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ANALYSIS AND ESTIMATION OF MODE LOCALIZATION IN MISTUNED ARCH-SUPPORTED RETICULATED SHELLS

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Abstract: As a new hybrid spatial structure, the arch-supported reticulated shell combines both advantages of a reticulated shell and an arch structure. A finite element model is developed to study the vibration mode localization in this structure. Effects of the stiffness detuning of the arch structures on the natural frequencies and mode shapes of the shell are investigated. The mode localization factors based on the modal strain energy are proposed to give a quantitative description of the degree of the localization. The influences of fixed end supported and fixed hinge supported boundary conditions on the mode localization factors are discussed. Analytical results indicate that the structural stiffness detuning does result in the strong mode localization phenomenon in arch-supported reticulated shell structures. The vibration localization factors based on modal strain energy can effectively estimate the degree of mode localization and help facilitate the analysis and design of such structures.

1. Introduction

The arch-supported reticulated shell structure is first proposed in 1996 by academician Shen Zuyan of Tongji University [1]. It combines advantages of both an arch structure and a reticulated shell, and thus its overall performance and bearing capacity are effectively improved. At present, the arch-supported reticulated shells have already been applied in architectures in China. For examples, the Affiliated Middle School Gymnasium of Shanghai Petrochemical and Heilongjiang Provincial Exhibition Hall both adopted an arch-supported cylindrical reticulated shell; while the new terminal building in the Ordos Airport and Zhuhai stadium both adopted the space truss arch-supported reticulated shell, and so on. These structures have enriched the types of the large-span space structure, and may be widely used in the future. Previous theoretical and experimental studies mostly focus on static analyses and studies on the dynamic performance of the arch-supported reticulated shell structure are comparatively rare [2, 3]. Early in the 1987, Bendiksen O.O. showed that, in the case of detuning, the mode localization phenomenon occurs in a large space reflector consisting of weak coupling substructures, and the mechanism of mode localization is also explained [4]. When the periodicity of the structure was destroyed, the distribution of vibration energy is not uniform in each substructure, and only focus on a few of substructures. It is known as the phenomenon of vibration mode localization. In recent years, many studies in this area demonstrate the influences of mode localization cannot be ignored for the structural safety and reliability [5-7]. The arch-supported reticulated shell is composed of arch structures with high rigidity and reticulated shells with rela-
tively weak rigidity. Can the mode localization phenomena occur in the arch-supported reticulated shell structure when it has the stiffness detuning?

In this paper, the modal analysis theory and the finite element method are applied to study the dynamic properties of the arch-supported reticulated shell structure. Different mistuned cases are analysed to identify the occurrence of mode localization in the purpose of improving the design and further applications of this new structure.

2. Modeling of the arch-supported reticulated shell structure

An arch-supported reticulated shell structure consisting of a K8 single-layer spherical reticulated shell and a cross arch structure is considered (Fig. 1). The span and rise span ratio of the reticulated shell are 40m and 1/5. The member cross-section of single-layer reticulated shells is Φ102mm×3.5mm, and the cross-section of arch structures is a box beam of 300mm×300mm×25mm. The density of the material Q345 steel is 7850kg/m³, and Young’s modulus is a constant equal to 206Gpa before detuning. The finite element model of the structure consists of 800 three-dimensional beam elements, 289 nodes and 1734 DOFs. The nodes of the seventh ring are treated as fixed end supports. All the elements are divided into 8 groups with the elements of every half a span arch structure. For example, the elements of the arch structure in the indicated ellipse form a group in Fig. 1. The stiffness detuning was introduced in the 8 groups of elements of arch structures by altering their stiffness K by an amount of $\Delta K/K$ as shown in Fig. 2. The stiffness detuning parameters are normally distributed with a mean of 0 and a standard deviation of $\sigma$. The mistuned cases are set to be Case 1, Case2, Case3 and Case4 whose standard deviation of the stiffness detuning parameters are 0.1, 0.2, 0.3 and 0.4, respectively. Using the finite element analysis software ANSYS, the vibrating modes are extracted with the Block Lanczos method for analysing the influence of stiffness detuning.

![Figure 1. Computational model of the arch-supported reticulated shell.](image)

![Fig. 2. Specific distribution of stiffness detuning of the 8 groups of arch elements](image)

![Fig. 3. Natural frequency comparison of the first fifty modes between the tuned and mistuned structure](image)
3. Modal analysis and quantitative description of mode localization

3.1 Influence of stiffness detuning on natural frequencies

As shown in Fig. 3, the first fifty order frequencies in the tuned and mistuned cases are intensive. The Case 0 represents the tuned case. The frequency differences between the four mistuned cases and Case 0 are not obvious. It indicates that the natural frequencies of the structure are insensitive to the small stiffness detuning, so the mode shape vectors are used for further analyse.

3.2 Sensitive mode

Modal Assurance Criterion (MAC) is a mathematical tool for evaluating the modal vector angle. It can be used to assess the correlation of the modal shape. The mathematical expression is as follows:

\[
MAC_{ij} = \frac{\left| \phi_i^T \phi_j \right|^2}{\left( \phi_i^T \phi_i \right) \left( \phi_j^T \phi_j \right)}
\]  

(1)

Where \( MAC_{ij} \) is the MAC matrix element of the \( i \)th row and the \( j \)th column, \( \phi_i \) and \( \phi_j \) are the \( i \)th and \( j \)th mode shape vectors. When \( MAC_{ij} \) equals 1, it indicates that the angle between the \( i \)th and \( j \)th mode shape vectors is 0 and the two vectors are of highest correlation; when \( MAC_{ij} \) equals 0, it indicates that the \( i \)th and \( j \)th mode shape vectors are orthogonal to each other. The modal assurance criteria matrices of the first 10 modes of the tuned case and the four different mistuned cases are constructed and shown in Fig. 4- Fig. 7.

![Fig. 4. MAC matrix between case 0 and case 1](image1)

![Fig. 5. MAC matrix between case 0 and case 2](image2)

![Fig. 6. MAC matrix between case 0 and case 3](image3)

![Fig. 7. MAC matrix between case 0 and case 4](image4)
As shown by Fig. 4, the fourth and the fifth elements of the main diagonal are close to 0. It can be determined that the 4th and 5th modes are sensitive mode for Case 1. In Fig. 5-Fig. 7, the analysis of the other three detuning condition can also get the similar conclusion. Therefore, in the following study of mode localization, the two order sensitive modes are used as the typical modes to analyze the dynamic characteristics of the arch-supported reticulated shell structure. It also can be seen from Fig. 4 and Fig. 5, the $MAC_{45}$ and $MAC_{54}$ are close to 1. It can be inferred that, compared with Case 0, the mode jumping phenomenon occurs between the 4th order and 5th order modes for Case1 and Case2. Fig. 6 and Fig. 7 show that, the mode jumping phenomenon doesn’t occur in Case3 and Case4 and thus further analysis is not performed for these two cases.

3.3 Vibration mode localization of the arch-supported reticulated shell

3.3.1 Changes of the mode shape for different detuning cases

The modal displacement nephograms of the sensitive modes for Case1, Case2, Case3 and Case4 are analyzed. The mode shapes for Case1 and Case2 are similar, and so are the mode shapes for Case3 and Case4. For brevity, only the mode displacement nephograms are given for Case0, Case1, and Case4. From Fig. 8 and Fig. 9, the similarities and differences in the 4th and 5th mode shapes between Case1 and Case0 can be found directly. The mode shape in Fig. 8 (b) is similar to the mode shape in Fig. 9 (a), and the mode shape in Fig. 9 (b) is similar to the mode shape in Fig. 8 (a). These indicate that the mode jumping phenomenon occurs between the 4th order and 5th modes for Case1 and Case0, and these confirm the results of Section 3.2. Note that similar results hold for Case2. As shown in Fig. 8 and Fig. 9, compared with Case0, the mode shapes of the two sensitive modes for case4 have changed greatly. The vibration mainly concentrates on the local area and the strong mode localization phenomenon has occurred. Similar results also hold for Case3.

![Fig. 8. Modal displacement nephograms of the 4th mode](image)

![Fig. 9. Modal displacement nephograms of the 5th mode](image)
The modal strain energy (MSE) is the function of the element stiffness matrix and the mode shape vectors. For the $j$th element in the $i$th mode, the elemental MSE of the tuned and mistuned state are expressed as:

$$MSE_{ij}^0 = \varphi_i^0 \mathbf{K} \varphi_j^0 / 2$$  \hspace{1cm} (2)$$

$$MSE_{ij}^1 = \varphi_i^1 \mathbf{K} \varphi_j^1 / 2$$  \hspace{1cm} (3)$$

where the superscript “0” and “1” denote the tuned and mistuned state, respectively.

According to Eq. (2) and Eq. (3), the modal strain energy of the elements is computed with the modal vector which is normalized with respect to the mass matrix in ANSYS. The maximum elemental strain energy for Case0, Case1, Case2, Case3 and Case4 are shown in Fig. 10.

The maximum element modal strain energy of the first 10 modes in Fig. 10 mostly increases with the increasing of the detuning strength. The maximum values of the element modal strain energy of the 4th and 5th modes are the most obvious of the 10 order modes.

The concentration of the distribution of elemental strain energy of reticulated shell can reflect through the ratio of the maximum elemental strain energy $MSE_{i\max}$ and the mean $M_i$ of the modal strain energy of the other elements. The $M_i$ is defined as the mean value of the MSE of all the elements except the largest MSE value of the $i$th order mode, which can be give by

$$M_i = \frac{1}{n-1} \sum_{j \neq j_{\max}}^n MSE_{ij} \hspace{1cm} (4)$$

The centralization diagram of the modal strain energy of the elements of the first ten order modes is shown in Fig. 11. The centralization of the 4th and 5th modes are the most obvious of the 10 order modes. With the increasing of the detuning paremeters, the concentration of the elemental strain energy is mostly increasing.

### 3.3.2 Localization factors

From the analysis above, for the sensitive modes, the maximum of elemental strain energy for mistuned cases changed greatly compared with Case 0, and the centralization extent of the modal strain energy is obvious. Considering the combined effect of these two aspects, the localization factor is constructed to quantitatively describe the degree of vibration mode localization. For the $i$th mode, the corresponding mode localization factor for estimating the degree of mode localization can be defined as

$$R_i = \left( MSE_{ij_{\max}} / MSE_{i_{\max}} \right) \left( MSE_{ij_{\max}} - M_i \right) / M_i \hspace{1cm} (5)$$
where $MSE_{i}^{\text{max}}$ and $MSE_{i}^{\text{0max}}$ are the largest $MSE$ values of all the elements of the $i$th mode for the mistuned state and tuned state, respectively.

### 3.3.3 Variation of the localization factor

Fig. 12. Localization factor graph of the former 10 modes

It can be seen from Fig. 12 that, the localization factor of the 4th and 5th modes are the most obvious in the first 10 order modes. It shows that the degree of mode localization of the 4th and 5th modes is the most serious, and this result is consistent with the analysis above. Localization factor defined in this paper can accurately identify the mode with the strong mode localization. It has clear physical meaning and can be used as the criterion of mode localization from the point of view of energy.

Fig. 13. Localization factors of the sensitive modes in fixed end supported condition

Fig. 14. Localization factors of the sensitive modes in fixed hinge supported condition

As shown in Fig. 14, the changes of the localization factors of the 4th and 5th modes deserve a detailed analysis. The localization factors of the 4th mode increased at first with the increasing of the detuning parameter, and then decreased slightly, which is up to a maximum value in the Case3. Localization factor of the 5th mode has been increasing with the increasing of stiffness detuning strength. The above analysis shows that the increasing of the detuning strength is not a sufficient condition for the increase of the mode localization degree, that is to say, the degree of mode localization is not always increased with the increasing of the detuning strength. And the localization of the 4th mode is more severe than that of the 5th mode in the arch-supported reticulated shell in this paper. But Phillip J. Cornwell and Oddvar O. Bendiksen found that the localization of large space reflector becomes more severe with increasing mode number in reference 5. This is the difference between the arch-supported reticulated shell and the large space reflectors. The analyses of the higher modes can also lead to the same conclusion and they are no longer elaborated here for brevity.
3.3.4 Localization factors of the arch-supported reticulated shell with fixed hinge support for different mistuned cases

The different boundary conditions will have a greater impact on the dynamic characteristics of the reticulated shell structure, so here the influences of simple supported on mode localization are also considered. The boundary condition of the arch-supported reticulated shell is changed from fixed end supported to fixed hinge supported, and the localization factors are calculated for the four mistuned cases.

As shown in Fig. 14, the localization factor of the 4th mode increase first and then decrease, and the localization factor of the 5th mode has been always increasing with the increasing of the detuning parameters, in the arch-supported reticulated shell structure with fixed hinge support.

<table>
<thead>
<tr>
<th></th>
<th>Boundary conditions</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>The 4th mode</td>
<td>Fixed end support</td>
<td>39.76</td>
<td>70.64</td>
<td>163.36</td>
<td>159.23</td>
</tr>
<tr>
<td></td>
<td>Fixed hinge support</td>
<td>30.85</td>
<td>112.49</td>
<td>214.66</td>
<td>183.06</td>
</tr>
<tr>
<td>The 5th mode</td>
<td>Fixed end support</td>
<td>35.59</td>
<td>75.89</td>
<td>114.82</td>
<td>138.38</td>
</tr>
<tr>
<td></td>
<td>Fixed hinge support</td>
<td>31.05</td>
<td>106.02</td>
<td>147.99</td>
<td>182.52</td>
</tr>
</tbody>
</table>

Undergoing the same stiffness detuning, the changing trend of localization factors for the fixed hinge support and the fixed end support is roughly the same, but the degree of localization is different. As shown in Tab.1, in general, the localization factors of the fixed hinge support are larger than that of the fixed end support. So the degree of localization of the fixed hinge support is more severe than that of the fixed end support in all of the 4th and 5th modes for Case3 and Case4. This shows that the fixed end support is more favourable to prevent the mode localization in the arch supported reticulated shell structure. The actual boundary condition of the arch-supported reticulated shell is between the fixed and hinged, and these conclusions above can provide a basis for the analysis of the vibration characteristics of the arch-supported reticulated shell structure.

4. Conclusion

1) The mode localization occurs when the stiffness detuning of the arch structure is relatively large, while the mode jumping occurs when the stiffness detuning of the arch structure is relatively small. 2) The localization factor based on modal strain energy defined in this paper can be used to effectively describe the degree of mode localization. 3) The analyses of mode localization of arch-supported reticulated shell structure with clamped and hinged boundary conditions, both demonstrate that the increasing of the detuning strength is not a sufficient condition for the increase of the degree of mode localization. 4) The degree of mode localization of arch-supported reticulated shell does not necessarily become more severe with increasing mode number.

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