Transverse Plane-Wave Analysis of Short Elliptical End-Chamber and Expansion-Chamber Mufflers

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The flow-reversal end chambers are used quite often in commercial automotive mufflers. The conventional axial plane-wave theory is not able to predict their acoustic performance because of the fact that the length of the end chambers is not enough for the evanescent three-dimensional modes generated at the junctions to decay sufficiently for frequencies below the cut-off frequency. Also, due to the large expansion ratio at the inlet, the first few higher-order modes get cut on even in the low-frequency regime. This necessitates a finite element or boundary element analysis, which is cumbersome and time-consuming. Therefore, an ingenious one-dimensional method has been developed. It models plane-wave propagation in the transverse direction between the incoming pipe and the return pipe, with the lateral-end cavities being modeled as variable-area quarter-wave resonators. Making use of this novel approach, the transfer matrices have been derived for elliptical and circular cross-section mufflers, which enable these elements to be analyzed along with the rest of the muffler elements by means of the transfer matrix-based muffler program. Through a comparison with a full, three-dimensional analysis on commercial software, it is shown that the one-dimensional approach presented in this paper is able to predict the transmission loss quite accurately up to about 1000 Hz for typical automotive mufflers.

NOMENCLATURE

c_0	–	sound speed in the medium
d_1, d_2	–	diameters of the inlet and outlet pipes
d_3	–	diameter of the pass tube in the flow-reversal muffler
D_1, D_2	–	major axis and minor axis of the elliptical muffler
D	–	diameter of the circular expansion chamber
j	–	$(-1)^{1/2}$
k_0	–	the wave number, ω/c_0
K_{1u}, K_{2u}, K_{3u}	–	stagnation pressure-loss coefficients at the inlet junction
K_{1d}, K_{2d}, K_{3d}	–	stagnation pressure-loss coefficients at the outlet junction
L	–	axial length of the muffler
M_d, M_u	–	Mach number at the outlet (U_o/c_0) and inlet (U_i/c_0), respectively
M_1, M_2	–	mean-flow Mach number at the muffler inlet (U_1/c_0) and outlet (U_2/c_0), respectively, perpendicular to the assumed plane-wave path
n	–	total number of small parts over which the region of interest is divided
$p(x)$	–	acoustic pressure field at any generic point in the muffler system
p_u, p_d	–	acoustic pressure at the upstream inlet pipe and point in the downstream outlet pipe, respectively
p_{r1}, p_{r2}	–	acoustic pressure at the inlet and outlet junctions of the quarter-wave resonators, respectively
p_1, p_2	–	acoustic pressure at the inlet and outlet junctions in the muffler at the beginning and end of the transverse plane path, respectively, for a short-length muffler with a staggered configuration
$S(x)$	–	cross-sectional area of muffler as a function of distance x, measured along the plane-wave propagation path
S_u, S_d	–	cross-section area of the inlet and outlet pipe, respectively
S_1, S_2	–	cross-section area of the muffler at the inlet and outlet, respectively, perpendicular to the assumed plane-wave path
$u(x)$	–	acoustic particle velocity along the direction of the plane-wave propagation at any generic point in the muffler system
U_i, U_o	–	mean-flow velocity at the inlet and outlet pipe, respectively
U_1, U_2	–	mean-flow velocity at the muffler inlet and outlet, respectively
v_u, v_d	–	acoustic-mass velocity at the upstream point and downstream point, respectively
v_{r1}, v_{r2}	–	acoustic-mass velocity at the quarter-wave resonators at inlet and outlet junctions, respectively
v_1, v_2	–	acoustic-mass velocity at the inlet and outlet junctions in the muffler at the beginning and end of the transverse plane path, respectively, for a short-length muffler with a staggered configuration
Y_d	–	characteristic impedance at downstream point in the outlet pipe, c_0/S_d
Y_u	–	characteristic impedance at the upstream point in the inlet pipe, c_0/S_u
Y_1, Y_2	–	characteristic impedance at the muffler inlet (c_0/S_1) and outlet (c_0/S_2), respectively, perpendicular to the assumed plane-wave path