tag{13}}$$

Define $\psi^p_k(\xi) = \psi^p_k(\xi)/\|\psi^k_2(\xi)\|_2$ as the orthogonal basis, which can be expressed as:

$$\psi^p_k(\xi) = \psi^p_k(\xi) = \frac{\psi^k_2(\xi)}{\|\psi^k_2(\xi)\|_2} \tag{14}$$

The orthogonal basis function needs to satisfy the following relation:

$$\int_0^1 \psi^p_k(\xi) \psi^s_k(\xi) d\xi = \delta_{ks}, (p = u, v, w); \tag{15}$$

where $\delta_{ks}$ is Kronecker function.

Substituting Eq. (10) into Eq. (6), and further simplifying the equation, we can obtain the expression of kinetic energy of the shell:

$$T = -\frac{\pi \rho H R L^2}{2} e^{-2j\omega t} \int_0^1 [U^2 + V^2 + W^2] d\xi. \tag{16}$$

Substituting Eq. (10) into Eq. (7), and further simplifying the equation, we can obtain the potential energy expression of the shell:

$$U_e = -\frac{L E H \pi}{2 R (1 - \mu^2)} e^{-2j\omega t} \int_0^1 \left( \frac{H^2 n^4}{12 R^2} + 1 \right) W^2 +$$

$$\left( \frac{n^2 H^2}{6 R^2} V^2 + \frac{n^3 H^2 + 2n}{6 R^2} W^2 \right) + (1 - \mu) \left( \frac{n^2 H^2}{6 L^2} W^2 - \frac{R}{L} V W_s \right) \tag{17}$$

Substituting Eq. (10) into Eq. (9), and further simplifying the equation, the elastic potential energy expression of the artificial spring can be expressed as:

$$\psi^u_{sp} = \left\{ \begin{array}{l}
    \tilde{k}_0 u[U(0)]^2 + \tilde{k}_0^2 V(0)]^2 + \\
    \tilde{k}_0 w[W(0)]^2 + \frac{1}{2} \left[ \frac{\partial W(0)}{\partial \xi} \right]^2 \end{array} \right. \tag{18}$$

Here, $\tilde{k}_0 u$, $\tilde{k}_0 v$, and $\tilde{k}_0 w$ are dimensionless stiffness, and these parameters can be expressed as follow:

$$\tilde{k}_0 u = \frac{R^2 (1 - \mu^2)}{E H L}; \quad \tilde{k}_0 v = \frac{R^2 (1 - \mu^2)}{E H L};$$

$$\tilde{k}_0 w = \frac{R^2 (1 - \mu^2)}{E H L}; \quad \tilde{k}_0 w = \frac{R^2 (1 - \mu^2)}{E H L}; \tag{19}$$

where $i = 0, 1$, and 0 refers to the fixed end while 1 refers to the free end.

According to Rayleigh-Ritz method, the dimensionless frequency of TCS can be written as follows:

$$(\omega^*)^2 = \frac{U^* + U^* s}{T^*} \tag{20}$$

Here,

$$\omega^* = \omega R \sqrt{\rho (1 - \mu^2)} / E \tag{21}$$
used to calculate the natural frequencies and modal shapes, so a model of TCS under elastic boundary is established, which is

\[ T^* = \int_0^1 \left[ U^2 + V^2 + W^2 \right] \, d\xi; \]  \tag{22}

\[ U^* = \int_0^1 \left( \frac{H^2n^4}{12R^2} + 1 \right) W^2 + \frac{n^2H^2}{12R^2} V^2 + \left( \frac{n^3H^2}{6R^2} + 2n \right) V^2 W + \frac{R^2}{L^2} U^2 \]
\[ + n^4V^2_s + (1 - \mu) \left( \frac{n^2U^2}{2} + \frac{n^2H^2}{6L^2} W^2 V^2 + \left( \frac{R^2}{2L^2} + \frac{H^2}{24L^2} \right) V^2 + \frac{n^2H^2}{6L^2} W^2 - n \frac{R}{L} V^2 U_s \right) + 2\mu R L U_s W + \frac{H^2R^2}{12L^4} W^n W - \mu m^2 H^2 \frac{s}{6L^2} W^n V_s + 2\mu R L U_s V_s \right] \, d\xi; \]  \tag{23}

Taking the partial of shape coefficients \( a_m, b_m, \) and \( c_m \) which are related to \( U(\xi), V(\xi), \) and \( W(\xi) \) in Eq. (11), we can have the characteristic equation in Eq. (25), and bring different \( k_{10}, k_{1w}, k_{01}, k_{w0} \) and \( k_{1e}, k_{w1}, k_{1c}, k_{c1} \) into this equation, we can calculate the natural frequency of TCS under elastic boundary by ASM.

\[ \frac{\partial \omega^*}{\partial a_m} = \frac{\partial \omega^*}{\partial b_m} = \frac{\partial \omega^*}{\partial c_m} = 0; \]  \tag{24}

\[ [K + K_{spr} - \omega^2 M] X = 0; \]  \tag{25}

\[ K = \begin{bmatrix} k_{aa} & k_{ab} & k_{ac} \\ k_{ba} & k_{bb} & k_{bc} \\ k_{ca} & k_{cb} & k_{cc} \end{bmatrix}; \]  \tag{26}

\[ K_{spr} = \begin{bmatrix} k_{saa} & k_{sab} & k_{sac} \\ k_{sba} & k_{sbb} & k_{sbc} \\ k_{sca} & k_{scb} & k_{scc} \end{bmatrix}; \]  \tag{27}

\[ M = \begin{bmatrix} M_{aa} & M_{ab} & M_{ac} \\ M_{ba} & M_{bb} & M_{bc} \\ M_{ca} & M_{cb} & M_{cc} \end{bmatrix}; \]  \tag{28}

Here, \( X = (a_m \ b_m \ c_m)^T \) is Ritz vector, \( K, K_{spr} \), and \( M \) are stiffness matrix, artificial spring stiffness matrix, and mass matrix, respectively, and the expression of the elements in the stiffness matrix, artificial spring stiffness matrix, and mass matrix can be seen in Appendix A, B, C. Besides, it should be noted that the numbers of row and column of \( K, K_{spr}, \) and \( M \) are \( 3NT \times 3NT \), where \( NT \) is the number of orthogonal polynomials. In this research in order to ensure calculation accuracy, we set \( NT = 7(3NT = 21) \), so the numbers of row and column of \( K, K_{spr}, \) and \( M \) are \( 21 \times 21 \).

### 2.3. Finite Element Method (FEM)

In this section, different thickness of rubber rings are used to simulate various types of elastic boundaries, and finite element model of TCS under elastic boundary is established, which is used to calculate the natural frequencies and modal shapes, so that vibration characteristics of elastically supported shell can be roughly mastered.

Finite element model of TCS under elastic boundary condition is established with Ansys Parametric Design Language (APDL) in ANSYS software, as seen in Fig. 3. SOLID186 element is used to create the model of the shell which consists of 6480 nodes and 960 elements, and MATRIX27 element is used as spring element to simulate different thickness of rubber rings in the elastic boundary by adjusting stiffness value in the \( X, Y, Z \) directions, which consists of a total of 8 spring elements and 80 nodes.

Eight spring elements with stiffness value of \( k = 1 \times 10^7, k = 1 \times 10^6, k = 1 \times 10^5 \) are used to simulate the elastically supported shell with the thicknesses of 1 mm, 2 mm, and 3 mm, as seen in Fig. 4 named as elastic boundary I, elastic boundary II, and elastic boundary III, respectively, and then the natural frequencies and modal shapes can be calculated by Block Lanczos method. The stiffness value of the eight spring elements is set to \( k = 1 \times 10^9 \), so as to simulate the free-clamped boundary condition (also see in Fig. 4), and the same method is employed to calculate the frequency and shape parameters of TCS under constraint boundary.

### 2.4. Theoretical Analysis Results and Conclusion

In the above section, ASM and FEM are used to establish the theoretical model of elastically supported shell. In this section, the theoretical analysis results are analysed and discussed in detail.

First, set stiffness value of eight spring elements to \( k = 1 \times 10^9, k = 1 \times 10^7, k = 1 \times 10^6, k = 1 \times 10^5 \) in the \( X, Y, Z \) directions, and finish the calculation work of frequency and shape parameters of TCS under constraint boundary as well as elastic boundary with different thickness of rubber ring. The first eight natural frequencies of modal shapes calculated by FEM are listed in Table 4 and Table 5. Then, set the support stiffness as \( k_u = k_v = k_w = k_\theta = 10^9, 10^7, 10^6, 10^5 \)
and ASM method is used to calculate frequency results under the above two kinds of boundary conditions, as seen in Table 4. Further, the differences between FEM and ASM are also given in Table 4, and Fig. 5 gives the relation between natural frequency and modal shape calculated by the two theoretical methods. It should be noted that in the shape results calculated by FEM, \( m \) represents the order of axial mode and \( n \) represents the number of circumferential waves of TCS.

From Table 4, Table 5, and Fig. 5, it can be found that:

(I) The elastic boundary has great influence on the inherent properties of the shell, which will lead to the decrease of the natural frequency, e.g., the reduced values of the first 8 natural frequencies are about 10 ~ 100 Hz. (II) For some high order modes, their decreased degrees are very small. Taking the 7th and 8th natural frequencies for an example, the decreased degrees are less than 1%. (III) Although frequencies and shapes of TCS will be changed by different kinds of elastic boundaries, the changing trend of natural frequencies with increased degrees are less than 1%. (IV) Although frequencies and shapes of TCS with different elastic boundaries, the changing trend of natural frequencies with increased degrees are less than 1%. (V) Although frequencies and shapes of TCS with different elastic boundaries, the changing trend of natural frequencies with increased degrees are less than 1%. (VI) Although frequencies and shapes of TCS with different elastic boundaries, the changing trend of natural frequencies with increased degrees are less than 1%.

3. TEST SYSTEM AND METHOD OF MODAL PARAMETERS OF TCS UNDER ELASTIC BOUNDARY

In Section 2, vibration characteristic of TCS under elastic boundary and its influence is analysed. But due to the complexity of elastic boundary, the real influence of such boundary on modal parameters, especially the damping characteristics of the shell cannot be accurately analysed by ASM or FEM. Therefore, it is necessary to employ experimental test to investigate the influence of elastic boundary on modal parameters of TCS. In this section, experiment system is first established to accurately measure modal parameters of the shell, and the corresponding test procedures and identification techniques suitable for the thin walled shell are also proposed.

3.1. Test System of Modal Parameters of TCS Under Elastic Boundary

On the one hand, due to light weight, closed modes, low level and complicated local vibration of TCS, traditional accelerometer will bring added mass and stiffness to the tested shell, which will severely affect the tested frequency and damping results.\(^2\) So laser Doppler vibrometer is used as non-contact response sensor to measure the vibration and frequency information of the shell. On the other hand, different excitation techniques also will result in test error, so the disadvantages of four common vibration excitation devices are compared in Table 6, and vibration shaker is finally chosen as excitation source with excitation level being precisely controlled, and test system of modal parameters of TCS under elastic boundary is given in Fig. 6. The instruments used in the test are as follows: (I) Polytec PDV-100 laser Doppler vibrometer; (II) king-design EM-1000F vibration shaker systems; (III) LongWei PS-305DM DC power supply; (IV) Aslong JGA25 DC geared motor; (V) 45° rotation mirror and 45° fixed mirror; (VI) LMS SCADAS Mobile Front-End and Dell notebook computer.

In these devices, LMS SCADAS Mobile Front-End and Dell notebook computer are responsible for recording and saving response signal from laser Doppler vibrometer. Dell notebook computer with Intel Core i7 2.93 GHz processor and 4G RAM is used to operate LMS Test.Lab 12B software and store measured data. For the frequency and damping test, sine sweep excitation is conducted with a closed loop control via accelerometer on the countertop of the vibration shaker, and point 1, point 2, and point 3 (being 120° with each other) are used to get response signal by adjusting laser point and average is used as the final results. In this test, natural frequency can be precisely determined through each resonant peak in frequency domain, and damping ratio can also be identified by the half-power bandwidth method which is calculated by measuring the bandwidth of the frequency curve (or approximately 3 dB) down from the resonant peak. For modal shape test, laser rotating scanning technique is used to get shape results of TCS. First, one of natural frequencies of TCS is employed to drive the tested shell under the resonance state by vibration shaker, and then DC power supply is used to provide stable voltage and current for DC geared motor. The motor is used to drive the 45° rotation mirror to complete a set of cross sectional scans with 360° circumferential coverage for the tested shell, and in this way modal shape data at certain modes can be obtained in a shorter amount of time than traditional test methods.

3.2. Test Method of Modal Parameters of TCS Under Elastic Boundary

In this section, the test and identification techniques suitable for elastically supported shell are described in detail, as seen in the following three key steps.

3.2.1. Accurately determine tightening torque under constraint boundary

Because modal parameters of TCS are closely related to constraint boundary, in actual test, we must ensure that one end of the tested shell will be effectively clamped. To this end, a torque wrench is used to determine the level of tightening torque on the M8 bolts of clamping-ring, as seen in Fig. 1, and the “pre-experiment” is adopted to determine the required tightening torque as well as to verify whether or not the tested shell is under constraint boundary. For instance, it should be done at least three times to test natural frequencies, and every time the same level of torque value should be applied on M8 bolts. If test results of the first three natural frequencies under three pre-experiments are close to each other (for example 1 ~ 3 Hz), we will regard this torque value as the determined tightening torque under constraint boundary. If the differences between each natural frequency are big, more than 5 ~ 20 Hz,
we need to increase torque value and repeat pre-experiments several times.

3.2.2. Measure modal parameters of TCS under constraint boundary

This step involves three different measurements and identification techniques. First, using sine sweep excitation by vibration shaker to test natural frequencies of TCS, and in order to get precise frequency results, the small-segment FFT processing technique is employed to deal with the measured sweep signal. The time domain signal involving the 3rd natural frequency of the tested shell is shown in Fig. 7(a). If FFT processing technique is directly applied on this sweep signal, we can obtain its frequency spectrum, as seen in Fig. 7(b), and the frequency of the response peak is 1024.8 Hz. However, if the whole time of sweep signal can be divided into small segments, and we conduct FFT on each segment of them (in this example, it is 1s with respect to the whole time of 68 s), the resulting frequency spectrum, as seen in Fig. 7(c), is plotted through the combination of the response peak of each segment (also treated with interpolation and smoothing). The frequency value related to the peak is 1025.7 Hz, which is truly accurate result of the 3th natural frequency. Therefore, for time-dependent sweep signal of TCS, it is necessary to use the small-segment FFT processing technique to accurately get frequency results.

Then, use the half-power bandwidth technique to identify each damping ratio of TCS from the frequency spectrum obtained by small-segment FFT processing technique. Because the resonant peak in the spectrum is already known, we can identify two half-power bandwidth points by measuring the bandwidth of the frequency curve (or approximately 3 dB) down from the resonant peak, consequently according to the damping formula to calculate the corresponding damping results based the MATLAB program. Figure 8 gives time waveform and frequency spectrum for the 3rd natural frequency and damping ratio of TCS at three measuring points. In order to

### Table 4. Calculated natural frequencies of TCS under different elastic boundaries by ASM and FEM.

<table>
<thead>
<tr>
<th>Modal order</th>
<th>Constraint boundary</th>
<th>Elastic boundary I</th>
<th>Elastic boundary II</th>
<th>Elastic boundary III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AEM</td>
<td>FEM</td>
<td>Difference (A-B)/B (%)</td>
<td>AEM</td>
</tr>
<tr>
<td>1</td>
<td>924.5</td>
<td>910.4</td>
<td>1.5</td>
<td>901.4</td>
</tr>
<tr>
<td>2</td>
<td>987.4</td>
<td>965.4</td>
<td>2.3</td>
<td>966.5</td>
</tr>
<tr>
<td>3</td>
<td>1006.4</td>
<td>986.5</td>
<td>2.0</td>
<td>988.3</td>
</tr>
<tr>
<td>4</td>
<td>1089.2</td>
<td>1068.7</td>
<td>1.9</td>
<td>1065.2</td>
</tr>
<tr>
<td>5</td>
<td>1323.6</td>
<td>1313.8</td>
<td>0.7</td>
<td>1312.5</td>
</tr>
<tr>
<td>6</td>
<td>1365.4</td>
<td>1355.9</td>
<td>0.2</td>
<td>1348.8</td>
</tr>
<tr>
<td>7</td>
<td>1618.1</td>
<td>1608.2</td>
<td>0.6</td>
<td>1584.6</td>
</tr>
<tr>
<td>8</td>
<td>2017.5</td>
<td>2000.8</td>
<td>0.8</td>
<td>2009.6</td>
</tr>
</tbody>
</table>

### Table 5. Calculated modal shapes of TCS under different elastic boundaries by FEM.

<table>
<thead>
<tr>
<th>Modal order</th>
<th>Constraint boundary</th>
<th>Elastic boundary I</th>
<th>Elastic boundary II</th>
<th>Elastic boundary III</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1,4)</td>
<td>(1,4)</td>
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<td>(1,4)</td>
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<tr>
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</tbody>
</table>
Table 6. Disadvantages of different vibration excitation devices for modal test of TCS under elastic boundary.

<table>
<thead>
<tr>
<th>Excitation device</th>
<th>Modal parameters of TCS</th>
<th>Disadvantage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Natural frequency</td>
<td>Mode shape ratio</td>
</tr>
<tr>
<td>Hammer</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Electromagnetic exciter</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Piezoelectric Ceramic exciter</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Vibration shaker</td>
<td>√</td>
<td>√</td>
</tr>
</tbody>
</table>

Pulse excitation level can not be precisely controlled and the excitation force varies for each measurement, and double hit can often lead to test errors.

The related force sensor will bring added mass and stiffness to TCS, which will severely affect test results of damping and natural frequency.

The excitation energy of piezoelectric ceramic exciter is often insufficient for TCS, which will result in poor response signal with low level of signal noise ratio.

Excitation frequencies are not that high, which are often limited to 1 Hz ~ 3000 Hz, and the test procedures are often complicated.

Figure 6. Schematic of test system of modal parameters of TCS under elastic boundary.

Improve accuracy of frequency and damping results, the final results is obtained by averaging the test results at these points.

Finally, using each natural frequency to excite TCS at resonance state, each modal shape with obvious reduction in time costs by laser rotating scan method is obtained.

3.2.3. Measure modal parameters of TCS under elastic boundary

After finishing the measurement work under constraint boundary, we can fill 1 mm, 2 mm, and 3 mm rubber ring made by ZN33 elastomer into the position between the clamping-ring and extension edge to simulate different elastic boundary conditions, which named as elastic boundary I, elastic boundary II, and elastic boundary III respectively. Then, experimental test is conducted to get natural frequencies, modal shapes, and damping ratios with the same test methods, such as the small-segment FFT processing technique, the half-power bandwidth technique, and laser rotating scan technique used in the above two steps. It should be noted that the excitation level and the position of the three measuring points must be the same as the ones under constraint boundary.

4. INFLUENCE ANALYSIS OF MODAL PARAMETERS OF TCS UNDER ELASTIC BOUNDARY

In this section, on the basis of both theoretical and experimental results, i.e., the simulation results calculated by ASM and FEM in Section 2 and the accurate measured data obtained by the test system and test method described in the Section 3, the influence on natural frequencies, modal shapes, and damping ratios of TCS under elastic boundary are analysed and discussed in detail.

4.1. Test Results of Modal Parameters of TCS Under Elastic Boundary

According to the proposed test method and procedures in Section 3, point 1, point 2, and point 3 are used as the response points, which are 120° with each other, and in the same cross section of the shell, the axial distance from this section to free end of TCS is about 5 mm, as seen in Fig. 6. For the natural frequency and damping test, the following set-ups and parameters are chosen: (I) excitation level of 1 g; (II) sweep rate of 1 Hz/s; (III) frequency resolution of 0.125 Hz; (IV) hanning window for sweep response signal with upward sweep direction; (V) frequency range of 0–2048 Hz. For the modal shape test, the following set-ups and parameters are chosen: (I) excitation level of 1 g ~ 3 g; (II) frequency resolution of 0.125 Hz; (III) rectangular window for stable response signal; (IV) sampling frequency of 12800 Hz; (V) rotated scan speed of 2 r/min. Besides, the first eight modal shapes of TCS are obtained in the test, and each modal shape is assembled from two sets of cross sectional scans, one is in the section which includes point 1, point 2, and point 3, the other is about 25 mm to the clamped end of the shell (restricted by the height of DC geared motor itself, but do not affect the test results when the...
order of axial mode $m = 1$).

The measured frequency, damping, and shape results under constraint boundary as well as different elastic boundaries are listed in Table 7, Table 8, and Table 9, respectively. Besides, in order to clearly describe the effect degree and trend of the elastically supported shell, the corresponding scattergrams of natural frequencies and damping ratios of TCS under such boundary are also given, as shown in Fig. 9 and Fig. 10, and Fig. 11 gives the relation between natural frequency and modal shape under elastic boundary.

4.2. Influence Analysis of Modal Parameters Under Elastic Boundary

4.2.1. Influence on natural frequencies of TCS

From Table 7 and Fig. 9, it can be found that: (I) When the boundary condition of TCS is changed into elastic boundary, the natural frequencies of most modes will decrease within the range of $1 \sim 54$ Hz, which are similar to the calculated results and they verify the correctness of theoretical analysis conclusion. (II) For the low order natural frequencies of the shell, they would decrease by much more, e.g., the decreased degree of the 1st frequency result can reach to 5%. (III) For the high order natural frequencies of the shell, they are basically not affected by elastic boundary. Taking the 7th and 8th natural frequencies for an example, they only change 0.3% compared with the ones under the fixed state. (IV) For some modes,
with the increase of the thickness of rubber ring, their natural frequencies will go up instead of decreasing. For example, with the increase of the thickness of rubber ring, the 3rd and 4th modal shapes of TCS changed. (II) As elastic boundary condition changes with increasing thickness of rubber ring, the 3rd modal shape is changed from (1,3) to (1,6) when the constraint boundary (or elastic boundary I) is turned into the elastic boundary II (or elastic boundary III). Although the calculated shape results can simulate the variation of experimental results, it is hard for the calculated frequency results to be very close to all of the experiment results. However, the decreased trend of natural frequency is true if we observe the whole 1–8 natural frequency results (except for the third mode) obtained by experiment in Tables 7.

4.2.2. Influence on damping ratios of TCS

From Table 8 and Fig. 10, it can be found that: (I) The elastic boundary has great influence on the damping characteristics of TCS; it will not only increase the damping of some modes, but also may lead to the decrease of the damping for part of modes of the shell. Therefore, in the process of vibration-reduction design, it is necessary to choose proper frequency range, otherwise elastic boundary may produce negative effects. (II) When the boundary condition of TCS is changed into elastic boundary, it would clearly reduce high order damping. Taking the 7th and 8th damping results for an example, the range of the decreased degree can reach to about 53% ~ 63%. (III) The elastic boundary may improve damping of the low and intermediate mode of TCS, especially for the 1st damping results, the increased degree is more than 480% compared with the ones under constraint boundary.

4.2.3. Influence on modal shapes of TCS

From Table 9 and Fig. 11, it can be found that: (I) When the support stiffness of elastic boundary is larger, i.e., the thickness of rubber ring is only 1 mm (which means that the boundary condition is just turned into elastic-support from the fixed-support), the resulting shapes of TCS can hardly be described by ASM and FEM to simulate such elastic boundary well, so the theoretical results may inevitably contain some errors. (II) the occurrence sequence of modal shape has changed such as the 3rd and 4th modal shape (for example, the 3rd modal shape is changed from (1,3) to (1,6) when the constraint boundary (or elastic boundary I) is turned into the elastic boundary II (or elastic boundary III)). Although the calculated shape results can simulate the variation of experimental results, it is hard for the calculated frequency results to be very close to all of the experiment results. However, the decreased trend of natural frequency is true if we observe the whole 1–8 natural frequency results (except for the third mode) obtained by experiment in Tables 7.

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Table 9. Measured modal shapes of TCS under different elastic boundaries

<table>
<thead>
<tr>
<th>Modal order</th>
<th>Constraint boundary A (m, n)</th>
<th>Elastic boundary I B (m, n)</th>
<th>Elastic boundary II C (m, n)</th>
<th>Elastic boundary III D (m, n)</th>
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</table>

TCS vary that different levels, but for medium and high order modes, such as the 6th, 7th, and 8th shape results, they are unchanged and still the same as the ones under constraint boundary. (III) Although frequencies and shapes of TCS will be changed by different kinds of elastic boundaries, the changing trend of natural frequencies with mode shapes is constant when the order of axial mode is \( m = 1 \), which shows frequency values are up after the decline with the increase of the number of circumferential waves \( n \), and usually frequency values related to \( n > 8 \) are higher than \( n = 2 \sim 7 \). This verified the correctness of theoretical analysis conclusion.

5. CONCLUSIONS

This research combines theory with experiment to investigate the influence of elastic boundary on modal parameters of TCS. Based on the analysis and experimental results, the following conclusions can be drawn:

1. FEM and ASM are adopted to roughly master vibration characteristics of shell structure, and the theoretical analysis results are helpful for us to determine measured frequency range, build experimental model, understand geographic distributions of some nodes or nodal lines etc.

2. Test system and method under elastic boundary is proposed to accurately measure modal parameters of TCS, and the following measurements and identification techniques are used
to get precise frequency, damping and shape results: (I) Non-contact laser Doppler vibrometer and vibration shaker with excitation level being precisely controlled are used in the test system; (II) “Pre-experiment” is adopted to determine the required tightening torque under constraint boundary; (III) The small-segment FFT processing technique is employed to accurately measure nature frequency; (IV) Laser rotating scanning technique is used to get shape results with high efficiency.

(3) The influence on natural frequencies, modal shapes, and damping ratios of TCS under elastic boundary are analysed and discussed. It can be found that elastic boundary can significantly affect modal parameters of TCS, which would reduce high order damping obviously. For example, the 7th and 8th damping can be decreased within the range of 53% ~ 63%. Besides, natural frequencies of most modes will also be decreased within the range of 1 ~ 54 Hz. However, high order natural frequencies and mode shapes are still the same as the ones under constraint condition, and the changing trend of natural frequencies with mode shapes is constant when the order of axial mode is \( n = 1 \), which agrees well with the results calculated by ASM and FEM.

**ACKNOWLEDGEMENT**

This study was supported by the National Natural Science Foundation of China granted No. 51505070, the Fundamental Research Funds for the Central Universities of China granted No. N150304011 and N160313002, and the Key Laboratory of Vibration and Control of Aero-Propulsion System Ministry of Education, Northeastern University, granted No. VCAE201603.

**CONFLICT OF INTERESTS**

The authors declare that there is no conflict of interests regarding the publication of this paper.

**APPENDICES**

Appendix A — The Elements in the Stiffness Matrix \( K \)

\[
k_{ij}^a = \int_0^1 \left[ -n(1 - \mu) \frac{R}{L} \varphi_i^u(\xi) \varphi_j^u(\xi) + 2\mu n \frac{R}{L} \varphi_i^v(\xi) \varphi_j^v(\xi) \right] \mathrm{d}\xi; \quad (29)
\]

\[
k_{ij}^{ab} = \int_0^1 \left[ -n(1 - \mu) \frac{R}{L} \varphi_i^u(\xi) \varphi_j^v(\xi) + 2\mu n \frac{R}{L} \varphi_i^v(\xi) \varphi_j^u(\xi) \right] \mathrm{d}\xi; \quad (30)
\]

\[
k_{ij}^{ac} = \int_0^1 \left[ 2\mu n \frac{R}{L} \varphi_i^u(\xi) \varphi_j^v(\xi) \right] \mathrm{d}\xi; \quad (31)
\]

\[
k_{ij}^{ab} = \int_0^1 \left[ \frac{n^2 H^2}{6R^2} + n^2 \right] \varphi_i^u(\xi) \varphi_j^v(\xi) \mathrm{d}\xi; \quad (32)
\]

\[
k_{ij}^{bb} = \int_0^1 \left[ \frac{n^2 H^2}{6R^2} + n^2 \right] \varphi_i^u(\xi) \varphi_j^v(\xi) \mathrm{d}\xi; \quad (33)
\]

Appendix B — The Elements in Artificial Spring Stiffness Matrix \( K_{SP} \)

\[
k_{ij}^{aa} = \tilde{k}_0 \varphi_i^u(\xi) \varphi_j^u(\xi) + \tilde{k}_1 \varphi_i^u(\xi) \varphi_j^u(\xi); \quad (38)
\]

\[
k_{ij}^{bb} = \tilde{k}_0 \varphi_i^v(\xi) \varphi_j^v(\xi) + \tilde{k}_1 \varphi_i^v(\xi) \varphi_j^v(\xi); \quad (39)
\]

\[
k_{ij}^{cc} = \tilde{k}_0 \varphi_i^w(\xi) \varphi_j^w(\xi) + \tilde{k}_1 \varphi_i^w(\xi) \varphi_j^w(\xi); \quad (40)
\]
Appendix C – The Elements in Mass Matrix $M$

\[ M_{ij}^{aa} = \int_{0}^{1} \int_{0}^{1} \varphi_i \varphi_j \frac{\partial^2 \sigma^0}{\partial \xi^2} \, d\xi; \quad (41) \]

\[ M_{ij}^{bb} = \int_{0}^{1} \int_{0}^{1} \varphi_i \varphi_j \frac{\partial^2 \sigma^0}{\partial \eta^2} \, d\eta; \quad (42) \]

\[ M_{ij}^{cc} = \int_{0}^{1} \int_{0}^{1} \varphi_i \varphi_j \frac{\partial^2 \sigma^0}{\partial \eta \partial \xi} \, d\eta \, d\xi; \quad (43) \]

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Extra Edge Damping as a Way to Improve Sound Insulation of Window Structures

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The article presents the experimental data of research of sound insulation of window structures using supplemental damping along the perimeter of the translucent portion of the protection with a strip of non-transparent vibration-damping material (edge damping). The authors considered a method for calculating double translucent protective structures with edge damping. They gave versions of possible design solutions for these protective structures.

**NOMENCLATURE**

IGU — insulating glass unit
ISO — International Organization for Standardization
NNGASU — Nizhegorodsky Gosudarstvenny Arkhitekturno-stroiteln Universitet (Nizhny Novgorod State University of Architecture and Civil Engineering)

1. INTRODUCTION

Nowadays the increasing sound pollution of urban areas makes it critical to improve sound insulation of outer protective structures of residential and public buildings, particularly those of windows. There are several ways to increase the sound insulation of windows without significantly increasing the material consumption and complexity of the design. Let us consider one of them.

2. WAYS TO INCREASE SOUND INSULATION USING THE ADDITIONAL DAMPING

Extra damping of glasses of protection is widely used in modern window structures, and specifically to improve their sound insulation. There are several methods to use extra damping to improve the sound insulation properties of translucent protection:

- method No.1 — damping with a transparent film on the outer surface of the pane;
- method No.2 — damping with a transparent material connecting two layers of the pane (triplex);
- method No.3 — use of non-transparent damping material in certain areas of glazing (e.g., along the perimeter of the structure).

The impact of damping methods No.1 and No.2 has been discussed by many researchers, in particular, by D.V. Murgygin, A.A. Kochkin, J.G. Lilly, N. Garg, U. Keller, J. Schimmelpenningh and others. In these and other researches, the results proving the efficiency of these ways of damping for improving sound insulation of various types of translucent enclosing structures are presented.

The third method is now scarcely studied, despite the fact that its application may allow to significantly improve the sound insulation due to the slight reduction of translucent area of the structure. In his paper I.I. Bogolepov discussed the double polymethylacrylic structure in which a special elastic material applied on the perimeter of the glazing, served for the acoustic separation of protection plates and prevention of occurrence of sound bridges; however, the effect of this material as a damping one has not been discussed.

Currently, there are effective self-adhesive damping materials based on different types of polymer mastics having a loss factor $\eta = 0.3–0.4$. It is proposed to use such materials for pasting the perimeter of the transparent portion of the protection. These materials are relatively inexpensive and their costs are low, since the glass area to be covered with these materials is small. Such materials include, in particular, “BiMast Bomb” ($\eta = 0.4$), which is proposed to be used in subsequent experimental studies of the method.

3. THEORETICAL STUDIES OF POSSIBLE INCREASE OF SOUND INSULATION USING ADDITIONAL DAMPING

Consider the theoretical possibility of pasting the strip of opaque vibration cushioning material around the perimeter of translucent walling (edge damping) to improve the design of sound insulation.

Since the single-layer translucent structures are practically not used in modern construction practice, the studies on the effect of regional damping on sound insulation of translucent walling should be carried out for the double structure with an air gap.

Theoretical studies of sound insulation of walling of buildings and structures in the NNGASU Laboratory of acoustics are carried out on the basis of the wave fields self-coupling theory (hereinafter — the WFSC theory) developed by a scientific school of Professor M.S. Sedov. Since there aren’t many materials on this subject in English sources, the following ground expressions describing the sound insulation of double walling with an air gap are demonstrated here.

According to the WFSC theory, the walling structure makes a wave motion under the influence of incident sound waves, which involves its own (free) and forced (inertial) waves.

Resonant passage of sound is determined by the degree of the wave fields self-coupling in front of and behind the walling and by the wave field made by own oscillations of the plate. The bigger the self-coupling of sound fields is, the more intense the sound will penetrate through the barrier. Inertial pas-
sage of sound depends on the surface density of the fence and its geometrical dimensions (length and width).

Given the independence of inertial and free waves in accordance with the principle of superposition of sound transmission, the coefficient for double walling with an air gap can be written as:

$$\tau = \tau_{s,i} + \tau_{s,r} + \tau_{11}T_{21} + \tau_{1r}T_{2r};$$

(1)

where

- $\tau_{s,i}$ is the coefficient of inertial transmission of sound through the structure as a system of plates with elastic connection between them;
- $\tau_{s,r}$ is the coefficient of resonant transmission of sound through the structure as a system of plates with elastic connection between them;
- $\tau_{1i}$ is the coefficient of inertial transmission of sound through the first plate;
- $\tau_{1r}$ is the coefficient of resonant transmission of sound through the first plate;
- $\tau_{2r}$ is the coefficient of resonant transmission of sound through the second plate.

In this expression $\tau_{s,i}$ coefficient is given by:

$$\tau_{s,i} = \frac{1}{\sqrt{n_0^2 c_0^2 f^2 (f^2 - 1) + 1}},$$

where $m'$ is the area density of the walling:

$$m' = m'_1 + m'_2;$$

(3)

wherein $m'_1$ is the area density of the first walling plate, $kg/m^2$;
- $m'_2$ is the area density of the second walling plate, $kg/m^2$;
- $f$ is the frequency, Hz;
- $n_0 c_0$ is the characteristic impedance;
- $f_0$ is the resonant frequency of the walling, as a system of “mass-elasticity-mass”, Hz.

$$f_0 = 60 \sqrt{\frac{m'_1 + m'_2}{dn_1m_2}}.$$  

(4)

In this expression, $d$ is the distance between the plates of the walling (the width of the air gap), m.

$F_{1i}$ is the response function of the first plate, on which the sound impinges.

$$F_{1i} = \frac{G_{mn}}{Q_{mn}}.$$  

(5)

Herein:

$$G_{mn} = \left\{ Q_m - \frac{\sin m \pi}{\pi (m^2 + n_1^2)} \cdot \left[ m_1 \sin m \pi + m \cdot e^{-m_1 \pi} + m \cdot \beta_a \cdot (1 + e^{-2m_1 \pi}) \right] \right\} \times$$

$$\left\{ Q_n - \frac{\sin n \pi}{\pi (n^2 + n_1^2)} \cdot \left[ n_1 \sin n \pi + n \cdot e^{-n_1 \pi} + n \cdot \beta_a \cdot (1 + e^{-2n_1 \pi}) \right] \right\};$$

$$\beta_a = \frac{e^{-m_1 \pi} - \cos m \pi}{1 - e^{-2m_1 \pi}};$$

$$\beta_0 = \frac{e^{-n_1 \pi} - \cos n \pi}{1 - e^{-2n_1 \pi}};$$

$$m = \frac{k_0 \cdot a}{\pi} \cdot \cos \alpha_x;$$

$$n = \frac{k_0 \cdot b}{\pi} \cdot \cos \alpha_y;$$

$$\cos \alpha_x = \frac{b \cdot \sin \theta_{av}}{\sqrt{a^2 + b^2}};$$

$$\cos \alpha_y = \frac{a \cdot \sin \theta_{av}}{\sqrt{a^2 + b^2}};$$

$$m_1 = \frac{k_0 \cdot a}{\pi} \cdot \sqrt{1 + \sin^2 \alpha_x};$$

$$n_1 = \frac{k_0 \cdot b}{\pi} \cdot \sqrt{1 + \sin^2 \alpha_y};$$

$$\cos^2 \alpha_x + \cos^2 \alpha_y + \cos^2 \theta = 1;$$

$$Q_{mn} = Q_m \cdot Q_n;$$

$$Q_m = \frac{1}{2} \cdot (1 + \frac{\sin 2m\pi}{2m\pi});$$

$$Q_n = \frac{1}{2} \cdot (1 + \frac{\sin 2n\pi}{2n\pi}).$$

Here:

- $a$ and $b$ is the walling dimensions on the plan (length and width), m;
- $\alpha_x$ and $\alpha_y$ — slip angles of the sound wave along the sides of the plate $a$ and $b$;
- $k_0$ — the wave number of the medium, $m^{-1}$;
- $\theta_{av}$ — the angle of incidence of the sound waves at the first walling plate (in diffused falling $\theta_{av} = 51.75759^\circ$).

The coefficient $\tau_{s,r}$ in the expression (1) is defined by:

$$\tau_{s,r} = \frac{1}{\sqrt{n_0^2 c_0^2 f^2 (f^2 - 1) + 1}};$$

(6)

where $A$ is the characteristic of self-coupling.

The value of characteristic of self-coupling depends on the field frequency range. Boundary frequencies of the areas of simple spatial resonance (hereinafter — SSR), incomplete spatial resonance (hereinafter — the ISR) and the full spatial resonance (hereinafter — FSR) is calculated by the following expressions.

Boundary frequency of SSR:

$$f_{frmn0} = \frac{c_0}{4 \cdot a};$$

(7)

where $c_0$ is the speed of sound in air, m/s.

This area is characterized by a mismatch between the speed vectors of sound waves in the plane of the plate and the own elastic waves.

Boundary frequency of ISR:

$$f_{frmn0} = \frac{c_0}{2 \cdot a \cdot \sin \alpha_{mn0}};$$

(8)

where

$$\sin \alpha_{mn0} = \frac{b}{\sqrt{a^2 + b^2}}.$$  

(9)

$\alpha_{mn0}$ is the angle between the sound waves impinging on the plate and waves of own oscillations of the plate.

In the ISR area the characteristics of own elastic and acoustic waves are not fully coincide: the speed of the spread of...
tracks of free and sound waves on one side of the plate are equal to each other, and on the other side they are in such a ratio, in which the response of the plate is largest.

Boundary frequency of FSR:

\[ f_{\text{FSR}} = \frac{c_0}{2\pi} \sqrt{m_1^2 / D_1}. \]  

(10)

In this frequency range the conditions of complete self-coupling of sound fields in front of and behind the walling structure and the wave field of the natural oscillations of the walling are met.

In the SSR range at frequencies below the boundary frequency of FSR, \((f < f_{\text{FSR}})\), the expression for the self-coupling characteristic will be:

\[ A_0^2 = m_{0\text{max}} \frac{n^2}{(n^2 - n_{0\text{av}}^2)^2} + n_{0\text{max}} \frac{m^2}{(m^2 - m_{0\text{av}}^2)^2}. \]  

(11)

In this expression:

\[ m_{0\text{max}} = b \sqrt{\frac{4f^2}{c_0^2} - \frac{1}{a^2}}; \]
\[ n_{0\text{av}}^2 = \left( \frac{n_{0\text{max}}}{2} \right)^2; \]
\[ m_{0\text{av}}^2 = \left( \frac{m_{0\text{max}}}{2} \right)^2; \]
\[ n^2 = b^2 \left( \frac{2}{\pi} \sqrt{m' / D - \frac{m_{0\text{av}}^2}{a^2}} \right); \]
\[ m^2 = a^2 \left( \frac{2}{\pi} \sqrt{m' / D - \frac{n_{0\text{av}}^2}{b^2}} \right). \]

In the frequency range \((f_{\text{FSR}} < f < f_{\text{FSR}})\), the self-coupling characteristic is given by:

\[ A_2^2 = A_0^2 + A_{01}^2. \]  

(18)

where the value \(A_{01}^2\) is defined by the expression (17), and the additional value of \(A_{01}^2\) is:

\[ A_{01} = m_{01\text{max}} \cdot \frac{n_1^2}{(n_1^2 - n_{01\text{av}}^2)^2} + n_{01\text{max}} \cdot \frac{m_1^2}{(m_1^2 - m_{01\text{av}}^2)^2}. \]  

(19)

Herein:

\[ m_1^2 = n_1^2 = \frac{2f}{\pi} \left( \frac{1}{a^2} + \frac{1}{b^2} \right) \sqrt{D / m_1}; \]
\[ m_{01\text{max}} = a \sqrt{\frac{4f^2}{c_0^2} - \frac{m_1^2}{b^2}}; \]
\[ n_{01\text{max}} = b \sqrt{\frac{4f^2}{c_0^2} - \frac{n_1^2}{a^2}}; \]
\[ n_{01\text{av}}^2 = \frac{n_{01\text{max}}^2}{2}; \]
\[ m_{01\text{av}}^2 = \frac{m_{01\text{max}}^2}{2}. \]

The coefficients \(\tau_{11}\) and \(\tau_{23}\) (1) describe the sound wave passing sequentially through the first plate, the air gap and the second plate. They are determined by the expressions:

\[ \tau_{11} = \frac{1}{\sigma_1^2 \frac{m_{12}^2 f^2 \cos^2 \theta_{11}}{v_{12}^2} + 1}; \]
\[ \tau_{23} = \frac{1}{\sigma_2^2 \frac{m_{23}^2 f^2 \cos^2 \theta_{23}}{v_{23}^2} + 1}. \]  

(20)

(21)

Here \(\theta_2\) is the angle of incidence of sound waves constituting the natural vibration shape of the air gap along the plane of the second plate:

\[ \cos \theta_2 = \frac{d}{\sqrt{a^2 + b^2} + d^2}. \]  

The nature of the resonance passage through the first and second plates is different for the following frequency areas:
The effective loss factor measured according to the expression (1) have the form:

\[ f < \frac{f_{mn}}{2} ; \quad \frac{f_{mn}}{2} < f < f_{mn} ; \quad f > f_{mn} . \]

In the frequency range, the \( f < \frac{f_{mn}}{2} \) coefficients of resonant passage from the expression (1) have the form:

for the first walling plate:

\[ \tau_{1r} = \frac{1}{8 \pi \rho \sqrt{d} A_{01} m_{2}^{2} f_{2}^{2} \eta_{1} \cos^{2} \theta_{av} + 1} ; \quad (22) \]

for the second walling plate:

\[ \tau_{2r} = \frac{1}{8 \pi \rho \sqrt{d} A_{02} m_{2}^{2} f_{2}^{2} \eta_{2} \cos^{2} \theta_{av} + 1} . \quad (23) \]

Here \( \eta_{1} \), \( \eta_{2} \) are the loss factor for the first and second plates, respectively.

For the frequency range \( f < f_{mn} \) (22) and (23) take the form:

for the first walling plate:

\[ \tau_{1r} = \frac{1}{8 \pi \rho \sqrt{d} A_{01} m_{2}^{2} f_{2}^{2} \eta_{1} \cos^{2} \theta_{av} + 1} ; \quad (24) \]

for the second walling plate:

\[ \tau_{2r} = \frac{1}{8 \pi \rho \sqrt{d} A_{02} m_{2}^{2} f_{2}^{2} \eta_{2} \cos^{2} \theta_{av} + 1} . \quad (25) \]

For the FSR area for the frequencies \( f > f_{mn} \) the coefficients of the resonant sound passage for the first and second plates are, respectively:

\[ \tau_{1r} = \frac{1}{8 \pi \rho \sqrt{d} A_{01} m_{2} f_{mn}^{2} \eta_{1} \cos^{2} \theta_{av} \sqrt{1 - \frac{f_{mn}}{f}} + 1} ; \quad (26) \]

\[ \tau_{2r} = \frac{1}{8 \pi \rho \sqrt{d} A_{02} m_{2} f_{mn} \eta_{2} \cos^{2} \theta_{av} \sqrt{1 - \frac{f_{mn}}{f}} + 1} . \quad (27) \]

In the above expressions the damping of the vibrations in the walling plate is taken into account by the coefficient of the material losses \( \eta \). Thus, it can be concluded that the additional damping affects the resonant oscillations of the walling plates to which it is applied.

To assess the impact of the edge damping on sound insulation of the structure it is proposed to introduce in the expressions (22)–(27) the value of the effective loss coefficient of construction \( \eta_{e,f} \). The effective loss coefficient is the rate of glass losses considering the damping with fragments of vibration cushioning material. Changing the effective loss coefficient will have an impact on the coefficients of the resonant transmission of sound: \( \tau_{1r} \) and \( \tau_{2r} \).

The values of \( \eta_{e,f} \) for glass with partial pasting with damping material can be measured experimentally by the Oberst method (standard method to measure the loss factor of the material). The results of measurements for glass 4 mm thick with different pasting area of vibration cushioning material “BiMast Bomb” conducted at the Acoustics Laboratory of NNSUACE are presented in Table 1.

As can be seen from the data shown in the table, the effective loss coefficient in case of the edge damping with the area 12.5% of structure area exceeds the loss coefficient of the glass by 3 times. It is proposed to consider the damping area of 12.5% as the most effective value, since with this relatively small pasting area a significant increase in the effective loss rate is achieved, and with further increase in the area of vibration cushioning material a significant increase in the values of the effective loss coefficient does not occur.

In order to estimate the expected increase in sound insulation due to the use of regional damping by the above procedure, there has been calculated the coefficients \( \tau_{1r} \) and \( \tau_{2r} \) and soundproofing coefficient R, dB for the double walling of silicate glass of a size 1.3 × 1.1 m with a glazing formula 4 + 24 + 4 mm (two sheets of silicate glass 4 mm thick each, separated by an air gap of 24 mm width). The calculation was made for two values of the effective loss factor:

1. \( \eta_{e,f} = 0.006 \) (glass without additional damping);
2. \( \eta_{e,f} = 0.019 \) (glass with an additional damping).

The calculation results are shown in Figures 1, 2, 3.

As seen from frequency characteristics in Fig. 1 and Fig. 2, with an increase in the loss factor from 0.006 to 0.019 the values of coefficients \( \tau_{1r} \) and \( \tau_{2r} \) significantly increase over the entire frequency range.

However, according to the WFSC theory the impact of \( \tau_{1r} \) and \( \tau_{2r} \) on sound insulation is largely dependent on the frequency range. These coefficients have the greatest impact on the structure sound insulation at frequencies \( f > \frac{f_{mn}}{2} \). At frequencies below \( \frac{f_{mn}}{2} \) the effect of \( \tau_{1r} \) and \( \tau_{2r} \) is slightly less (the inertial passage of sound has more influence in this case). At frequencies close to the resonance frequency of the structure as a system of “mass-elasticity-mass” \( f_{0} \), the influence of these coefficients is minimal. The sound insulation here is mainly determined by coefficients, that considering the construction as a whole: \( \tau_{s,i} \) and \( \tau_{s,rf} \).

Thus, for walling with glazing formula 4 + 24 + 4 mm, we should expect a substantial increase of sound insulation at frequencies above 1600 Hz, and a slight increase in the range of 315–1600 Hz, which is confirmed by calculating the insulation.
The sound insulation of studied protection structures has been studied in large reverberant chambers of the NNGASU Laboratory of acoustics according to standard procedure ISO 10140-2 «Acoustics — Laboratory measurement of sound insulation of building elements — Part 2: Measurement of airborne sound insulation». The measurements were made using a precision acoustic instrumentation “Larson & Davis” (2900V spectrum analyzer, 1/3-s measuring microphones, type 2559). Measuring microphones were sequentially installed in eight points in high- and low-level chambers (in this case n = 8 for each chamber); $L_i$, dB is the sound pressure level in the ith point.

Sound insulation of the enclosing structures $(R, \text{dB})$ when exposed to air noise was calculated by the formula 10:

$$R = L_{m1} - L_{m2} + 10 \cdot \log \frac{S}{A};$$

where $L_{m1} \cdot L_{m2}$, dB — the average sound pressure levels in a high level chamber and in the low level chamber, respectively; $S$, m$^2$ — the area of the tested enclosure; $A = \frac{0.16 \cdot V}{m}$, m$^2$ — the equivalent acoustic absorption area of low-level camera; $V$, m$^3$ — the volume of low level chamber; $T$, s — the average time of reverberation in the low level chamber.

For each tested type of the enclosing structures, the measurement of sound insulation was carried out three times. Accuracy of measurements was $\pm 1.0$ dB.

### 5. THE DESCRIPTION OF STUDIED TRANSLUCENT WALLING

The translucent structures based on insulated glass units are widely used in construction practice, so to conduct a research on the effect of the edge damping on sound insulation of double walling the insulated glass unit was manufactured with the glazing formula $4 + 24 + 4$ mm with plan dimensions of $1.3 \times 1.1$ m.

The specific material “BiMast Bomb” manufactured by “Standartplast” LLC (Russia) was used for additional edge damping. This material has one of the highest damping characteristics ($\eta = 0.4$) among opaque self-adhesive damping materials, that are common in the Russian market.

The sound insulation of double glazed unit with varying degrees of regional damping was measured in the course of these experimental studies.

The scheme of the studied structure is presented in Figure 4.

### 6. RESULTS OF THE PERFORMED EXPERIMENTAL STUDIES

Frequency characteristics obtained during the experiment are presented in Figure 5. The experimental results confirm the data of a theoretical calculation of the frequency range in which can be expected the increased sound insulation of the structure when using edge damping.

The main gain in sound insulation when applying edge damping occurs in the frequency range $315–3150$ Hz. The most effective way to improve sound insulation proved to be damping both panes of glass with strip material “BiMast Bomb” $40$ mm wide around the perimeter (at that, area occupied by the vibration cushioning material is $13\%$ of the total area of glazing structure), which roughly corresponds to the “effective” area of damping material $12.5\%$ obtained by determining the effective loss factor. The main gain in sound insulation is $2–8$ dB when applying edge damping in the frequency range $315–3150$ Hz. In the frequency range below $315$ Hz the...
visible gain in sound insulation is not observed. With a smaller area of vibration damping material the gain in sound insulation is much less, but there it is observed in the same frequency range.

The lack of increase in sound insulation at frequencies below 315 Hz supports the conclusion that the additional damping is inefficient at frequencies close to the resonance frequency of “mass-elasticity-mass”.

It is necessary to draw attention to the fact that the application of the regional damping leads to increase in weight of the structure. However, this mass change is very small and does not exert any significant influence on the sound insulation of enclosure. For the investigated structure with damping using 40 mm wide strip around the perimeter of both glasses, the weight is increased by 12% of the original weight of the glazing without damping.

It should also be noted that the design of double translucent enclosing structures based on glass units provides the additional damping element by applying a sealing material for insulating glass perimeter. However, as follows from the results of the experiment, the additional edge damping improves the sound insulation even in this case.

Also the additional damping arising during construction of the measuring aperture can influence the sound insulation of the structure. However, during these measurements the method of installing glass in the structure did not change from measurement to measurement, so the effect of damping, which occurs during the installation of the structure, was always constant and was not reflected in the increase of sound insulation arising from the application of the edge damping.

The edge damping is only one factor affecting the sound insulation of translucent structures. When combined with other similar factors (including, for example, the number and thickness of glass, the width of the air gap, etc.) there can be expected further increase of sound insulation of the structure. The aim of the study on the current stage is to study the possibility of using the edge damping in particular to increase insulation, combine it with other factors to improve the result achieved — the goal of the next phase of the work.

In Fig. 6 there is a comparison of the theoretical calculation data in accordance with the above-mentioned method and experimental results for the construction without edge damping.

In Fig. 7 there is a comparison of the theoretical calculation data in accordance with the above-mentioned method and experimental results for the construction with edge damping of both glasses with use of material “BiMast Bomb” by a stripe of 20 mm along the perimeter.
the whole standardized frequency range of 100–3150 Hz (i.e. within the areas of SSR (simple spatial resonance) and ISR (incomplete spatial resonances). Identification of the reasons for discrepancies between theoretical and experimental indices of sound insulation that are observed at frequencies higher than the standardized range (above 3150 Hz, i.e. in the area of SSR) will be the goal of our further research.

7. **DESIGN SOLUTIONS OF WINDOWS WITH EDGE DAMPING**

Based on the above, as well as other experimental and theoretical data obtained by the staff of the NNGASU Laboratory of acoustics, design solutions of window units with IGUs with edge damping have been proposed in the course of studies of the capabilities of edge damping[11][12] (see. Fig. 8). At that, it can be seen on the schemes that in this case it is possible to reduce the area of the translucent part of enclosure occupied by edge damping by placing the glazing parts coated with damping material inside the window unit housing.

An application for a patent for the invention was filed for the described method to improve sound insulation, as well as design solutions of structures based on it.[13]

8. **CONCLUSIONS**

Based on the results of the study, it can be concluded that the edge damping with opaque materials is a promising way to improve the sound insulation of windows and other glass structures. The increase of sound insulation due to the edge damping occurs in a wide range of frequencies of practically significant range (at frequencies of 315–8000 Hz for constructions of 4 mm glass). In the field of a resonance of structure as a system “mass-elasticity-mass” the extra edge damping is ineffective. This study established the most effective area of pasting the vibration damping material (1/8 of the glass area). In the future we plan to study the possibility of using regional damping in combination with other structural measures, allowing to increase the sound insulation of the structure.

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![Webe João Mansur](image)

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![S. Josephine Kelvina Florence](image)

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![K. Renji](image)

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