EXPERIMENTAL RESEARCH ON THE VIRTUAL ABSORBED ENERGY IN MULTI-CHANNEL DECENTRALIZED VELOCITY FEEDBACK CONTROL OF A PLATE WITH PIEZOELECTRIC PATCH ACTUATORS

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Virtual absorbed energy of the piezoelectric patch actuator is a new cost function for the optimal feedback gain which has been proved theoretically. In this work, its performance is validated by experiments on the reduction of the vibration and sound radiation of a smart panel with 9 decentralized control loops. Each control unit consists of a collocated piezoelectric patch actuator and accelerometer sensor with a single channel digital controller. Since the system is not unconditionally stable, a phase lag compensator is designed to guarantee the stability for larger feedback gains. The stability of the multi-channel decentralized feedback control system has been assessed by the eigenvalue locus of the open loop sensor-actuator frequency response function matrix. The control effectiveness of the reduction of the panel kinetic energy has been assessed by the energy at 12 error sensors. It is observed that the feedback gains minimizing the kinetic energy and maximizing the broadband virtual energy are nearly the same.

1. Introduction

Active vibration and noise control of large structures generally requires the strategy of multi-channel feedback control when the disturbance is broadband at audio frequencies [1-4]. Due to the complexity of processing multi-channel signals with a single controller, decentralized control strategy has attracted extensive attention recently. Decentralized feedback control is less complicated since it uses several independent control loops. For each control unit, the error signal measured by the sensor is filtered by the controller and used to drive the collocated actuator. Since the control loops are unconditionally stable, the failure of one control unit has no effect on the stability of the entire control system [4-8]. Thus the decentralized velocity feedback control offers a more robust approach to control.

Control gain is a key factor in decentralized velocity feedback control, which determines the performance and stability of the control system. There is an optimal feedback gain that can be obtained by minimizing the kinetic energy or the radiated sound energy of the structure [9-10]. However, the physical information of the structure and its sound radiation generally cannot be acquired conveniently. Therefore, several researchers focused on this problem and discovered some other cost functions to find the optimal feedback gain instead of minimizing the kinetic energy [11-13]. Virtual absorbed energy of the piezoelectric patch actuator is a new cost function for the optimal
feedback gain which has been proved theoretically. Compared to the kinetic energy or the sound radiation power, the virtual absorbed energy is much easier to measure. Furthermore, when the disturbance is broadband white noise, the feedback gain obtained by minimizing the kinetic energy and maximizing the virtual energy absorption are nearly the same [14].

In this work, an experiment of 9 channels decentralized velocity feedback control system is performed. The feedback gain, obtained by maximizing the virtual absorbed energy, is implemented in the multi-channel control system. Since the open loop actuator-sensor frequency response function is not strictly positive real at higher frequencies and the analogue phase lag compensator can enhance the stability of the control system but not good enough, a digital phase lag compensator is designed so that larger feedback gains could be achieved.

2. Digital controller design

In this work, multi-channel decentralized velocity feedback control of a plate with piezoelectric patch actuators is discussed. There are nine decentralized control units. Each control unit consists of a collocated accelerometer sensor and piezoelectric patch actuator, with a single channel digital controller. In order to implement larger control gains, and guarantee the stability of the system, a phase lag compensator has been designed according to the open loop actuator-sensor frequency response function.

2.1 Open loop actuator-sensor frequency response function

In this section, the measured open loop frequency response functions of the sensor/actuator pairs are analyzed. The open loop frequency response function (0-10kHz) of a sensor/actuator pair is depicted in Figure 1. When the actuator and sensor are collocated, the open loop frequency response function has some particular properties. At the frequency band of 0-2.7kHz, the phase of the frequency response function is between 0-180 degree, which can be used to guarantee the stability of the directly feedback control system. However, at higher frequencies, the actuation and sensing are not collocated and also the moment actuation is not compatible to the transverse velocity sensing, so the frequency response function is not positively definite [6-7].

![Figure 1. Open loop frequency response function (0-10kHz) of a sensor/actuator pair](image)

2.2 Design of the 9 channels feedback controller

The open loop frequency response function is not positive real at higher frequencies, and additionally, the digital system also produces phase lag, contributing to the instability of the control unit.
Hence, in order to ensure the stability of the control system at higher frequencies, a phase lag compensator is required. In general, the compensator cannot perfectly make up the phase lag, as a consequence that the controller ought to be similar to a low-pass filter. In this work, an initial filter with phase restraint is designed, and corresponding to this initial filter, LMS method in the frequency domain has been used to design the first order IIR filter, which can be solved by the \textit{fdesign.arbmagnphase} function in Matlab. However, the amplitude of the initial filter is arbitrary, which may lead to the instability of the IIR filter. So \textit{fmincon} function (Matlab), namely the active-set method, is used to optimize it. The frequency response of the final IIR filter is shown in Figure 2. The compensator acts as a low-pass filter to reduce the non-collocated effect of the sensor and actuator pair. Also, the phase of the controller frequency response function increases up to $-43^\circ$, in order to make up the phase lag and enhance the stability of the control system.

![Figure 2. The frequency response of controller](image)

2.3 Stability assessment

The block diagram of the multi-channel decentralized velocity feedback control system is shown in Figure 3. The matrix $G(j\omega)_{9\times9}$ is the transfer function matrix between the sensors and the actuators. The control system is decentralized and the feedback control function $H(j\omega)_{9\times9}$ is diagonal. The stability of the multi-channel decentralized feedback control system has been assessed by the eigenvalues loci of the open loop frequency response matrix $GH$ in a frequency range of 0-10kHz, and the 7th eigenvalue locus is shown in Figure 4. It can be noticed that the locus does not enclose the Nyquist point (-1,0) at the frequency band of 0-10kHz, and actually other 8 eigenvalue loci share the same properties. It proves that the multi-channel feedback control system is stable, until the feedback gain increases more than 7dB.
\[
G(j\omega)e(t) + \sum \rightarrow \sum - H(j\omega) + \Sigma + \text{Plant} \rightarrow \text{Disturbance}
\]

\[
\text{Controller} \rightarrow \text{Error} + \text{Sensor noise}
\]

**Figure 3.** The block diagram of the multichannel decentralized velocity feedback control system

**Figure 4.** The 7th eigenvalue locus of matrix \( G(j\omega)H(j\omega) \)

### 3. Experimental implementation of virtual absorbed energy

#### 3.1 Experimental system

In this work, an aluminous panel has been mounted on the rectangular cavity with rigid walls, which is shown in Figure 5. The plate is clamped by a pair of rigid aluminium frames on the open side of the box, so that the plate is simply supported. The box is made of perspex, with the dimensions of cavity \( l_x \times l_y \times l_z = 500\text{mm} \times 400\text{mm} \times 400\text{mm} \), and plate \( l_x \times l_y \times l_z = 500\text{mm} \times 400\text{mm} \times 2\text{mm} \). At the corner of the box, there is a loudspeaker to generate the primary disturbance. As shown in Figure 5, a \( 3 \times 3 \) array of piezoelectric actuators, of dimensions \( a_x \times a_y \times a_z = 61\text{mm} \times 35\text{mm} \times 0.5\text{mm} \), have been bonded on one side of the plate. On the other side of the panel, a \( 3 \times 3 \) array of accelerometers has been arranged corresponding to the centres of the piezoelectric actuators.

As depicted in Figure 5, the 9 error sensors are placing on one side of the plate and the actuators on the other side. There are other 12 sensors used to measure the kinetic energy of the plate. Each control unit consists of a collocated accelerometer sensor (Lance, ULT2052) and piezoelectric patch actuator (PI, P876), with a single channel digital controller. The digital system is comprised of an ADC (TI, ADS8422), a DAC (TI, DAC8820), and a FPGA (Alter, Cyclone III). The feedback control filter is accomplished in the FPGA, with the sample frequency of 1MHz to ensure less digital delayed time.
3.2 Virtual absorbed energy

In this section, the feasibility of virtual absorbed energy for optimal feedback gain is confirmed. When the piezoelectric patches are used as actuators, the energy they have absorbed is proportional to the moment induced by the piezoelectric patches and the modal velocity of the plate. If replacing the angular velocity with the velocity signal, then there is a possibility for the quantity, which is the product of the amplitude of the line moment and the velocity signal from the velocity sensor, to be used to represent the energy absorption, and this quantity is termed the virtual energy absorption [14]. As the line moment of the actuator is nearly proportional to the output voltage of controller, therefore the total virtual absorbed energy of the piezoelectric patches can be depicted as follows:

\[
E_a = |U_s^H V_c|
\]

where \(U_s\) is the output voltage vector of controller; \(V_c\) is the velocity vector measured by the sensors; \((\cdot)^H\) denotes the Hermitian.

When the panel is disturbed by any broadband white noise from the speaker in the cavity, the velocity at the element centres and at the control points can be derived as:

\[
\begin{align*}
V_e(\omega) &= D_e(\omega) + H_{es}(\omega) U_s(\omega) \\
V_c(\omega) &= D_c(\omega) + H_{cs}(\omega) U_s(\omega)
\end{align*}
\]

where \(V_e(\omega)\) and \(V_c(\omega)\) are the velocity vectors at the element centres and control points, respectively. Likewise, \(D_e(\omega)\) and \(D_c(\omega)\) are the initial velocity vectors at the element centres and control points respectively, when there is only the acoustic disturbance without control. \(H_{es}(\omega)\), \(H_{cs}(\omega)\) are the frequency response function from the piezoelectric amplifier input to the output of accelerometer sensors at the element centres and control points respectively.

When the control loop is implemented, the output voltage of controllers can be expressed as:

\[
U_s = -H(\omega)V_c(\omega)
\]

Here \(H(\omega)\) denotes the controller, which includes the designed feedback control filter, the analogy filter of the digital system, and the delay of digital sampling. It should be noted that \(H(\omega)\) is a diagonal matrix due to the decentralized control method.
Hence, the velocity at the element centres can be derived as:

\[ \mathbf{V}_e(\omega) = \mathbf{D}_c(\omega) - \mathbf{H}_{cs}(\omega)\mathbf{H}(\omega)[\mathbf{I} + \mathbf{H}_{cs}(\omega)\mathbf{H}(\omega)]^{-1}\mathbf{D}_c(\omega) \]  

(5)

The kinetic energy of the plate is given by:

\[ E_k(\omega) = \frac{m}{2N} \mathbf{V}_e^H(\omega)\mathbf{V}_e(\omega) \]  

(6)

Here \( m \) is the mass of the plate; \( N \) is the number of the element.

The virtual absorbed energy is given by:

\[ E_a(\omega) = \text{diag}(\mathbf{H}(\omega)[\mathbf{I} + \mathbf{H}_{cs}(\omega)\mathbf{H}(\omega)]^{-1}\mathbf{D}_c(\omega))^{\text{H}}[\mathbf{I} + \mathbf{H}_{cs}(\omega)\mathbf{H}(\omega)]^{-1}\mathbf{D}_c(\omega) \]  

(7)

According to equations (5)-(7), the kinetic energy and the virtual absorbed energy can be derived by the measured plant frequency response functions and the plate original disturbance. As shown in Figure 6, when there are 9 control units, the variation of the kinetic energy of the plate and the virtual absorbed energy of the 5th, 7th and 8th control unit against the variation of the feedback gain.

![Figure 6](image_url)

**Figure 6.** The variation of the kinetic energy and the virtual energy absorption against the variation of the feedback gain of control unit 5 \( h_5 \), unit 7 \( h_7 \), and unit 8 \( h_8 \) (a) unit 5; (b) unit 7; (c) unit 8

Since the optimal feedback gain of a control unit is not changed as the feedback gain of the other control unit, the optimal feedback gain of each control unit can be found with their own virtual energy absorption [14]. In this figure, the feedback gain maximizing the virtual absorbed energy and minimizing the kinetic energy are nearly the same, and all the feedback gains are approximately 4dB to 6dB. Additionally, the other 6 units can get the same conclusions. So, it is feasible to find the optimal feedback gain by maximizing the virtual absorbed energy.

### 4. Control results

The control effectiveness of this system has been assessed with reference to the primary excitation of a loudspeaker in the cavity. The four plots in Figure 7 show the kinetic energy of the plate measured by 12 error sensors. The optimal gain is obtained by the virtual energy absorption as mentioned above. Particularly, the effectiveness of the number of decentralized feedback control unit has been discussed in these experiments. According to the Parseval’s theorem, the total kinetic energy of the plate has been measured between 0-1kHz where reductions of about 5.5dB, 5.8dB,
6.5dB, and 7.5dB for 2 control loops, 4 control loops, 6 control loops, and 9 control loops are achieved respectively.

The first three resonances, associated with the modes of the panel (1,1), (1,3) and (3,1), are well coupled to the low-frequency volumetric response of the cavity, and can be controlled effectively. In contrast, the resonances controlled by the cavity natural modes are not controlled [6]. For instance, the fourth resonance frequency, due to the natural mode of the cavity (1,0,0), which occurs at 345Hz, cannot be controlled by the decentralized velocity control.

![Graphs](a)(b)(c)(d)

**Figure 7.** The kinetic energy of the plate with the excitation of the loudspeaker in the cavity between 0-1kHz without control (dashed line), and when the 2 control loops (a), 4 control loops (b), 6 control loops (c), 9 control loops (d) are implemented (solid line).

### 5. Conclusions

In this work, an experimental research on virtual absorbed energy for optimal gain in decentralized velocity feedback control is implemented. Compared to the feedback gain obtained by minimizing the kinetic energy of the plate, the virtual absorbed energy is much more convenient to measure, and results in almost the same gain even in 9 channels control system. As the open loop actuator-sensor response function is non-positive real at higher frequencies and the stability of the control system is not good enough, although the analogue phase lag compensator enhances it, a digital phase lag compensator is designed so that larger feedback gains could be achieved. 9 channels feedback control, with the optimal feedback gain maximizing the virtual absorbed energy, can reduce the total kinetic energy up to 7.5dB.

### REFERENCES