# **Study of Automobile Suspension System Vibration Characteristics Based on the Adaptive Control Method**

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An automotive suspension determines both the driving stability and comfort of the vehicle occupants. This paper establishes two kinds of two degrees of freedom for the automobile suspension vibration model and uses the PID controller to establish an automobile suspension adaptive open-loop and closed-loop control system. Respectively, by step interference, white noise and sinusoidal interference for the input, studying the vibration characteristics of the vibration model in the vertical direction. By numerical simulation, we obtain the suspension of the vertical displacement and acceleration-time graphs. The simulation results show that the vibration characteristics of the first model are more in accordance with the actual situation of the car, and the closed-loop control is better than the open-loop control. The adaptive closed-loop control system can reduce the output displacement of automobile suspension to around 1% of the interference road input displacement. The output acceleration value is small, and the acceleration changes smoothly. The results verify the rationality and validity of the automobile suspension model and adaptive control system, which provides a theoretical foundation for the design and optimization of the automobile suspension system.

### **1. INTRODUCTION**

Suspensions are an important part of the car chassis as they directly determine the car's ride comfort and handling stability, but because of their high costs and high energy demand, their application is restricted. Hence, the semi-active suspension system is now used in the automobile suspension structure.

Over the years, many studies on suspension systems have been done.<sup>1-15</sup> Tomoaki Mori<sup>1</sup> put forward an adaptive damper controller for compensating the nonlinear hysteresis dynamics of the MR damper. Kayhan Gulez,<sup>2</sup> Ali Ahmed Adam,<sup>12</sup> and Hua Li<sup>14</sup> use an algorithm method to research the nonlinear characteristics for the semi-active suspension of automobiles. Enrico Pellegrini<sup>3</sup> and Zhao Cheng<sup>4</sup> build a semi-active suspension system model based on control strategy in order to describe the damper behaviour. Li Shaohua<sup>5</sup> and Liang Shan<sup>6</sup> studied the two degrees of freedom 1/4 automobile suspension model; the former used a multi-scale method to study the combination of multi-frequency excitation resonance characteristics, and the latter researched the suspension model chaotic vibration occurring in road roughness excitation. Ren Chenglong<sup>7</sup> established a single degree of freedom model of automobile suspension under random excitation; the numerical simulation verifies the automobile suspension chaotic vibration characteristic.<sup>7,11</sup> Jiammin Sun and Oingmei Yang<sup>8</sup> established a suspension model and analysed the comfort and safety based on the adaptive filter theory. Abu-Khudhair,<sup>9</sup> Changizi,<sup>10</sup> and Barr<sup>13</sup> use a fuzzy logic technique to build the semi-active automobile suspension systems control, which has a significantly fewer number of rules in comparison to existing fuzzy controllers. Hung Yichen<sup>15</sup> proposes a functional-approximation based adaptive sliding controller with fuzzy compensation for an active suspension system.

The above studies on automobile suspension do not simultaneously propose two vibration models based on the adaptive control method so they can't prove which model is more suitable for an automobile suspension system. Hence, this paper gives two automobile suspension system vibration models using the adaptive control method and two control algorithms consisting of PID controllers. This paper also studies suspension system vibration characteristics with step input, whitenoise input, and sinusoidal input as the external excitation. By numerical simulation, this study obtains the displacement and acceleration characteristics. Comparing the results of the two models, we find that when examining the engine and the suspension as a whole, the suspension system vibration characteristics more closely match the actual conditions, which is conducive for designing the most appropriate automobile suspension model.



Figure 1. The first model.



Figure 2. The second model.

# 2. AUTOMOBILE SUSPENSION SYSTEMS

# 2.1. Vibration System Modelling

A car is mainly composed of wheels, suspension, and engines, and the automobile body vibration will directly affect the handling and safety. The vibration in the vertical direction of the suspension determines the performance of the body. This study establishes two kinds of two degrees of freedom automobile suspension vibration models as shown in Fig. 1 and Fig. 2. The first model depicts the engine and suspension as a whole, the second model considers the engine and suspension separately. Suspension vertical displacement and acceleration is an important indicator of automotive comfort and handling. This article will focus on the suspension vertical displacement and acceleration.<sup>3,5,6,8–10</sup>

According to the vibration model in Fig. 1, the following dynamic differential equation is obtained:<sup>14, 15</sup>

$$\begin{cases} m_1 \ddot{x}_1 = k_s (x_2 - x_1) + b \left( \frac{dx_2}{dt} - \frac{dx_1}{dt} \right) + u \\ m_2 \ddot{x}_2 = -k_s (x_2 - x_1) - b \left( \frac{dx_2}{dt} - \frac{dx_1}{dt} \right) - u + k_t (w - x_2). \end{cases}$$
(1)



Figure 3. Adaptive linear combiner.



Figure 4. Adaptive transversal filter.

Make  $x = x_1 - x_2$ , by the Laplace transform can be obtained:

$$\begin{cases} X(s) = [U(s) - G_f(s)W(s)] G_p(s) \\ G_f(s) = \frac{m_1k_ts^2}{(m_1 + m_2)s^2 + k_t} \\ G_p(s) = \\ \frac{m_1k_ts^2}{m_1m_2s^4 + b(m_1 + m_2)s^3 + k_tm_1s^2 + k_s(m_1 + m_2)s^2 + bk_ts + k_sk_t}. \end{cases}$$

According to the vibration model in Fig. 2, the following dynamic differential equation is obtained:<sup>14,15</sup>

$$\begin{cases} m_1 \ddot{x}_1 + c_1 \left( \frac{dx_1}{dt} - \frac{dx_2}{dt} \right) + k_1 (x_1 - x_2) = 0\\ m_2 \ddot{x}_2 + c_2 \left( \frac{dx_2}{dt} - \frac{dx_3}{dt} \right) + k_2 (x_2 - x_3) + \\ c_1 \left( \frac{dx_2}{dt} - \frac{dx_1}{dt} \right) + k_1 (x_2 - x_1) - u = 0\\ m_3 \ddot{x}_3 + c_2 \left( \frac{dx_3}{dt} - \frac{dx_2}{dt} \right) + k_2 (x_3 - x_2) + \\ k_3 (x_3 - x_r) + u = 0. \end{cases}$$
(3)

Make  $x = x_2 - x_3$ , by the Laplace transform can be obtained:

$$\begin{cases} X(s) = [U(s) + G_f(s)X_r(s)] G_p(s) \\ G_f(s) = \frac{k_3c_2s + k_2k_3 - k_3}{s^2 + k_3} \\ G_p(s) = \\ \frac{t_8s^8 + t_7s^7 + t_6s^6 + t_5s^5 + t_4s^4 + t_3s^3 + t_2s^2}{n_{10}s^{10} + n_9s^9 + n_8s^8 + n_7s^7 + n_6s^6 + n_5s^5 + n_4s^4 + n_3s^3 + n_2s^2 + n_1s + n_0}. \end{cases}$$
(4)



Figure 5. The automobile suspension open-loop control system (without PID).



Figure 6. The automobile suspension closed-loop control system (with PID).

#### 2.2. Adaptive Control Method

The adaptive control method is useful when great changes have taken place with the input signal or disturbance signal and the system automatically adjusts to maintain the output that meets the necessary requirements. The adaptive linear combiner structure shown in Fig. 3,  $x_0, x_1, \ldots, x_L$  are signal vectors, and  $w_0, w_1, \ldots, w_L$  is a set of adjustable weight. Fig. 3 depicts an input corresponding to an adjustment of weight and the final output of the system of linear combinations of the input. As can be seen from this figure, the system is linear.<sup>8</sup>

From the adaptive control method and Fig. 3, the following equation can be established:<sup>8</sup>

$$\begin{cases} X^{T}(n) = [x(n), x(n-1), \dots, x(n-L+1)] \\ W^{T} = [w_{1}, w_{2}, \dots, w_{L}] \\ y(n) = \sum_{j=1}^{L} w_{j}x(n-j+1) = W^{T}X(n) \\ e(n) = d(n) - y(n) \\ E\left[e^{2}(n)\right] = E\left[d^{2}(n)\right] 2R_{xd}^{T}W + W^{T}R_{xx}W; \end{cases}$$
(5)

where  $X^T$  is input vector,  $W^T$  is weight vector, L is the length of filter, y(n) is the output of the filter, e(n) is the error signal,  $R_{xd}$  is the cross-correlative function of x(n) and d(n), and  $R_{xx}$  is the auto-correlative function of x(n).

When an input of a filter is formed from a series of delayed samples of some signal, such filter structure is called adaptive transversal filter structure,<sup>8</sup> as shown in Fig. 4.

The following equation<sup>8</sup> can be established according to Fig. 4:

$$\begin{cases} W(n+1) = W(n) - \mu \nabla e^{2}(n) \\ \Delta e^{2}(n) = -2e(n)x(n) \\ y(n) = W^{T}X(n) \\ e(n) = d(n) - W^{T}X(n) \\ W(n+1) = W(n) + 2\mu e(n)X(n); \end{cases}$$
(6)

where W(n + 1) is the next moment that equals the weight value of w(n),  $\mu$  is the gain constant, which controls adaptive speed and stability,  $\nabla e(n)$  is the error function, and  $\Delta e(n)$  is the error gradient function.

# 2.3. Constructed Automobile Suspension Control System

Random road is the disturbance input of the suspension system, which decides the car driving performance. So it is important to build a useful control system to reduce or eliminate bad interference. The following control system is built on the assumption that the system's expected output is X = 0<sup>-1,2,12,13</sup>

$$G_c = \frac{K_d s^2 + K_p s + K_i}{s} \tag{7}$$

where  $G_c$  is PID controller.  $K_d = 5$ ,  $K_i = 8$ ,  $K_p = 95$ , Fig. 7 is the Bode diagram of  $G_c$ .

Figure 7 gives the suspension system bode diagram without correction, with correction, and after correction. From the figure, it can be obtained that without correction, the system phase changes dramatically near degree 180, and at that time, if the system is to increase the gain or suffer some kind of disturbance, the system phase may become negative, making the system unstable. After the correction, the system phase beats near degree 0, which effectively improves the stability of the system.

# 3. AUTOMOBILE SUSPENSION VIBRATION MODEL RESPONSE

## 3.1. Step Disturbance Response Characteristics in the First Model

Take the system parameters:  $m_1 = 2500 \text{ kg}$ ,  $m_2 = 320 \text{ kg}$ ,  $k_s = 10000 \text{ N/m}$ , b = 140000 Ns/m into Eq. (2). The simulation results as shown in the Fig. 8 and Fig. 9.

As can be seen from Fig. 8 and Fig. 9, the response speed of the open-loop control system is faster than that of the closedloop system, but the closed-loop system has the function of adaptive control, the timely adjustment of the deviation of system, the reduction of the vibration of the system, and the improvement of the car's stability and security.

## 3.2. Step Disturbance Response Characteristics in the Second Model

Take the system parameters:  $m_1 = 100 \text{ kg}$ ,  $m_2 = 200 \text{ kg}$ ,  $m_3 = 30 \text{ kg}$ ,  $k_1 = 400000 \text{ N/m}$ ,  $k_2 = 20000 \text{ N/m}$ ,  $k_3 = 180000 \text{ N/m}$ ,  $c_1 = 4000 \text{ Ns/m}$ ,  $c_2 = 500000 \text{ Ns/m}$  into Eq. (4) and the simulation results as shown in Fig. 10 and Fig. 11

From Fig. 10 and Fig. 11, we can see that the response of two models is similar. The amplitude of closed-loop control system is smaller than the open-loop system, and the closedloop control system response speed is slower than that of the open-loop system. At the time of automobile start, the amplitude of the automobile is 0, which indicates that the system has good stability, and ensures a smooth start of the car. But shortly after running the car, there will be a more serious jitter, which greatly affects the normal running of the car.

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Figure 7. The bode diagram.



Figure 8. Step interference open-loop system response.



Figure 9. Step interference closed-loop system response.



Figure 10. Step interference open-loop system response.



Figure 11. Step interference closed-loop system response.



Figure 12. White-noise signal.



Figure 13. Integral white-noise signal.



Figure 14. Automotive suspension displacement-time curve under white noise.



Figure 15. Automotive suspension acceleration-time curve under white noise.

By comparing the two models, it can be found that in the first model a serious quiver appears at first, but the jitter time is short, and the car can realize the adaptive control within a few minutes, quickly reducing the car jitter, and ensuring that the car is moving. The first model better reflects the vibration characteristics of a vehicle suspension system, so this paper will do further research of this model.

As the interference of the system-step response has already been studied, the following will study the vehicle suspension



Figure 16. Sinusoidal interference signal.



Figure 17. Integral sinusoidal interference signal.



Figure 18. Automotive suspension displacement-time curve under the sinusoidal interference signal.



Figure 19. Automotive suspension acceleration-time curve under the sinusoidal interference signal.

system response characteristics under white noise and sinusoidal excitation.

Figure 12 to Fig. 15 shows the suspension system in the white noise excitation. By the adaptive closed-loop control, the amplitude of the suspension system is reduced to about 1% of the excitation, the acceleration of the suspension is about 4 m/s<sup>2</sup>, the acceleration change is relatively stable, and larger acceleration values occur at only very few moments. The adaptive closed-loop control system has a good white-road noise

interference protective effect, affectively reducing the amplitude and acceleration of the suspension, guaranteeing the car to drive normally.

Figure 16 to Fig. 19 depict the suspension system in the sinusoidal interference signal excitation. In these figures it can be seen that the maximum displacement of the vertical direction is reduced to about 1% of the disturbance input. The acceleration of the overall image changes as a sinusoidal, and the maximum acceleration is  $0.4 \text{ m/s}^2$ . Relative to the high-speed operation of the car, the car's acceleration has little effect on normal driving. By comparing the results, we can find that the adaptive closed-loop control system being used in sinusoidal interference signal excitation is more effective than in white-noise interference excitation, and the results once again validate the practicality and effectiveness of the adaptive closed-loop control system.

# 4. CONCLUSIONS

(1) This paper establishes two kinds of two degrees of freedom automobile suspension vibration models; two models responses are studied under step and white-noise interference in both an open and closed-loop system. By contrast and analysis, the numerical simulation results show that the engine and suspension as a whole suspension vibration model is more likely to meet the needs for the actual operation of the automobile.

(2) This paper uses proportional, integral, and differential elements to form the PID controller, and makes up the adaptive closed-loop control system to study the vertical displacement and acceleration response of the automobile suspension system.

(3) Numerical simulation results show that the adaptive closed loop control system can reduce the vertical direction amplitude of the suspension to 1% of the original interference input displacement, while the maximum acceleration is no more than 5 m/s<sup>2</sup>, and the acceleration changes smoothly. The results demonstrate the rationality and effectiveness of the design of the adaptive closed-loop control system and provide a theoretical basis for the design of automobile suspension control systems.

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