Vibration Control for a Cantilever Beam with an Eccentric Tip Mass Using a Piezoelectric Actuator and Sensor

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A novel model using the transfer matrix method for multibody system (TMMMS) is put forward to describe the dynamic characteristics of a cantilever beam that has a concentrated mass at its tip under axial excitations. The theoretical analysis and numerical results demonstrate that this model has some advantages, such as for a small matrix and a higher computational speed. Based on this model a control system, which is composed of a LQG controller, a piezoelectric actuator, and a sensor for the cantilever beam is proposed, theoretically analyzed, and experimentally verified. The experimental results show that the proposed controller with the piezoelectric actuator can effectively reduce the vibration of the cantilever beam with an eccentric tip mass. The piezoelectric sensor can measure vibration responses with high-accuracy. Therefore, this new model gives a broad range of possibilities for model-based controller design and implementation.

1. INTRODUCTION

A cantilever beam with an eccentric tip mass is a familiar dynamic model for mechanical systems. For instance, it may be used to investigate flexible robot arms,1 mast antenna structures,1,2 wind tunnel stings carrying an airplane,3 and Stockbridge dampers used for damping out aeolian vibrations on high-voltage transmission lines.4 Therefore, it is critical to analyze its vibration characteristics and to design an active vibration controller.5 The mechanical vibrations of cantilever beams have attracted plenty of attention from researchers over the past several years. The majority of the literature has focused on deriving and solving the exact frequency equation for the particular case of a concentrated mass and/or moment of inertia at the tip.6

However, some problems still exist; the first problem of mechanical vibrations on a cantilever beam with a tip mass continues to attract the attention of the research community due to a wide range of practical situations for which such a mechanical system is a reasonable idealization. The second problem is that it is difficult to design a proper vibration controller based on those previous works due to the large size of the matrix of the dynamic equations.7

With the developments in sensor/actuator technologies, many researchers have concentrated on vibration control using smart materials such as shape memory alloys (SMAs), and piezoelectric transducers. Piezoelectric materials have been applied in structural vibration control as well as in structural acoustics because of their fast response, large force output, and because they generate no magnetic field in the conversion of electrical energy into mechanical motion.8–12 Positive position feedback (PPF) was devised by Goh and Caughey13 and has several distinguished advantages.14 It has proven to be a solid vibration control strategy for flexible systems with smart materials, particularly with the piezoelectric actuator.14–17 PPF is essentially a second-order filter that is used to apply high frequency gain stabilization by improving the frequency rolloff of the system.18 Alternatively, PPF works by using a second-order system which is forced by the position response of the structure. This response is then fed back to give the force input to the structure. To apply PPF, the natural frequencies of the structure should be known. The effectiveness of PPF will deteriorate if the natural frequencies are not well known or have changed for some reason, such as the presence of a tip mass.

As such, in order to reduce the size of matrix of the dynamic equations, we introduce TMMMS to describe the dynamic characteristics of the system and design a LQG control system, which is composed of a LQG controller, a piezoelectric actuator, and a sensor.

TMMMS is a new method of multibody system dynamics (MSD) developed by Rui and his co-workers.19,20 The highlights of this method are as follows: study MSD without global dynamics equations of the system, keep a low order of the system matrix, and avoid the difficulties in computation caused by high-order matrices. Nowadays, TMMMS has been widely applied to various engineering cases such as for such features without the global dynamics equations of a system, low order of involved system matrix, fast computational speed, and high automation in programming.

2. DYNAMIC MODELING

A sketch of a uniform cantilever beam carrying a rigidly mounted mass is shown in Fig. 1, which is consist of a rigid body (1) and a beam (2). The mass and inertia relative to the centroid of the rigid body are m and Jc, respectively. The beam is an Euler-Bernoulli beam, which has a flexural rigidity, linear density, and length are EI, m, and L, respectively.

2.1. State Vectors

The state vectors of the connection point among any bodies and hinges vibrating are defined as:

\[
z_{k,j} = [y, \theta_z, m_z, q_y]^T ; \quad (1)
\]

\[
Z_{k,j} = [Y, \Theta_z, M_z, Q_y]^T ; \quad (2)
\]