Nonlinear Vibrations and Chaos in Rectangular Functionally Graded Plates with Thermo-mechanical Coupling

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We analyze the nonlinear dynamics of a simply supported, rectangular, and functionally graded plate in terms of a newly derived coupled system of thermo-elasticity and energy equations, which is then expanded here in derivations and explored for chaotic responses through a parameter study in the state space.[1] The plate properties vary linearly in thickness. Three-dimensional stress-strain relations are considered in general case and nonlinear strain-displacement relations are deployed to account for the plate's large deflection. A lateral harmonic force is applied on the plate, and there is a heat generation source within it and the surfaces are exposed to free convection. By integrating over the thickness, four new thermal parameters are introduced, which together with the mid-plane displacements constitute a system of seven partial differential equations. These equations are changed into ordinary differential equations in time using Galerkin's approximation and solved by using the 4$^{\text{th}}$ order Runge-Kutta method. Finally, a parameter study is performed and the appropriate conditions resulting in chaotic solutions are determined by using numerical features such as the Lyapunov exponent and power spectrum.

NOMENCLATURE

$\overline{u}, \overline{v}, \overline{w}$	Particles displacements in x, y, z directions
u, v, w	Midplane displacements in x, y, z directions
A, B, C	Dimensionless amplitudes of u, v, w
V	Dimensionless amplitude of $\dot{w}$
ε_{ij}	Strain components
σ_{ij}	Stress components
θ	Point-wise plate temperature
θ_0	Initial plate temperature
T	Dimensionless amplitude of θ
$N_\theta, M_\theta, P_\theta, Q_\theta$	Temperature 0$^{\text{th}}$ to 3$^{\text{th}}$ moments
D, E, F, G	Dimensionless amplitudes of N, M, P, and Q
E	Modulus of elasticity
ν	Poisson's ratio
α	Thermal expansion coefficient
ρ	Density
k	Thermal diffusivity
c	Specific heat capacity
r	Heat generation rate per unit mass
λ	Plate side length to thickness ratio
h	Plate thickness
a, b	Plate side lengths in x and y directions
ω	Frequency of the external force
H	Heat transfer coefficient
q_0	Amplitude of the external force

1. INTRODUCTION

Functionally graded materials (FGMs) are widely used in aeronautic structures, where they are simultaneously subjected to mechanical and thermal loadings. Thus, presenting a model that combines the mechanical and thermal characteristics increase our understanding of their behavior. In this paper, the coupled problem of thermo-elasticity for the nonlinear dynamics of FGM plates was studied, the governing equations were derived, and after achieving the solution, some parameter values for which trajectories show chaotic behavior were determined.

In the field of linear thermo-elasticity Nowacki has made several fundamental contributions, among which was the derivation of the equations of thermo-elastic vibrations of plates in the coupled case.[2] He solved the problem of transverse vibrations when the temperature field varied harmonically with time. Kawamura et al. derived the governing equations for thermally induced vibrations of an FGM plate exposed to sinusoidally varying surface temperature.[3] Xiang and Melnik presented a numerical approach for the general thermo-mechanical problems, which was based on the reduction of the original system of partial differential equations to a system of differential algebraic equations.[4] They tested the method for a two-dimensional, thermo-elasticity problem. Yang and Shen performed an analysis on free and forced vibrations of initially stressed FGM plates with temperature dependent material properties.[5] They also studied partially distributed impulsive loads on FGM plates resting on elastic foundations.[6] Gupta investigated non-linear thickness variation on the thermally-induced vibration of a rectangular plate using a spline technique.[7]