Stability Control of Linear and Nonlinear Dynamic Systems

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(Received 5 November 2014, accepted: 11 March 2016)

The behavior of linear or nonlinear dynamic systems depends on different parameters (identifiable or free) that are involved in their definition. The stability analysis of such dynamical systems is realized by using a domain of selected free parameters. In this paper, we discuss specific theorems that concern the stability of linear dynamical systems, the stability of nonlinear dynamical systems in terms of "first linear approximations", and other stability criteria. We study the stable/unstable separation property in the free parameters domain and present a rigorous mathematical justification of this property with specific examples from various branches of science. Furthermore, we investigate specific conditions when the separation property is passed on to the nonlinear dynamical system from its first order linear approximation. The stable/unstable separation property is also emphasized as an important property of the environment that can contribute to its mathematical modeling.

1. INTRODUCTION

In this paper, we analyzed the multiple aspects of the stability control of linear or nonlinear dynamical systems ensured by the property of separation between stable and unstable regions of the free parameters domain.

Numerous authors have studied the problems of dynamic systems stability. We have surveyed some of the relevant literature here.[1–8, 11–13]

Any dynamical system can be considered in terms of its defining parameters without fixing their values as geometrical parameters, physical parameters (in particular mechanical parameters), and possible economical or biological parameters.

Another important idea is that many real-life dynamical systems are considered in the literature (e.g. the Mathieu equation, the Hill equation, the harmonic vibration equation, etc.) and have the property of separation between the stable and unstable region in a selected domain of free parameters. The stable and unstable regions are separated by a boundary in the domain of the free parameters. The property of separation can be described by the fact that the stable and unstable regions, except the points on the boundary, are open sets. This separation aspect, which is considered in this paper, creates the freedom of stability control on a neighborhood of fixed stable point in the open stable region of the dynamical system.

We discovered some mathematical conditions of the stability regions existence for dynamical systems using various results from matrix theory, real analysis, stability theory, and others.

The property of separation of stability regions is an important property of the environment, as one refers to a specific dynamical system locally implemented in the environment.

Our study has not exhausted the subject of stability control. New results in matrix theory, in the linear or nonlinear dynamical system stability theory, and in real analysis will provide further direction.

2. ON THE CONTINUITY OF THE REAL MATRIX EIGENVALUES

The real matrix in the discussion was the matrix that defined the linear dynamical system, or the "first approximation", of the nonlinear dynamical system depending on some parameters. The components of the real matrix were assumed to be continuous or piecewise continuous functions of the system parameters (time could also be considered as a parameter).

The dependence of the spectrum of this matrix on the matrix components properties is discussed in the following paragraphs.

2.1. QR Algorithm for Hessenberg Form of the Real Matrix

In what follows, we assumed that the $n \times n$ matrix had distinct eigenvalues. The QR algorithm was formulated for the matrices of Hessenberg form, meaning that its entries satisfy $a_{ij} = 0$ for $2 < i \leq n, j < i - 1$.