Response of a Cylindrical Shell with Finite Length Ring Stiffeners

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This paper derives a spatial domain wave propagation solution of a cylindrical shell that contains periodically spaced ring stiffeners. Previous work in this area has modeled the stiffeners as having a very short or very long length. This paper models the stiffeners as finite length inclusions with forces that have spatial extent in three-dimensions. Furthermore, there is a well-defined separation distance between each stiffener. The new model uses Donnell shell equations with the stiffener forces applied in three-dimensions using Heaviside step functions. These equations of motion are orthogonalized in both the angular and longitudinal directions, resulting in a double fixed index matrix equation. These indices can be varied, which yields a set of double indexed matrix equations that are written together as a single global matrix. This global matrix can be solved, which results in a solution to the system displacements. Two specific external loading cases are investigated and convergence criteria are discussed. One of the models is verified with a comparison to finite element analysis.

1. INTRODUCTION

Reinforced shells were used in a variety of applications. They can be found in undersea vehicles, industrial pipes, hydraulic lines, and marine piers. Reinforcement was typically added to these structures as a method to increase stiffness without adding significant mass. Adding reinforcement changes the structural response of almost any system, and the forces that are introduced by the reinforcement need to be included in an analytical or numerical model that predicts the corresponding response. Unreinforced isotropic thin cylindrical shell models have existed in the literature for a long time and can be found in textbooks on acoustics and applied mechanics.[1,2] Isotropic thick shell cylindrical shells models were also derived for shells without reinforcement.[3] Shell models were extended to include transversely isotropic behavior[4] and general orthotropic behavior.[5]

The inclusion of ring stiffeners in the cylinder increased the stiffness in all three cylindrical directions and changed the character of the infinite cylinder response from a (purely propagating) single longitudinal term expression to a (partially reflective) multi-longitudinal term expression due to the forces of the stiffeners interacting with the wave motion. Historically, research in this area has been divided into two separate approaches: (1) where the length of the stiffener was very short compared to the periodicity of the stiffeners or (2) where the length of the stiffener was relatively long compared its periodicity. Work in the first area (i.e. short stiffeners) was abundant, and various systems were analyzed. Free wave propagation of periodically ring stiffened shells has been studied using finite element analysis[6] applied to various different ring geometries that all had relatively small spatial extent and to determine natural frequencies and modes shapes of ring-stiffened shells.[7] The free vibration analysis of cylindrical shells with ring stiffeners that had non-uniform eccentricity and unequal spacing were investigated using a Ritz analytical method, experimental testing, and finite element analysis,[8] where the stiffeners had a relatively small spatial extent.

A Laplace transfer numerical method to analyze ring stiffened circular thin shells was developed,[9] where the spatial dimensions of the rings were small and the structure was loaded with a transient pressure load. The problem of acoustic radiation from fluid-loaded, ring-supported thin shells subjected to a point forces has been solved[10] in the wavenumber domain. In this paper, both single and double periodic ring supports were considered. The theory of vibrations of a cylinder reinforced by periodically spaced circular T-section ribs along its length has been derived.[11] A method for obtaining the propagation constants of a thin uniform periodically stiffened cylindrical shells with an emphasis on the stop and pass bands of free wave motion has been developed.[12]

Work in the second area (i.e. long stiffeners) generally used wave propagation approaches to model finite length ring-stiffened cylindrical shells by assuming that the structure behaves as an orthotropic shell, a method that is sometimes referred to as "smearing". This was studied for initial hydrostatic pressure using Flugge equations of motion[13] and for fi-