FREQUENCY RESPONSE FUNCTION OF A MOVING SENSOR MEASUREMENT

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Generally, it is time consuming to confirm operating deflection shape or modeshape of a structure by experiment. In order to overcome this problem, recently, a researcher proposed Hilbert Huang Transform (HHT) technique to extract modeshape from the measurement which continuously measures vibration of an interesting region of a structure with non-contact laser sensor. In this previous research, however, two technical processes were required before applying HHT to obtain un-damped impulse response for each mode. The purpose of this study is to improve and complement the previous research; modal analysis approach is adapted to obtain an accurate un-damped impulse response of each mode for continuous measurement instead of using the two technical processes. For this purpose, Frequency Response Functions (FRFs) for each type of beam are derived, hence, making it possible to measure continuous measurement in a straight profile. In this process, technical limitation and drawback of the proposed damping compensation technique presented in the reference paper are pointed out; in addition, separation of damping phenomenon which occurs in the continuous measurement and Doppler Effect as discussed in several previous researches is also observed. The reliability of the proposed method is quantitatively verified by comparing it with the results obtained from conventional approach to estimate modeshape with impulse response.

1. Introduction

Dynamic characteristics (natural frequency, damping, modeshape) of mechanical system are important parameters to analyze and to design a system. The general method to estimate the characteristics of a system is to use the frequency response function (FRF), which requires two signals. These two signals are force applied by an impact hammer or a magnetic exciter and the vibration response measured by contact sensor (e.g. accelerometer or PZT displacement sensor), respectively. In case of contact sensor, several measurements at different positions of the structure are required to confirm the modeshape, however, this increases the number of sensor used and hence in turn will increase the cost of the system. Alternatively, one can also use only one sensor for several measurements but this will increase the time consumption. In addition, contact sensors affect the characteristics of the system and works as a mass loading on the structure. It acts as a disturbance factor in the process of identifying the characteristics of the system. If one measures the vibration of the structure with optical sensor such as laser doppler vibrometry (LDV) instead of contact sensor, it is possible to estimate the dynamic characteristics in a faster way without mass loading effect.
Recently, an advanced technique is conducted that measures the vibration of the structure in interesting region by moving the laser sensor continuously instead of measuring the vibration by positioning the laser at individual interesting nodes. Kang et al. measures a beam, which is vibrated by a single frequency, by moving the laser continuously from one end to the other end with constant speed. This measurement is used to estimate the operating deflection shape (ODS) of the structure. Recently, Kyung et al. developed a method to extract the modeshapes of a beam vibrated by an impulse hammer by continuously measuring it from one end to the other. The above mentioned references adopted HHT technique to extract ODS and modeshape results from continuous measurement, the profile of the laser during continuous measurement was a straight line moving from one end to the other. In this research, a beam which is vibrated by an impulse is measured continuously in a straight line which is similar to Kyung’s work; HHT is applied to this measurement to extract the modeshape. In the reference, however, two technical processes had to be carried out as prerequisite to apply HHT to the continuously measured impulse response, in this study; more accurate modeshape is identified by applying HHT to continuous measurement which is reconstructed by modal analysis.

This study is aimed to enhance the research of Kyung’s paper, the procedure of HHT to extract modeshape in the previous research is introduced in a simplified manner and the phenomenon (Doppler Effect) which occurred when continuous measurement is conducted is also introduced. In section 3, numerical configurations of FRF of a clamped-clamped beam and a cantilever beam for a continuous scanning measurement are derived, and the relation between obtained FRFs and HHT is investigated in detail. In section 4 conclusions drawn in this study have been highlighted.

2. Procedure to reconstruct mode shape by HHT

HHT has been applied to analyze vibration signal and to extract modeshape since Huang introduced this method to evaluate linear and non-linear signals. Recently, Kang et al. introduced the method to identify ODS result by applying HHT to continuous measurement and Kyung et al. expands Kang’s work to reconstruct modeshape of a continuous measurement excited by an impulse. In this section, detail theoretical background of HHT is not included but the procedure in the reference paper of extracting the modeshape is introduced with simply supported beam example. In addition, a certain phenomenon (Doppler Effect) which was not explained in the reference but should be considered for the continuous measurement will be explained.

\[ u(x, x_f, t) = \sum_{n=1}^{N} Y_n(t, \omega_n) \phi_n(x_s) \phi_n(x_f) \]  

Where, 

\[ Y_n(t, \omega_n) = \frac{1}{M} e^{-z_n \omega_n t} \sin(\omega_{dn} t + \theta_n), \quad \omega_{dn} = \omega_n \sqrt{1 - \zeta_n^2} \]  

\[ \phi_n(x) = \sqrt{2} \sin(k_n x), \quad k_n = \frac{n \pi}{L} \]

Equation (1) represents impulse response of a simply supported beam, here; x indicates position in x coordinate while subscript s and f represents the observed position and the forcing position, respectively. Furthermore, Y represents modal displacement, \( \phi \) represents modeshape, \( \omega_n \) represents natural frequency and subscript n represents mode number. In equation (1a), M indicates mass per unit length, \( \zeta \) is modal damping, \( \omega_n \) is resonance frequency and \( \Theta \) is phase difference. In equation (1c) L is the length of the beam. If the forcing position is chose and the vibration propagation from this position is continuously measured from x = 0 to L by constant speed, then the observing x_s position can be represented as
\[ x_t = L \Omega t \]  

Here, \( \Omega \) is scanning frequency. If equation (1) can be represented after applying equation (2),

\[ u(t) = \sum_{n=1}^{N} A_n e^{-\xi \omega_n t} \sin(\omega_n t + \theta_n) \sin(n \pi \Omega t) \]  

That is, impulse response function as previously expressed by a function of time and space is represented here by a function of time only. In this equation, \( A \) is a constant weighting factor of each mode, which is constituted by the mass of the beam and modeshape in the forcing position. By using this equation, one can get the result as shown in figure 1 by using \( n = 2, \omega_n = 220\text{Hz} \) and \( L = 0.45\text{m} \) for un-damped case, while measuring it with 13Hz as \( \Omega \).

![FIG. 1. Continuous measurement of 2nd mode for un-damped impulse response](image)

As shown in the figure, un-damped impulse response for a certain mode can be used to obtain modeshape through HHT. This procedure is shown below.

![FIG. 2. (a) Schematic diagram of HHT (b) from top to bottom, 1st IMF of impulse response of 2nd mode, instantaneous frequency, envelope, obtained sign from instantaneous frequency, reconstructed modeshape (■) and analytic modeshape (···)](image)

Figure 2 (a) shows the schematic diagram of the process to extract modeshape by HHT. First, as obtained in figure 1, the un-damped impulse response is applied to Huang theory; one can get 1st IMF (Intrinsic mode function) through EMD (Empirical Mode Decomposition) as the first basis in
time. IMF is used to obtain the envelope by Hilbert transform and the peak position of instantaneous frequency is used to decide the sign of the envelope, by envelope and sign, modeshape can be obtained. Figure 2 (b) shows results obtained in every step of figure 2 (a) with the result in figure 1. From top to bottom, the first plot shows 1st IMF of the result in figure 1 through EMD process and the second plot shows instantaneous frequency which is also the result of the EMD process. The third plot shows the obtained envelope by applying Hilbert transform to the 1st IMF and the forth plot shows the sign decided by the instantaneous frequency. Here, one can see the change of the sign at the location of maximum peak value of the instantaneous frequency, in fact this sign can be changed depending on how the first value has been selected, for instance, if one assign -1 as a starting value, the sign should change to +1 at the peak location of the instantaneous frequency but the final reconstructed modeshape would remain the same either +1 is used as the starting value or -1. Moreover, physical meaning of the peak value in the instantaneous frequency stands for nodal position of vibration and one can expect abrupt change of the instantaneous frequency as the sensor is passed through this position.

Finally, the fifth plot shows reconstructed modeshape by combining the envelope and the sign and the analytic modeshape from equation (1-3). As shown in the plot, the reconstructed result from HHT is in well agreement with the analytical result. Figure 2 shows overall process to obtain modeshape in the reference4. In the reference, however, they implemented 2 additional techniques (ideal band pass filter and compensate damping) to continuously by scan the measurement of an impact test in order to obtain the un-damped impulse response but in the present study, only modal analysis method has been used for more accurate result (for detail explanation please refer to next section). Next, we tried to find out the inherent features that are included in the continuous measurement of an impact testing. Equation (3) represents the measurement of an impact through continuous measurement that can be re-represented by applying trigonometric identities.

\[
 u(t) = \sum_{n=1}^{N} \frac{1}{2} A_n e^{j\omega_n t} \left[ \cos \theta_n \left\{ \sin \left( \left( \omega_n + n\pi \Omega \right) t \right) - \sin \left( \left( \omega_n - n\pi \Omega \right) t \right) \right\} \right. \\
 \left. - \sin \theta_n \left\{ \cos \left( \left( \omega_n + n\pi \Omega \right) t \right) - \cos \left( \left( \omega_n - n\pi \Omega \right) t \right) \right\} \right]
\]  (4)

From equation (4), one can see the fact that the resonance frequency in non-continuous measurement has been shifted to plus and minus direction in continuous measurement by an amount \( n\pi \Omega \). It means that resonance frequency in non-continuous measurement is shown by peak at single point in frequency domain, while, in continuous measurement two peaks are shown at the shifted position of the original resonance frequency by an amount \( \pm n\pi \Omega \), this phenomenon is called Doppler Effect. Even though this effect was not considered in the reference paper, however, it is important consideration in the present study (refer to next section). In addition, it is confirmed from this equation that the two frequency components are combined with same phase (sine, cosine), the beating phenomenon can occurred by two frequencies nearby the resonance frequency of the structure. The envelope frequency of this beating phenomenon can be represented as below6.

\[
 f_m = \frac{1}{2} \left( \left( \omega_n + n\pi \Omega \right) - \left( \omega_n - n\pi \Omega \right) \right) = n\pi \Omega
\]  (5)

It is confirmed from equation 5 that the envelope frequency by beating phenomenon is represented by the combination of mode number and scanning frequency only, from this, during one scanning revolution (from one end to the other end of the beam 0 ~ L) n numbers of the envelope variation would be expected. Eventually, HHT is nothing but a technical method to extract the mode information (n) from the continuous measurement of an un-damped impulse response.

3. Frequency response function

the impulse response of a clamped-clamped beam and a cantilever beam in time domain6-7 are given by
\[ u(x_s, x_f, t) = \sum_{n=1}^{N} -A_n e^{-\zeta_n \omega_n t} \phi_n(x_s) \phi_n(x_f) \sin(\omega_{dn} t + \theta_n) \]  \hspace{1cm} (6)

Where,

\[ \phi_n(x) = (\cosh k_n x - \cos k_n x) - \alpha_n (\sinh k_n x - \sin k_n x) \]  \hspace{1cm} (6a)

\[ \alpha_n = \frac{\cosh k_n L - \cos k_n L}{\sinh k_n L - \sin k_n L}, \cos k_n L \cosh k_n L = 1 \]  \hspace{1cm} (6b)

\[ \alpha_n = \frac{\sinh k_n L - \sin k_n L}{\cosh k_n L + \cos k_n L}, \cos k_n L \cosh k_n L = -1 \]  \hspace{1cm} (6c)

The clamped-clamped beam and the cantilever beam is represented by equation (6). FRF for continuous measurement can be obtained by applying Laplace transformation to equation (6) by substituting equation (2).

\[ U(s) = \sum_{n=1}^{N} -A_n' \{ C_n(s) - D_n(s) \} \]  \hspace{1cm} (10)

\[ C_n(s) = \frac{1}{2} \left\{ \frac{(1-\alpha_n) \sin \theta_n s + (1+\alpha_n) \omega_d \cos \theta_n}{(s + \kappa)^2 + \omega_d^2} + \frac{(1+\alpha_n) \sin \theta_n s + (1-\alpha_n) \omega_d \cos \theta_n}{(s + \chi)^2 + \omega_d^2} \right\} \]  \hspace{1cm} (10a)

\[ D_n(s) = -\frac{1}{2} \left[ \left\{ (\sin \theta_n + \alpha_n \cos \theta_n) s + (\cos \theta_n - \alpha_n \sin \theta_n)(\omega_d + k_n \Omega) \right\} \left( \frac{1}{(s + \zeta_n \omega_n)^2 + \omega_d^2} + \frac{1}{(s + \zeta_n \omega_n)^2 + (\omega_d - k_n \Omega)^2} \right) \right] \]  \hspace{1cm} (10b)

4. Conclusion

This paper is intended to enhance and complement the reference paper\textsuperscript{4} in which continuous scanning measurement of impulse response is used to extract modeshape. For this, the procedure in the reference to extract modeshape of each mode by using IHT is briefly introduced and interpretation of the physical meaning of every stage of the procedure is explained.
REFERENCES