# Electromechanical Coupled Vibration for Double Coupled Micro Beams


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In this paper, the electromechanical coupled dynamic equations for the double coupled micro beams are presented. The linearisation of the dynamic equations is made. From the linear dynamic equations, natural frequencies and vibration modes of the double coupled micro beams are investigated. The forced responses of the double coupled micro beams to voltage excitation are derived. A number of useful results are given. These results are useful in the design and manufacture of the micro-electro-mechanical systems (MEMS) with multi-coupled micro beams.


## 1. INTRODUCTION

Micro-electro-mechanical systems (MEMS) are integrated systems consisting of microelectronics, microactuators, and, in most cases, microsensors.[1,2] Typical MEMS structures consist of thin beams with cross-sections in the order of microns ($\mu m$) and lengths in the order of ten to hundreds of microns. Sometimes, MEMS structural elements are thin plates or thin rings. Examples of devices that utilise vibrations of such plates are synthetic microjets and microspeakers.[3,4] Another example is micromechanical gyroscopes that are exclusively comprised of vibrating types, including double-gimbal structures, cantilever beam structures, tuning-fork structures, and vibrating ring structures.[5-8]

The dynamics of these microsystems is an important subject that should be developed. Understanding the dynamic behaviour of MEMS is necessary for developing new MEMS devices and controlling their performance. Electrostatic actuation is the most frequently applied principle combing versatility and simple technology. Then, the focus of the microsystem dynamics is the electromechanical coupled dynamics of the micro elements under electrostatic field. Rafael Nadal-Guardia used a one-dimensional (1-D) lumped model of parallel plate electrostatic transducers and studied its dynamic behaviour.[9] Lizhong Xu presented an electromechanical coupled continuous body dynamic model of the micro beam under electrostatic force and investigated natural frequencies and vibration modes of the micro beam.[10] Slava Krylov and Ronen Maimon investigated pull-in dynamics of an elastic beam actuated by continuously distributed electrostatic force.[11]

Rotational and linear micromotors are often found to be a key part of micro electromechanical systems. In the electrostatic micro motors, the comb drives are presented.[1] In the comb drives, the coupled dynamics of the multi-micro beams should be developed.

In this paper, the electromechanical coupled dynamic equations of the double coupled micro beams are presented. The linearisation of the dynamic equations is made. From the linear dynamic equations, natural frequencies and vibration modes of the double coupled micro beams are investigated. The forced responses of the double coupled micro beams to voltage excitation are derived. The results show that compared with the single micro beam, the natural frequencies of the double coupled micro beams are larger. As the order of the modes increases, the increase in the natural frequencies becomes smaller. The vibration modes of the two micro beams are coupled to each other. For different modes, the forced response magnitude relationship between the corresponding points of the two beams is invariable. For the double coupled micro beams, the vibrating magnitudes of each micro beam at the same mode are all smaller than those of the single micro beam. The reason is that for the double coupled micro beams, the electrostatic restraint applied to each micro beam is stronger than that applied to the single micro beam.

## 2. ELECTROMECHANICAL COUPLED DYNAMIC EQUATION

Figure 1 illustrates an electromechanical coupled dynamic model of the two micro beams coupled by electrostatic force. The dynamic equation of the two micro beams subjected to electrostatic force is

$$EI\frac{\partial^4 \delta_i(x,t)}{\partial x^4} + \rho_l \frac{\partial^2 \delta_i(x,t)}{\partial t^2} = q(x,t)\,, \qquad (1)$$

where $\delta_i(x,t)$ is transverse displacement of the micro beam $i$ ($i = 1$ or 2), $x$ is the axial coordinate of the micro beam, $t$ is the time, $\rho_l$ is the material density per unit length of the beam, $E$ is the modulus of elasticity of the micro beam material, $I$ is the moment of inertia of the beam, $I = bh^3/12$ ($b$ is the effective width of the two beams, $h$ is the thickness of each beam), and $q(x,t)$ is the load per unit length of the beam.

The capacitance between the two micro beams can be calculated by

$$C = \frac{\varepsilon_0 \varepsilon_r A}{u - \delta(x)}, \qquad (2)$$

where $\varepsilon_0$ is permittivity constant of free space, $\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2/Nm^2$, $\varepsilon_r$ is the relative dielectric constant of the insulating layer, $u$ is the initial clearance between two micro beams, $A = Lb$ is the effective area between the two micro beams, and $L$ is the length of the beam.