# Stability and Accuracy of Aeroacoustic Time-Reversal using the Pseudo-Characteristic Formulation

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This paper investigates the stability and accuracy of the aeroacoustic Time-Reversal (TR) simulation using the Pseudo-Characteristic Formulation (PCF). To this end, the forward simulation of acoustic wave propagation in 1-D and 2-D computational domain with a uniform mean flow was implemented using the PCF of the Linearised Euler Equations (LEE). The spatial derivatives in the opposite propagating fluxes of the PCF were computed using an overall upwind-biased Finite-Difference (FD) scheme and a Runge-Kutta scheme was used for time-integration. The anechoic boundary condition (ABC) was implemented for eliminating spurious numerical reflections at the computational boundaries, thereby modelling a free-space. The stability of 1-D forward and TR (with only timereversed acoustic pressure as the input at the boundary nodes) simulations were analysed by means of an eigenvalue decomposition, wherein it was shown that opposite upwinding directions must be considered while using the overall upwind-biased FD scheme. Furthermore, the implementation of ABC was found to be crucial for ensuring the stability of the forward simulation over a large time duration and the 2-D TR simulations. The overall central Dispersion-Relation Preserving (DRP) FD schemes were however, found to be unstable and unsuitable for TR simulation. The accuracy of both the forward and the TR simulations using the PCF was assessed by comparing the simulation results against the corresponding analytical solutions of a spatially and temporally evolving Gaussian pulse. It was shown that numerically reversing the mean flow direction during TR (using the PCF) and only the time-reversed acoustic pressure as input at the boundaries is sufficient to accurately back-propagate the waves and localise the initial emission point of the pulse in 1-D or 2-D computational domain.

## **1. INTRODUCTION**

The acoustic Time-Reversal (TR) method, developed by Fink, et al.,<sup>1,2</sup> is a promising method to localise acoustic sources in time-domain and is explained by the following two-step procedure:

- 1. In the first step, the acoustic pressure field radiated by the source(s) is recorded by microphone line arrays (LAs) in a Time-Reversal Mirror (TRM) during experiments, or stored at the boundary nodes (virtual microphones) during forward simulations, either (a) over LAs completely enclosing the sources, or (b) over a limited angular aperture LA(s) that only partially encloses the source(s).
- 2. In the second step, the recorded acoustic pressure timehistory is reversed in time followed by emission from "numerical sources" at the boundary nodes. The backpropagated acoustic pressure signals undergo a constructive interference to form spatio-temporal maxima<sup>3</sup> during TR simulations, which corresponds to the spatial location of the source(s). Method (a) which uses the enclosing LAs, can account for almost the total acoustic power radi-

ated; therefore, back-propagation from this configuration yields the most accurate prediction of the source location, characteristics, and strength. Method (b), however, can account for only a fraction of the acoustic power radiated, thereby limiting the ability of TR to identify the location and nature of sources.<sup>4</sup>

Fink, et al.<sup>2</sup> provide an excellent review of the TR method and discuss its applications in various fields such as hydrodynamics, ultrasound medical imaging, and diagnostic and non-destructive testing. The TR method has also been used for long-range communication in deep underwater acoustics,<sup>5</sup> structural dynamics for health monitoring,<sup>6</sup> in the presence of a reflecting surface,<sup>7</sup> and in electromagnetic wave propagation.<sup>8</sup> Different methods have been presented to enhance the focal-resolution of TR, namely an active cancellation technique called the Time-Reversal Acoustic Sink (TRAS), developed by Bavu, et al.,<sup>9</sup> and a passive radial damping approach mimicking an acoustic sink called the Point-Time-Reversal-Sponge-Layer (PTRSL), developed by the present authors.<sup>10</sup> The application of the TR method in Computational Aeroacoustics (CAA) for localising sound sources in a flow field is,