INVESTIGATION OF FREQUENCY DOMAIN ADAPTIVE FILTER IN SHORT DUCT ACTIVE NOISE CONTROL SYSTEM

Jing Lu, Xin Mao and Kai Chen

Key Lab of Modern Acoustics, Institute of Acoustics, Nanjing University, Nanjing, China 210093

e-mail: lujing@nju.edu.cn

The active noise control system within a short duct has many potential applications. However, it has been noticed that the causal constraint of this system is always broken due to the extra delay caused by the AD/DA converters and the downsampling process. Although the frequency domain adaptive algorithm has the benefits of fast convergence speed and low computational burden, it has been noticed that it suffers from the deterioration of steady state behavior in noncausal conditions. In this paper, the performance of the frequency domain adaptive filter in short duct active noise control system is investigated and a modified algorithm that can guarantee optimal performance is proposed. The benefit of the proposed algorithm is validated by both simulations and experiments.

1. Introduction

The finite impulse response (FIR) filter is always used as the control filter for the feedforward active noise control (ANC) system [1]. However, when the noise to be controlled is of broad bandwidth, the normal time domain algorithm will suffer from high computational burden of a very long FIR filter. Thus frequency-domain block least-mean-square (FBLMS) algorithm that is more computationally efficient than the time domain algorithm is preferred [2]. Due to the feedforward controller’s strict sensitivity to time delay, a delayless FBLMS algorithm should be used, in which the filter coefficients updating is carried out in frequency domain, while the output calculation is carried out in time domain. The convergent property of the FBLMS algorithm has attracted many researchers’ attention. It has been noted that the normally used bin-normalized FBLMS algorithm cannot converge to the optimal Wiener filter [3-4], especially when noncausal circumstances are considered [5]. In feedforward ANC scenario, the causal constraint cannot be guaranteed when a compact control structure is needed. In this paper, a unified framework of the FBLMS algorithm without any assumptions on the signal and system model is proposed, which can be used to comprehensively analyze the steady-state behavior of the algorithm. Based on this framework, a modification is proposed on the existing algorithm that guarantees optimal steady-state behavior. The efficacy of the proposed algorithm in ANC systems is demonstrated by experiments.
2. Analysis of FBLMS steady-state behavior based on a unified framework

Let \( x(k) = [x(kN-N), x(kN-N+1), \ldots, x(kN+N-1)]^T \) be the reference signal vector, where the superscript T represents the transpose operation, \( w(k) = [w_0(k), w_1(k), \ldots, w_{N-1}(k)]^T \) be the \( N \)-tap filter, and \( d(k) = [d(kN-N), d(kN-N+1), \ldots, d(kN+N-1)]^T \) be the desired signal vector. Then the error vector in the frequency domain can be described as

\[
e_i(k) = FG_{N,N}F^{-1}[d_i(k) - X_i(k)w_i(k)]
\]

where \( F \) represents a \( 2N \times 2N \) discrete Fourier transform (DFT) matrix, \( d_i(k) = F[\mathbf{0}_{1 \times N}, d(k)]^T, X_i(k) = \text{diag}[x_i(k)] = \text{diag}[Fx(k)], w_i(k) = F[w^T(k), \mathbf{0}_{1 \times N}]^T \), and

\[
G_{0,N} = \begin{bmatrix} 0_{N \times N} & 0_{N \times N} \\ 0_{N \times N} & I_{N \times N} \end{bmatrix}
\]

(2)

There are two kinds of FBLMS algorithms: constrained and unconstrained [2]. The unconstrained FBLMS algorithm is more computationally efficient by removing the constrained operations; however, the alias caused by circular convolution leads to poorer convergence behavior [8]. Therefore this paper focuses only on the constrained algorithm.

The constrained filter update equation in the frequency domain is given by [2]

\[
w_i(k + 1) = w_i(k) + FG_{N,N}F^{-1}F(\mu M_iX^H_i(k)e_i(k)
\]

(3)

where the superscript \( H \) represents the conjugate transpose operation, \( \mu \) is a constant step size, \( M_i = \text{diag}[\xi] \) is a diagonal matrix with \( \xi \) representing a vector containing the normalizing factors for each frequency bin, and

\[
G_{N,0} = \begin{bmatrix} I_{N \times N} & 0_{N \times N} \\ 0_{N \times N} & 0_{N \times N} \end{bmatrix}
\]

(4)

Note that the non-causal part of the filter coefficients is not affected by the updating process, due to the constrained operation in (3). Therefore, multiplying both sides of (3) by \( F^{-1} \) yields

\[
\begin{bmatrix} w(k + 1) \\ 0_{N \times 1} \end{bmatrix} = \begin{bmatrix} w(k) \\ 0_{N \times 1} \end{bmatrix} + \mu G_{N,0}M X(k)\begin{bmatrix} 0_{N \times 1} \\ e(k) \end{bmatrix},
\]

where \( e(k) = [e(kN), e(kN+1), \ldots, e(kN+N-1)]^T \),

\[
X(k) = F^{-1}X^H_i(k)F = \begin{bmatrix} X_1 & X_2 \\ X_2 & X_1 \end{bmatrix}
\]

(6)

is a circulant matrix whose first row is \( x(k) \), and

\[
M = F^{-1}M_iF = \begin{bmatrix} M_1 & M_2 \\ M_2 & M_1 \end{bmatrix}
\]

(7)

is also a circulant matrix whose first column is \( F^{-1}\xi \) (the inverse Fourier transform of the normalizing vector).

With simple derivation, (5) becomes

\[
w(k + 1) = w(k) + \mu [M_iX_2 + M_2X_1]e(k)
\]

(8)

where

\[
e(k) = d(k) - X_1^Tw(k).
\]

Taking expectation on both sides of (8) and using the independence assumption with respect to the reference signal and filter coefficients [1,2] yields

\[
E\left[w(k + 1)\right] = \left[I_{N \times N} - \mu M_1R - \mu M_2\hat{R}\right]E\left[w(k)\right] + \mu M_1r + \mu M_2\hat{r},
\]

with
\[
\mathbf{R} = E\left[\mathbf{X}\mathbf{X}^T\right] = N\mathbf{R}_s
\]
\[
\hat{\mathbf{R}} = E\left[\mathbf{X}\mathbf{X}^T\right]
\]
\[
\mathbf{r} = E\left[\mathbf{X}_s\mathbf{d}(k)\right] = N\mathbf{r}_{dt}
\]
\[
\hat{\mathbf{r}} = E\left[\mathbf{X}_s\mathbf{d}(k)\right]
\]

where \(\mathbf{R}_s\) represents the autocorrelation matrix of the reference signal and \(\mathbf{r}_{dt}\) represents the correlation vector between the reference signal and the desired signal, and both of these are needed for the Wiener solution. The steady-state solution of (10) is

\[
E\left[\mathbf{w}_s(k)\right] = \left[\mathbf{M}_1\mathbf{R} + \mathbf{M}_2\hat{\mathbf{R}}\right]^{-1}\left[\mathbf{M}_1\mathbf{r} + \mathbf{M}_2\hat{\mathbf{r}}\right].
\]

(12)

Equation (12) is a unified description without any assumption on the signal and system models, based on which the steady-state behavior of the FBLMS algorithm can be investigated.

If a constant normalizing factor is used in the frequency domain, i.e., \(\mathbf{M}_1 = \xi_1\mathbf{I}_{2N \times 2N}\), then \(\mathbf{M}_2 = \mathbf{0}_{N \times N}\) according to (7), so that

\[
E\left[\mathbf{w}_s(k)\right] = \mathbf{R}^{-1}\mathbf{r} = \mathbf{R}_r\mathbf{r}_{dt},
\]

(13)

which is exactly the causal Wiener filter \([1,2]\).

To accelerate the convergence, the normalizing factors can be set as the reciprocal of the reference signal power spectrum as

\[
\mathbf{M}_1 = \text{diag}\left[\frac{1}{P_0}, \frac{1}{P_1}, \ldots, \frac{1}{P_{2N-1}}\right],
\]

(14)

where \(P_i\) represents the power of the \(i\)th frequency bin. For the bin-normalized FBLMS algorithm, the optimal solution can still be achieved under the following two conditions.

(a) Sufficient filter length

When the adaptive filter is of sufficient length, the desired signal can be described as

\[
\mathbf{d}(k) = \mathbf{X}_s^T\left[\mathbf{h}^T, \mathbf{0}_{(N-L)}\right]^T + \mathbf{u}(k),
\]

(15)

where \(\mathbf{u}(k)\) is an independent additive noise sequence and \(\mathbf{h} = [h_0, h_1, \ldots, h_{L-1}]^T\) is the \(L\)-tap \((L \leq N\) ) impulse response of the unknown system. It can be easily found that

\[
\mathbf{r} = \mathbf{R}\left[\mathbf{h}^T, \mathbf{0}_{(N-L)}\right]^T
\]

(16)

and

\[
\hat{\mathbf{r}} = \hat{\mathbf{R}}\left[\mathbf{h}^T, \mathbf{0}_{(N-L)}\right]^T.
\]

(17)

Substituting (16) and (17) into (6), the steady-state solution simplifies to \(E[\mathbf{w}_s(k)] = [\mathbf{h}^T, \mathbf{0}_{1 \times (N-L)}]^T\). This means that there is a perfect match between the adaptive filter and the unknown system.

(b) White noise reference signal

The normalizing matrix simplifies to \(\mathbf{M}_1 = \mathbf{I}_{2N \times 2N}/P\) for a white noise reference signal, where \(P\) stands for the constant power spectrum level of the signal. This results in convergence to the Wiener filter since the normalizing factor is uniformly constant.

For a colored reference signal, if the reference signal lags behind the desired signal or the control filter is of insufficient length, (12) cannot be simplified to the Wiener solution, which indicates a deterioration of the steady-state behavior of the bin-normalized FBLMS algorithm. These two phenomena have been addressed in [5] (for linear prediction) and [4] (for system identification), respectively. The contribution of this paper is that a unified framework as described above is proposed for analyzing the steady-state behavior of the FBLMS algorithm without making any assumption on the signal and system model, so it can be used to inspect the behavior in all circumstances. Based on the proposed framework, some modifications on the existing algorithm can also be made to improve the performance, as will be shown below.
3. Modification of FBLMS Algorithm

From (8) it can be found that the term $M_2X_t$ forms an obstacle to convergence of the FBLMS algorithm. Actually, if the updating of the causal part of the filter can be described as

$$w(k + 1) = w(k) + \mu M_2X_t e(k),$$

(18)

then taking expectation on both sides of (18) yields

$$E[w(k + 1)] = \left[I_{N \times N} - \mu M_2R\right]E[w(k)] + \mu M_2r.$$

(19)

The steady state solution of (19) is $E[w_{\infty}(k)] = R^{-1}r$, and obviously, this means an unconditional convergence to the Wiener solution. The updating equation of the FBLMS algorithm needs to be changed to obtain (18). After careful inspection of (3) and (5), it can be found that a simple change of the position of the normalizing factor matrix will achieve this goal. The modified updating equation is

$$w_{t}(k + 1) = w_{t}(k) + \mu M_2FG_{N,2} X_t^H(k)e_t(k).$$

(20)

Applying inverse Fourier transformation on both sides of (20) leads to

$$\begin{bmatrix} w(k + 1) \\ w_{nc}(k + 1) \end{bmatrix} = \begin{bmatrix} w(k) \\ w_{nc}(k) \end{bmatrix} + \mu M_2G_{N,2} X(k) \begin{bmatrix} 0_{N \times 1} \\ e(k) \end{bmatrix},$$

(21)

where $w_{nc}(k)$ represents the non-causal part of the adaptive filter, which does not influence the filtered output. From (21), the updating of the causal part of the filter can then be described exactly in the form of (18).

Comparing (3) and (20), it can be found that the computational load of the proposed algorithm for the updating of the filter coefficients is the same as that of the ordinary FBLMS algorithm. However, some extra constraint operations are needed to eliminate the influence of the non-causal part $w_{nc}(k)$ on the output calculation, as depicted in (1). The modified algorithm is easy to implement and the increase of the whole computational load is moderate with only one more FFT/IFFT pair in each block.

4. Simulation

To demonstrate the effectiveness of the proposed method, two simulations are conducted with the same setup as those described in [5] (non-causal case) and [4] (deficient filter-length case) respectively. For brevity of description, the bin-normalized FBLMS algorithm is abbreviated as NFBLMS, and the proposed modified bin-normalized method as MFBLMS. To compare the convergence of these algorithms, a parameter of coefficient-deviation (CD) is defined as the norm of the difference between the steady-state filter coefficients and the optimal Wiener solution, i.e.,

$$CD = \sqrt{(w_{\infty} - w_0)^T (w_{\infty} - w_0)},$$

(22)

where $w_{\infty}$ represents the filter coefficients after convergence and $w_0$ represents the Wiener solution.

4.1 Non-causal case

In this example, the reference signal was generated by passing Gaussian white noise with unit variance through a low pass filter with transfer function $H(z) = [(1 - 0.5z^{-1}) / (1 - 0.6z^{-1})]^6$. The desired signal was one sample ahead of the reference signal, resulting in a typical linear prediction problem. A white noise signal, uncorrelated with the reference signal, was added to the desired signal so that the maximum attenuation of the mean-square error (MSE) obtained by the Wiener solution is about 18.7 dB. The simulation results were averaged over 100 independent trials. The adaptive filter length $N$ was 128 and a 256-point FFT was used. The step sizes for the NFBLMS algorithm and the MFBLMS algorithm were 0.001 and 0.00005 respectively, both of which were close to the upper limits to guarantee both the fastest convergence and stable steady state behavior.

Figure 2 presents the convergence curves of these two algorithms. It is clearly shown that the steady-state performance of the NFBLMS algorithm deteriorates seriously, with only 2.8 dB atten-
The evaluation of the MSE. The convergence speed of the MFBLMS algorithm is initially very fast with 18 dB attenuation obtained within 1500 samples. After that, the slow mode dominates the convergence process and the optimal steady state performance is achieved at about 20000 samples. The steady-state filter coefficients are depicted in Fig. 3. Note that only the first 9 coefficients are depicted since the rest of the coefficients are all very close to 0. The deviation of the NFBLMS algorithm is obvious, with the CD of 1.0458, while the MFBLMS algorithm converges to the Wiener solution, with the CD of 0.0722.

4.2 Deficient filter-length case

In this example, the reference signal was generated by passing Gaussian white noise with unit variance through a 4-tap FIR filter with coefficients [0.1 0.2 0.4 0.7]. The desired signal was generated by passing the reference signal through a 16-tap FIR filter with coefficients [0.01 0.02 0.04 0.08 0.15 0.3 0.45 0.6 0.6 0.45 0.3 0.15 0.08 0.04 0.02 0.01]. A 10-tap adaptive filter was used, resulting in a typical deficient-filter-length scenario. The step size of both algorithms was 0.0001. The simulation results were also averaged over 100 independent trials.

From the convergence curve shown in Fig. 4, it can be found that the two algorithms have roughly the same convergence speed for the system modeling problem. However, the MFBLMS algorithm benefits from a lower MSE. As depicted in Fig. 5, the steady-state solution of the NFBLMS algorithm deviates from the Wiener solution, especially at the last several points, while the MFBLMS algorithm converges precisely to the Wiener solution. The CDs of the NFBLMS algorithm and the MFBLMS algorithm are 0.2618 and 0.0063 respectively. It should be pointed out that both algorithms perform poorly with less than 10 dB MSE reduction because the filter-length is significantly shorter than that of the unknown model. With the increase of the filter-length, both algorithms perform better, and the steady-state filter coefficients of the NFBLMS algorithm still deviates more than that of the MFBLMS algorithm as long as the filter is of deficient length. For example, for the filter-length of 13, the MSE reduction level of the NFBLMS algorithm is 25.6 dB with the CD of 0.0366, while the MSE reduction level of the MFBLMS algorithm is 27.1 dB with the CD of 0.0006.

These simulations clearly demonstrate the superior steady-state behavior of the proposed MFBLMS algorithm in both non-causal and deficient filter-length circumstances. Although the convergence speed might be slower than that of the NFBLMS algorithm in some situations, the guaranteed optimal steady state behavior still makes this algorithm a good option for many applications.

5. Experiment

A compact duct ANC system is established, with noise source 43 cm away from the error microphone, and the control source 15 cm away from the error microphone. The sampling frequency is 4 kHz. The extra time delay caused by the anti-alias filtering is 5 sampling time intervals; therefore the whole system is destined to be non-causal. The noise used in the experiment is transportation noise captured near a highway. The noise reduction level of different algorithms is shown in Table 1. Note that the FXLMS algorithm is used as a benchmark. It can be seen that the MFBLMS

<table>
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<th>N</th>
<th>32</th>
<th>64</th>
<th>128</th>
<th>256</th>
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algorithm always performs better than the NFBLMS. More interestingly, with the increase of the filter length, the NFBLMS algorithm even shows the trend of deterioration, which is in accordance with the analysis in [6].

6. Conclusions

The behaviour of the FBLMS algorithms has been analysed and it has been proved that the commonly used NFBLMS algorithm suffers from the deterioration of the steady-state behaviour. A efficient modified FBLMS algorithm is proposed in this paper, which can theoretically be proven to converge exactly to the Wiener solution. Both simulations and experiments demonstrate the superiority of the proposed algorithm when used in an active noise control system. For a noncausal short duct active noise control system the proposed algorithm always performs better than the NFBLMS algorithm.

ACKNOWLEDGMENTS

This work was supported by National Natural Science Foundation of China No. 11374156 and No. 11204130

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