
Vibration Interaction Analysis of Non-uniform Cross-Section Beam Structure under a Moving Vehicle

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One of an engineer's concern when designing bridges and structures under a moving load is the uniformity of stress distribution. The dynamic behavior of a vehicle on a flexible support is also of great importance. In this paper, an analysis of a variable cross-section beam subjected to a moving load (such as a concentrated mass), a simple quarter car (SQC) planar model, and a two-axle dynamic system with four degrees of freedom (4DOF) is carried out. The finite element method with cubic interpolation functions is used to model the structure based on the Euler-Bernoulli beam and a direct integration method is implemented to solve time dependent equations implicitly. The effects of variation of a cross-section and moving load parameters on the deflection, natural frequencies, and longitudinal stresses of the beam are investigated. The interaction between vehicle body vibration and the support structure is also considered. The obtained results indicate that using a beam of parabolically varying thickness with a constant weight can decrease the maximum deflection and stresses along the beam while increasing the natural frequencies of the beam. The effect of moving mass inertia at a high velocity of a moving vehicle is also investigated and the findings indicate that the effect of inertia is significant at high velocities.

1. INTRODUCTION

The analysis of structures carrying moving loads is of considerable practical importance. Bridges on which vehicles or trains travel, trolleys of cranes that move on their girders, and many modern machining operations, such as high-speed precision drilling, can be modeled as a moving load problem.

Since the middle of the last century, when railway construction began, the problem of oscillation of bridges under traveling loads has interested engineers. Contributions towards a solution of this problem were initially made by Stoke¹ and Robert Willis.² Timoshenko³ found an analytical solution for the case of a concentrated force moving with a constant velocity along a beam, neglecting the damping effect.

A comprehensive treatment of the subject of vibrations of structures due to moving loads that contain a large number of related cases is that of Fryba.⁴ In a dynamic analysis of structures subjected to moving loads involving a large moving mass, neglecting inertia may cause a considerable error. When the mass of either the moving load or the structure cannot be ignored, the dynamic analysis of moving load problems becomes more involved. The first attempt to include the mass of both the beam and the moving load was given by Jefcott.⁵ Calculating the response of beams affected by moving mass involves solving sufficiently complex partial differential equations that the analytical methods are not almost applicable. Therefore, the numerical methods have been used frequently to solve various boundary conditions and complicated cases such as variable speed moving load, multiple span beam, damping within the beam, sprung mass, et cetera. Akin and Mofid⁶ developed an analytical-numerical method to determine the behavior of beams carrying a moving mass. Esmailzadeh and Ghorashi⁷ analyzed the Timoshenko beam traversed by a uniform partially distributed moving mass. Esmailzadeh and Jalili^{8,9} investigated the dynamic interaction of moving vehicles on uni-

form suspension bridges. They modeled the vehicle as a half-car planar model with six degrees of freedom.

The finite element method was applied to overcome some of the limitations in analytical analysis. The finite element method was first used by Cook and Fleming.^{10,11} Filho¹² surveyed the application of the finite element method as a simply supported beam subjected to a constant-velocity two degrees of freedom system with various mass ratios. Lin and Trethewey¹³ analyzed the dynamics of an elastic beam that was subjected to dynamic loads induced by the arbitrary movement of a spring-mass-damper system, which was based on a finite element formulation and solved it with a Runge-Kutta integration scheme. The analysis of a beam with a non-uniform cross-section was completed by Gutierrez and Laura.^{14,15} It dealt with the approximate determination of the vibration of a beam traversed by a time varying concentrated force. Zheng et al.¹⁶ studied the vibration behavior of a multi-span continuous bridge modelled as a multi-span non-uniform continuous Euler-Bernoulli beam under a set of moving loads using different assumed mode shapes. Wu and Dai¹⁷ and Henchi and Fafard¹⁸ used the same Euler-Bernoulli beam and the finite element transfer-matrix approach.

Ahmadian et al.¹⁹ also considered the analysis of a variable cross-section beam subjected to a moving concentrated force and mass by using the finite element method. Dyniewicz²⁰ dealt with the vibrations of structures subjected to a moving inertial load using the velocity approach to the space-time finite element method. Zhai and Song²¹ were concerned with the transient vibration analysis of the railway-ground system under fast moving loads formulating a 3D finite element method in a connected coordinate system moving with the load together with viscous-elastic transmitting boundary conditions in order to limit the finite element mesh. Azizi et al.²² employed the spectral element method in a frequency domain to analyze continuous beams and bridges subjected to a moving load. Samani