Exact Solution for a Free Vibration of Thermoelastic Hollow Cylinder Under GNIII Model

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(Received 12 April 2014; accepted 17 February 2015)

The exact analytic solutions are obtained with the use of the eigenvalue approach for a free vibration problem of a thermoelastic hollow cylinder in the context of Green and Naghdi theory (GNIII). The dispersion relations for the existence of various types of possible modes of vibrations in the considered hollow cylinder are derived in a compact form and the validation of the roots for the dispersion relation is presented. To illustrate the analytic results, the numerical solution of various relations and equations has been carried out to compute the frequency, thermoelastic damping and frequency shift of vibrations in a hollow cylinder of copper material with MATHEMATICA and MATLAB software.

1. INTRODUCTION

In the literature concerning thermal effects in continuum mechanics, several parabolic and hyperbolic theories for describing the heat conduction were developed. These hyperbolic theories were also called theories of second sound and there the flow of heat was modelled with finite propagation speed, which contrasts with the classical model based on the Fouriers law leading to infinite propagation speed of heat signals as in.[1–9] Green and Naghdi[10,11] proposed GNII and GNIII, which is a generalized thermoelasticity theory based on entropy equality rather than the usual entropy inequality. An important feature of this theory, which was not present in other thermoelasticity theories, was that it does not accommodate the dissipation of thermal energy. GN theory seems to be idealistic from a physical point of view. The genesis lies in the fact that the thermoelastic model of the GN theory was an idealized material model. During the last years, different problems were considered by using Green and Naghdi theories, as in Abd El-Latief et al.,[12] Youssef,[13] Mukhopadhyay et al.,[14] Sharma et al.,[15] Prasad et al.,[16] Othman et al.,[17] Abbas,[18] and Abbas et al.[19] A survey article of representative theories in the range of generalized thermoelasticity is given by Hetnarski and Ignaczak.[20]

The vibrations in thermoelastic materials have many applications in various fields of science and technology, namely aerospace, atomic physics, thermal power plants, and chemical pipes. The cylinders were frequently used as structural components and their vibrations were obviously important for practical design. Abbas studied the natural frequencies of a poroelastic hollow cylinder.[21] Abd-alla and Abbas investigated the magnetoelastic longitudinal wave propagation in a transversely isotropic circular cylinder.[22] Mykityuk studied the thermoelastic vibrations of a thick-walled cylinder of time-varying thickness.[23] Zhitnyaya analyzed an uncoupled problem of the thermoelastic vibrations of a cylinder.[24] Marin and Lupu studied the harmonic vibrations in thermoelasticity of micropolar bodies.[25] Erbay et al. investigated thermally induced vibrations in a generalized thermoelastic solid with a cavity.[26] Sharma et al. solved the vibration analysis of a transversely isotropic hollow cylinder by using the matrix Frobenius method.[27] Nayfeh and Younis presented a model for thermoelastic damping in microplates.[28,29] Rezazadeh et al. studied the thermoelastic damping in a micro-beam resonator using modified couple stress theory.[30]

The present article is devoted to study the frequency, frequency shifts and damping due to thermal variations in homogenous isotropic hollow cylinder, in the context of Green and Naghdi of type III model of non-classical (generalized) thermoelasticity.

2. BASIC EQUATION AND FORMULATION OF THE PROBLEM

Following Green and Naghdi, the basic equations of the thermoelasticity theory for homogeneous isotropic material in the absence of body forces and heat sources were considered as the equations of motion[10,11]:

$$\sigma_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2}; \tag{1}$$

where ρ was the density of the medium, t was the time, σ_{ij} were the components of stress tensor, and u_i were the components of displacement vector. The equation of heat conduction is:

$$\left(K^*_{ij} T_{,j} + K_{ij} \dot{T}_{,j}\right) = \frac{\partial^2}{\partial t^2}\left(\rho c_e + \gamma T_0 e\right); \tag{2}$$

where T is the temperature, c_e was the specific heat at constant strain, K_{ij} was the thermal conductivity, K^*_{ij} was the material constant characteristic of the theory, T_0 was the reference temperature; $\gamma = (3\lambda + 2\mu)\alpha_t$, α_t was the coefficient of linear thermal expansion. The constitutive equations were given by:

$$\sigma_{ij} = 2\mu e_{ij} + \left[\lambda e - \gamma (T - T_0)\right] \delta_{ij}; \tag{3}$$

with $e = e_{ii},\ i, j = r, \theta, z$, where λ, μ were the Lame's constants and δ_{ij} was the Kronecker symbol.

Let us consider an elastic hollow cylinder of an isotropic homogeneous medium whose state could be expressed in terms of the space variable r and the time variable t. In a cylindrical coordinate system (r, θ, z), for the axially symmetric problem $u_r = u_r(r, z, t)$, $u_\theta = 0$, $u_z = u_z(r, z, t)$. Furthermore, if only the axisymmetric plane strain problem was considered, we had $u_r = u(r, t)$ and $u_\theta = u_z = 0$. Thus, the strain-displacement relations are

$$e_{rr} = \frac{\partial u}{\partial r}, \quad e_{\theta\theta} = \frac{u}{r}, \quad e_{zz} = e_{rz} = e_{r\theta} = e_{\theta z} = 0. \tag{4}$$

The stress-strain relations are

$$\sigma_{rr} = 2\mu \frac{\partial u}{\partial r} + \lambda \left(\frac{\partial u}{\partial r} + \frac{u}{r} \right) - \gamma (T - T_0); \tag{5}$$

$$\sigma_{\theta\theta} = 2\mu \frac{u}{r} + \lambda \left(\frac{\partial u}{\partial r} + \frac{u}{r} \right) - \gamma (T - T_0). \tag{6}$$

It was assumed that there were no body forces and heat sources in the medium, the equation of motion and energy equation hadthe form:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho \frac{\partial^2 u}{\partial t^2} \tag{7}$$

$$K^* \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + K \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial^2 T}{\partial t \partial r} \right) = \frac{\partial^2}{\partial t^2} \left(\rho c_e T + \gamma T_0 \left(\frac{\partial u}{\partial r} + \frac{u}{r} \right) \right). \tag{8}$$

It was convenient to change the preceding equations into the dimensionless forms. To do this, the dimensionless parameters were introduced as

$$(r', u') = \frac{(r, u)}{c\chi}, \quad t' = \frac{t}{\chi}, \quad \omega' = \omega \chi,$$
$$(\sigma'_{rr}, \sigma'_{\theta\theta}) = \frac{1}{\lambda + 2\mu} (\sigma_{rr}, \sigma_{\theta\theta}), \quad T' = \frac{T - T_0}{T_0}, \tag{9}$$

where, $c^2 = \frac{\lambda + 2\mu}{\rho}$, $\chi = \frac{K}{\rho c_e c^2}$. From Eq. (9) into Eqs. (5) to (8), one may obtain (here dashes are ignored for convenience):

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} - a_2 \frac{\partial T}{\partial r} = \frac{\partial^2 u}{\partial t^2}; \tag{10}$$

$$\left(\varepsilon_1 + \frac{\partial}{\partial t} \right) \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) = \frac{\partial^2}{\partial t^2} \left(T + \varepsilon_2 \left(\frac{\partial u}{\partial r} + \frac{u}{r} \right) \right); \tag{11}$$

$$\sigma_{rr} = \frac{\partial u}{\partial r} + a_1 \frac{u}{r} - a_2 T; \tag{12}$$

$$\sigma_{\theta\theta} = a_1 \frac{\partial u}{\partial r} + \frac{u}{r} - a_2 T \tag{13}$$

where $a_1 = \frac{\lambda}{\lambda + 2\mu}$, $a_2 = \frac{\gamma T_0}{\lambda + 2\mu}$, $\varepsilon_1 = \frac{K^*}{\rho c_e c^2}$, $\varepsilon_2 = \frac{\gamma}{\rho c_e}$. The boundary conditions for stress free and isothermal surfaces of the cylinder may be expressed as:

$$\sigma_{rr}(a, t) = \sigma_{rr}(b, t) = 0; \quad T(a, t) = T(b, t) = 0; \tag{14}$$

where a and b are the inner and outer radii of the cylinder respectively.

3. THE EXACT SOLUTION OF THE MODEL

We considered cylindrical time-harmonic vibrations so that:

$$u(r, t) = \bar{u}(r) e^{i\omega t}, \quad T(r, t) = \overline{T} e^{i\omega t}; \tag{15}$$

where ω was the non-dimensional circular frequency of vibrations. By placing Eq. (15) into Eqs. (10) and (11), we get:

$$\frac{d^2 \bar{u}}{dr^2} + \frac{1}{r} \frac{d\bar{u}}{dr} - \frac{\bar{u}}{r^2} = -\omega^2 \bar{u} + a_2 \frac{d\overline{T}}{dr}; \tag{16}$$

$$\frac{d^2 \overline{T}}{dr^2} + \frac{1}{r} \frac{d\overline{T}}{dr} = -\omega^2 \left(\varepsilon_3 \overline{T} + \varepsilon_4 \left(\frac{d\bar{u}}{dr} + \frac{\bar{u}}{r} \right) \right) \tag{17}$$

where $\varepsilon_3 = \frac{1}{\varepsilon_1 + i\omega}$ and $\varepsilon_4 = \frac{\varepsilon_2}{\varepsilon_1 + i\omega}$. By differentiating Eq. (17) with respect to r and using Eq. (16) we got:

$$\frac{d^2}{dr^2} \left(\frac{d\overline{T}}{dr} \right) + \frac{1}{r} \frac{d}{dr} \left(\frac{d\overline{T}}{dr} \right) - \frac{1}{r^2} \left(\frac{d\overline{T}}{dr} \right) = - \omega^2 \left(-\omega^2 \varepsilon_4 \bar{u} + (\varepsilon_3 + \varepsilon_4 a_2) \frac{d\overline{T}}{dr} \right). \tag{18}$$

Equations (16) and (18) could be written in a vector-matrix differential equation as follows:

$$L\mathbf{V} = \mathbf{A}\mathbf{V} \tag{19}$$

where $L \equiv \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2}$ was the Bessel operator, $\mathbf{V} = \begin{bmatrix} \bar{u} & \frac{d\overline{T}}{dr} \end{bmatrix}^T$ and $\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, with $A_{11} = -\omega^2$, $A_{12} = a_2$, $A_{21} = \omega^4 \varepsilon_4$, $A_{22} = -\omega^2 (\varepsilon_3 + a_2 \varepsilon_4)$.

Let us now proceed to solve Eq. (19) by the eigenvalue approach proposed by Das at al.,[31] Abbas,[32–34,36] and Youssef et al.[35] The characteristic equation of the matrix $\mathbf{A}$ takes the form:

$$A_{11} A_{22} - A_{12} A_{21} - (A_{22} + A_{11}) \lambda + \lambda^2 = 0. \tag{20}$$

The roots of the characteristic Eq. (20), which were also the eigenvalues of matrix $\mathbf{A}$, were of the form $\lambda = \lambda_1$, $\lambda = \lambda_2$. The eigenvector $\mathbf{X} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$, which corresponded to the eigenvalue λ, could be calculated as:

$$x_1 = A_{12}, \quad x_2 = \lambda - A_{11}. \tag{21}$$

From Eq. (20), we could easily calculate the eigenvector $\mathbf{X}_j$, which corresponded to the eigenvalue λ_j, $j = 1, 2$. For further reference, we shall use the following notations:

$$\mathbf{X}_1 = [\mathbf{X}]_{\lambda = \lambda_1}, \quad \mathbf{X}_2 = [\mathbf{X}]_{\lambda = \lambda_2}. \tag{22}$$

The solution of Eq. (20) could be written as follows:

$$\mathbf{V} = \mathbf{X}_1 \left(A_1 I_1(p_1 r) + A_2 K_1(p_1 r) \right) + \mathbf{X}_2 \left(A_3 I_1(p_2 r) + A_4 K_1(p_2 r) \right); \tag{23}$$

where $p_1 = \sqrt{\lambda_1}$, $p_2 = \sqrt{\lambda_2}$, I_1, K_1 were the modified of Bessels functions and A_1, A_2, A_3, A_4 were arbitrary constants to be determined. Upon using Eq. (23), the displacement and temperature gradient were obtained as:

$$\bar{u}(r) = x_1^1 \left(A_1 I_1(p_1 r) + A_2 K_1(p_1 r) \right) + x_1^2 \left(A_3 I_1(p_2 r) + A_4 K_1(p_2 r) \right); \tag{24}$$

$$\frac{d\bar{T}}{dr} = x_2^1 \left(A_1 I_1(p_1 r) + A_2 K_1(p_1 r)\right) + x_2^2 \left(A_3 I_1(p_2 r) + A_4 K_1(p_2 r)\right); \quad (25)$$

where x_i^j was the component number i of the eigenvector number j. Thus, the exact solutions of field variables could be written for r and t as:

$$u(r,t) = \Big[x_1^1 \left(A_1 I_1(p_1 r) + A_2 K_1(p_1 r)\right) + x_1^2 \left(A_3 I_1(p_2 r) + A_4 K_1(p_2 r)\right)\Big] e^{i\omega t}; \quad (26)$$

$$T(r,t) = \left[\frac{x_2^1}{p_1} \left(A_1 I_0(p_1 r) - A_2 K_0(p_1 r)\right) + \frac{x_2^2}{p_2} \left(A_3 I_0(p_2 r) - A_4 K_0(p_2 r)\right)\right] e^{i\omega t}; \quad (27)$$

$$\begin{aligned}\sigma_{rr}(r,t) ={}& A_1 \left[\frac{(p_1^2 x_1^1 - \beta x_2^1)}{p_1} I_0(p_1 r) + \frac{x_1^1(\xi - 1)}{r} I_1(p_1 r)\right] + \\ & A_2 \left[\frac{(\beta x_2^1 - p_1^2 x_1^1)}{p_1} K_0(p_1 r) + \frac{x_1^1(\xi - 1)}{r} K_1(p_1 r)\right] + \\ & A_3 \left[\frac{(p_2^2 x_1^2 - \beta x_2^2)}{p_2} I_0(p_2 r) + \frac{x_1^2(\xi - 1)}{r} I_1(p_2 r)\right] + \\ & A_4 \left[\frac{(\beta x_2^2 - p_2^2 x_1^2)}{p_2} K_0(p_2 r) + \frac{x_1^2(\xi - 1)}{r} K_1(p_2 r)\right]; \quad (28)\end{aligned}$$

$$\begin{aligned}\sigma_{\theta\theta}(r,t) ={}& A_1 \left[\frac{(p_1^2 \xi x_1^1 - \beta x_2^1)}{p_1} I_0(p_1 r) - \frac{x_1^1(\xi - 1)}{r} I_1(p_1 r)\right] + \\ & A_2 \left[\frac{(\beta x_2^1 - p_1^2 \xi x_1^1)}{p_1} K_0(p_1 r) - \frac{x_1^1(\xi - 1)}{r} K_1(p_1 r)\right] + \\ & A_3 \left[\frac{(p_2^2 \xi x_1^2 - \beta x_2^2)}{p_2} I_0(p_2 r) - \frac{x_1^2(\xi - 1)}{r} I_1(p_2 r)\right] + \\ & A_4 \left[\frac{(\beta x_2^2 - p_2^2 \xi x_1^2)}{p_2} K_0(p_2 r) - \frac{x_1^2(\xi - 1)}{r} K_1(p_2 r)\right]. \quad (29)\end{aligned}$$

4. DISPERSION RELATIONS

We assumed that the thermoelastic hollow cylinder was subjected to traction-free and isothermal boundary conditions, Eq. (15), at its surfaces ($r = a, b$). By applying boundary conditions, which were Eqs. (15), (27), and (28), we obtain a system of four homogeneous linear algebraic equations in unknowns A_1, A_2, A_3, and A_4. This system would have a nontrivial solution if and only if the determinant of the coefficients A_1, A_2, A_3, and A_4 vanished and such a requirement of nontrivial solution lead to dispersion equations given by:

$$\Delta = \det(L_{ij}) = 0, \quad i,j = 1,2,3,4; \quad (30)$$

where,

$$L_{11} = \frac{(p_1^2 x_1^1 - a_2 x_2^1)}{p_1} I_0(p_1 a) + \frac{x_1^1(a_1 - 1)}{a} I_1(p_1 a);$$

$$L_{12} = \frac{(a_2 x_2^1 - p_1^2 x_1^1)}{p_1} K_0(p_1 a) + \frac{x_1^1(a_1 - 1)}{a} K_1(p_1 a);$$

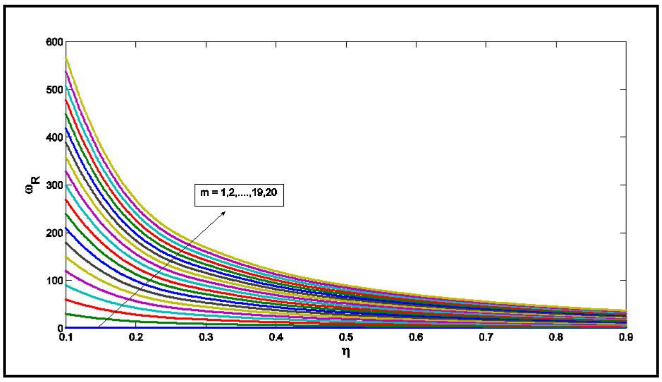

Figure 1. Non-dimensional frequency ω_R verses the length to mean radius ratio η.

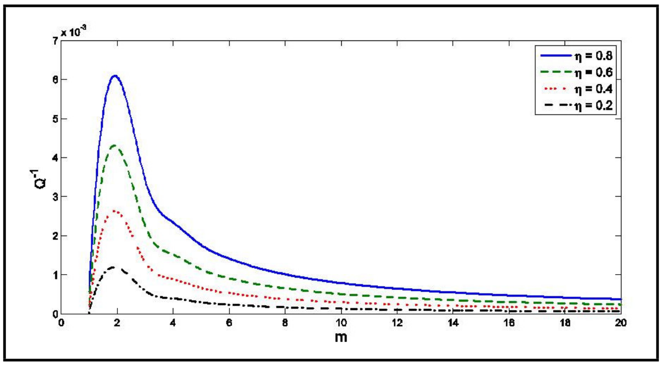

Figure 2. Thermoelastic damping Q^{-1} versus m for different values of η.

$$L_{13} = \frac{(p_2^2 x_1^2 - a_2 x_2^2)}{p_2} I_0(p_2 a) + \frac{x_1^2(a_1 - 1)}{a} I_1(p_2 a);$$

$$L_{14} = \frac{(a_2 x_2^2 - p_2^2 x_1^2)}{p_2} K_0(p_2 a) + \frac{x_1^2(a_1 - 1)}{a} K_1(p_2 a);$$

$$L_{21} = \frac{(p_1^2 x_1^1 - a_2 x_2^1)}{p_1} I_0(p_1 b) + \frac{x_1^1(a_1 - 1)}{b} I_1(p_1 b);$$

$$L_{22} = \frac{(a_2 x_2^1 - p_1^2 x_1^1)}{p_1} K_0(p_1 b) + \frac{x_1^1(a_1 - 1)}{b} K_1(p_1 b);$$

$$L_{23} = \frac{(p_2^2 x_1^2 - a_2 x_2^2)}{p_2} I_0(p_2 b) + \frac{x_1^2(a_1 - 1)}{b} I_1(p_2 b);$$

$$L_{24} = \frac{(a_2 x_2^2 - p_2^2 x_1^2)}{p_2} K_0(p_2 b) + \frac{x_1^2(a_1 - 1)}{b} K_1(p_2 b);$$

$$L_{31} = \frac{x_2^1}{p_1} I_0(p_1 a), \qquad L_{32} = -\frac{x_2^1}{p_1} K_0(p_1 a)$$

$$L_{33} = \frac{x_2^2}{p_2} I_0(p_2 a), \qquad L_{34} = -\frac{x_2^2}{p_2} K_0(p_2 a)$$

$$L_{41} = \frac{x_2^1}{p_1} I_0(p_1 b), \qquad L_{42} = -\frac{x_1^2}{p_1} K_0(p_1 b)$$

$$L_{43} = \frac{x_2^2}{p_2} I_0(p_2 b), \qquad L_{44} = -\frac{x_2^2}{p_2} K_0(p_2 b)$$

5. NUMERICAL RESULTS AND DISCUSSION

The copper material had been chosen for the purposes of numerical evaluations in the space-time domain. From the material constants, we got the non-dimensional values of the problem as Abbas:[37]

$$\mu = 3.86 \times 10^{10} (\mathrm{kg})(\mathrm{m})^{-1}(\mathrm{s})^{-2};$$

Table 1. The validation of the roots for the dispersion relation.

m	$\eta = 0.4$		$\eta = 0.8$	
	ω	Δ	ω	Δ
1	$0.7062 + 0.00002\mathrm{i}$	$4.2 \times 10^{-16} - 1.5 \times 10^{-17}\mathrm{i}$	$0.5628 + 0.00016\mathrm{i}$	$9.9 \times 10^{-16} + 1.1 \times 10^{-15}\mathrm{i}$
2	$6.3247 + 0.00825\mathrm{i}$	$-3.2 \times 10^{-16} - 6.9 \times 10^{-16}i$	$2.4331 + 0.00738\mathrm{i}$	$-1.0 \times 10^{-16} + 1.8 \times 10^{-17}\mathrm{i}$
5	$25.143 + 0.00839\mathrm{i}$	$5.1 \times 10^{-19} - 2.4 \times 10^{-19}\mathrm{i}$	$9.4425 + 0.00834\mathrm{i}$	$5.0 \times 10^{-16} - 7.1 \times 10^{-17}\mathrm{i}$
10	$56.553 + 0.00839\mathrm{i}$	$-23 \times 10^{-18} - 4.4 \times 10^{-19}\mathrm{i}$	$21.213 + 0.00839\mathrm{i}$	$-2.0 \times 10^{-18} + 7.1 \times 10^{-18}\mathrm{i}$
15	$87.967 + 0.0084\mathrm{i}$	$7.3 \times 10^{-19} + 1.3 \times 10^{-18}\mathrm{i}$	$32.991 + 0.00839\mathrm{i}$	$1.7 \times 10^{-17} + 4.8 \times 10^{-18}\mathrm{i}$
20	$119.38 + 0.0084\mathrm{i}$	$-3.1 \times 10^{-19} - 1.8 \times 10^{-18}\mathrm{i}$	$44.771 + 0.00839\mathrm{i}$	$-2.7 \times 10^{-17} + 1.5 \times 10^{-18}\mathrm{i}$

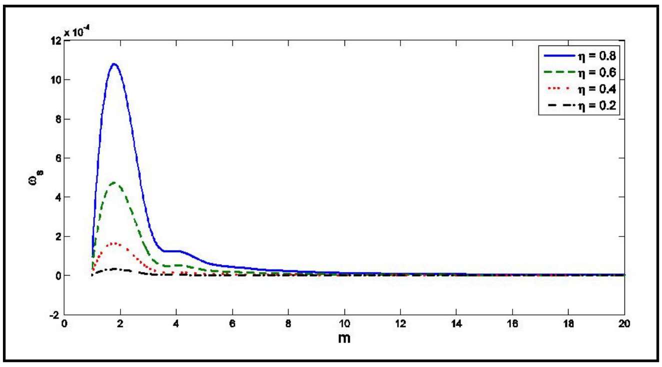

Figure 3. Frequency shift ω_s versus m for different values of η.

$$\begin{aligned}
&\lambda = 7.76 \times 10^{10}(\mathrm{kg})(\mathrm{m})^{-1}(\mathrm{s})^{-2};\\
&c_e = 3.831 \times 10^{2}(\mathrm{m})^{2}(\mathrm{K})^{-1}(\mathrm{s})^{-2};\\
&K = 3.68 \times 10^{2}(\mathrm{kg})(\mathrm{m})(\mathrm{K})^{-1}(\mathrm{s})^{-3};\\
&\rho_0 = 8.954 \times 10^{3}(\mathrm{kg})(\mathrm{m})^{-3};\\
&\alpha_t = 17.8 \times 10^{-6}(\mathrm{K})^{-}1;\\
&T_0 = 293(\mathrm{K}); \quad a = 1.
\end{aligned} \tag{31}$$

The numerical computation had been carried out with the help of MATHEMATICA and MATLAB files for the length to mean radius ratio $\left(\eta = \frac{b-a}{R}\right)$, where $a = 1$, $b > a$, $R = \frac{a+b}{2}$. Due to the presence of a dissipation term in the heat conduction equation, the frequency equation in the general complex transcendental equation provided us with a complex frequency value, $\omega^m = \omega_R^m + i\omega_I^m$, where m was the mode number that corresponded to the roots of the transcendental Eq. (30) and ω_R^m and ω_I^m were the real part and imaginary parts of frequency ω^m. The thermoelastic damping factor was given by:

$$Q^{-1} = 2\left|\frac{\omega_I^m}{\omega_R^m}\right|.$$

The frequency shift due to thermal variations was defined as:

$$\omega_s = \left|\frac{\omega_R^m - \omega_o^m}{\omega_o^m}\right|;$$

where ω_o^m was the frequency in elastic hollow cylinder.

The calculation of the roots of the dispersion relation Eq. (30) represented a major task and required a rather extensive effort of numerical computation. The frequency spectrum ω^m versus the different values of the length to mean radius ratio η for the first twenty modes was computed by the interval halving method. The validation of the roots for the dispersion relation is presented in Table 1.

The first twenty modes of non-dimensional frequency ω_R verses the length to radius ratio η are presented in Fig. 1. It was observed that the non-dimensional frequency ω_R decreased with an increasing length to mean radius ratio. The thermoelastic damping Q^{-1} versus m for different values of the length to mean radius ratio η are presented in Fig. 2, from which it is seen that the thermoelastic damping Q^{-1} increased initially to attain its maximum peak value at the second mode before it decreased in order to become ultimately asymptotic with increasing m. Figure 3 shows the variation of the frequency shift ω_s versus m for different values of the length to mean radius ratio η. It could be inferred that the frequency shift ω_s increased sharply to attain its maximum peak value at the second mode and then decreased to become ultimately asymptotic with increasing m.

6. CONCLUSIONS

The exact solution for a free vibration of thermoelastic hollow cylinder under GNIII model has been done with the help of the eigenvalue approach. The eigenvalue approach is applied successfully to get an explicit, totally analytic, and uniformly valid solution for the current problem. The validation of the roots for the dispersion relation is also presented. The closed form solution obtained here opens the scope of further studies in mathematics, science, and engineering disciplines.

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