Dynamic Characteristics of a Cantilever Beam with Transverse Cracks

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It has long been observed that the dynamic response of a structure changes due to the presence of a crack. Scientific analysis of such phenomena can be utilised for fault diagnosis and the detection of cracks in structures. The present investigation is an attempt in that direction. Theoretical expressions have been developed in order to determine the natural frequencies and mode shapes for an elastic cantilever beam with two cracks using flexibility influence coefficients and a local stiffness matrix. The numerical results for the beams without cracks, with one crack, and with two cracks are compared. It has been observed from the numerical results that there are appreciable changes in the vibration characteristics of the cantilever beam with and without cracks. This method can be utilised for multi crack identification of structures.

Nomenclature

a_1 – depth of crack
a_2 – depth of crack
A – cross-sectional area of beam
$A_i,\ i = 1, 18$ – unknown coefficients of matrix A
B – width of the beam
W – depth of beam
B_1 – vector of exciting motion
$c_u = (E/\rho)^{1/2}$
$c_y = (EI/\mu)^{1/2}$
E – Young's modulus of elasticity
$F_i,\ i = 1, 2$ – experimentally determined function
i,j – variables
J – strain-energy release rate
$K_{Ii},\ i = 1, 2$ – stress intensity factors for P_i loads
$\bar{k}_u = \omega L/c_u$
$\bar{k}_y = (\omega L^2/c_y)^{1/2}$
K_{ij} – local flexibility matrix element
L – length of beam
L_1, L_2 – location of the crack from fixed end
$M_i,\ i = 1, 4$ – compliance constant
$M_{ij} = M_i/M_j$
$P_i,\ i = 1, 2$ – axial force ($i = 1$), bending moment ($i = 2$)
K – stiffness matrix for free vibration
K', K'' – stiffness matrix of first and second crack locations
$u_i,\ i = 1, 3$ – normal functions (longitudinal) $u_i(x)$
x – co-ordinate of the beam
y – co-ordinate of the beam
Y_0 – amplitude of the exciting vibration
$y_i,\ i = 1, 3$ – normal functions (transverse) $y_i(x)$
ω – natural circular frequency
β_1 – relative crack location (L_1/L)
β_2 – relative crack location (L_2/L)
$\mu = A\rho$
ρ – mass density of the beam
ν – Poison's ratio
ξ_1 – relative crack depth (a_1/W)
ξ_2 – relative crack depth (a_2/W)
$|Q|$ – determinant of Q

1. INTRODUCTION

For the last several years, a considerable amount of research work has been undertaken to investigate the faults in structures. It has been observed that most of the structural members fail due to the presence of cracks. The cracks are developed mainly due to fatigue loading. Therefore the detection of cracks is an important aspect of structural design. A crack that occurs in a structural element causes some local variation in its stiffness, which affects the dynamic behaviour of the element and the whole structure to a considerable degree. The frequencies of natural vibration, amplitudes of forced vibration, and areas of dynamic stability change due to the existence of such cracks.[1-5] An analysis of these changes makes it possible to identify the magnitude and location of the crack. This information enables one to determine the degree of sustainability of the structural element and the whole structure.

Regarding the above problem, Cawley et al.[6] have combined sensitivity analysis with FEM to determine crack location. The coupling of vibration modes of vibration of a clamped – free circular cross-section Timoshenko beam with a transverse crack was investigated by Papadopoulos and Dimarogonas.[7] The crack is simulated using a 6×6 local flexibility matrix. The nondiagonal terms of this matrix cause coupling between the longitudinal, torsional, and bending vibrations. The researchers observed that the method used is very sensitive even in the case of small cracks.

Gudmundson[8] has investigated the transverse vibration of a cracked beam experimentally to validate a perturbation method which he had developed. He observed that depending upon the crack location, a crack may remain completely open, partially open, or closed if the vibration amplitudes are