# Instabilities of Compressible Fluid Flow over a Plate

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This paper considers the linearised dynamics of flow of a compressible fluid over a plate. In particular, we study the stability of the flow, and identify the types of instability that occur. The zones of stability/instability in the Mach number-velocity ratio plane have been obtained. Two regions, namely convectively unstable and absolutely unstable, are observed to exist. Thus, the flow is always unstable. It is found that low values of Mach number and velocity ratio yield a convectively unstable flow. At high Mach number and/or velocity ratio, the system becomes absolutely unstable.

## 1. INTRODUCTION

Problems involving fluid-loaded structures are essentially of two kinds: one in which there is no mean flow of the fluid, and the other with a mean flow. These two problems are characteristically different. In the case of a structure in a static fluid, one is usually interested in modelling the fluid-structure interaction through the determination of the added mass and radiation damping parameters for the structure. There is another class of literature that deals with the response of locally excited fluid-loaded structures in the absence of a mean flow (see for example reference[1]). In such problems, the objective is to understand the spatio-temporal evolution of waves of different wave numbers when the structure is excited by a space-localised harmonic signal. On the other hand, in the case of flow of the ambient fluid past a flexible structure, there is a possibility of the structure deriving energy from the flowing fluid, and hence stability of the structure is the primary point of investigation.

Interaction of structures with flowing fluids are commonly found in engineering applications. Some examples are the wings of an aircraft in flight, a high-rise building interacting with the wind, and a high-speed rotating disc in a computer. There have been numerous studies on the stability of fluids flowing over compliant surfaces like membranes and plates. However, most of these studies have considered the fluid to be incompressible. On the other hand, the study of acoustics of vibrating structures has gained importance lately. This analysis requires the fluid to be compressible. Hence, it is of interest to study the stability of compressible fluid flow over compliant surfaces.

Fluid flow over compliant surfaces introduces considerable complications. Such problems have been studied previously, and important contributions have been made by Benjamin[2,3] and Landahl[4]. The classification of waves and their stability properties in reference[3] has led to the theory of positive and negative energy waves. Cairns[5] studied the effect of negative energy waves in hydrodynamic stability problems. Classically, investigations on the stability of flow over compliant surfaces study the evolution of an initial disturbance (shape and velocity) with a real wave-number on the surface. This is essentially an initial value problem with non-local initial disturbance. However, disturbances are usually localised, and the classical stability analysis fails to capture the transient dynamics of such disturbances. For example, when a disturbance develops at a point, it may be washed downstream faster than it can spread (convective instability). On the other hand, it may spread fast enough to contaminate the whole medium (absolute instability). These concepts are discussed further in Section 3.2.

As far as locally excited fluid-loaded structures are concerned, in general it is possible that multiple solutions to a single frequency excitation exist. Such cases are handled using the concept of causality,[6] which existed in the literature on plasma instability problems.[7] A comprehensive summary of the methods used can be found in reference[8]. The instability of (incompressible) fluid flow over an elastic plate was first studied by Brazier-Smith and Scott[9] largely through numerical examinations of a (quintic) dispersion relation which can be obtained analytically. They showed that at normalised flow speeds, $U$, greater than a critical value $U_c$, the systems become absolutely unstable. In that case, the response to any forcing diverges exponentially in time at all spatial locations of the plate. For $U < U_c$, the system was shown to have different characteristics for different harmonic forcing frequency ranges. In a certain range $0 < \omega < \omega_s(U)$, the system was found to be convectively unstable, neutrally stable (with some anomalous features) for $\omega_s(U) < \omega < \omega_p(U)$, and stable and behaving essentially as in the absence of a mean flow for $\omega > \omega_p(U)$. An analytical solution for the above problem has been presented by Crighton and Oswell,[10] where asymptotic expressions are found for $U_c$ and the frequencies $\omega_s(U)$ and $\omega_p(U)$. An energy equation governing the behaviour is derived. This is used to illuminate properties of the convectively unstable and neutral waves. Anomalous propagation is investigated further from the viewpoint of the theory of negative energy waves. The stability of a fluid flow over a membrane has been studied by Metcalfe.[11] However, he has also considered an incompressible fluid.

Stability of elastic waves that develop in elastic structures unbounded in the streamwise direction (such as infinite plates, membranes, shells and pipes) has been considered in the