SIMPLE METHOD TO DETERMINE THE TRANSMISSION LOSS OF GYPSUM PANELS

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In recent years, a new technique for the determination of the transmission loss of sandwich panels has been applied to simple and composite building materials. The method is based on a model which, given an anisotropic panel, requires physical and mechanical properties of the materials making the structure and the natural frequencies of two beams cut from the plate, orthogonal directions, to determine its sound transmission loss. A simple test procedure can be used to measure the frequency response functions (FRF) of the two specimens. The beams are suspended in free-free conditions and an accelerometer is placed at one side. An impact hammer is used to excite the specimen to the other side, while an analyser correlates the data from the transducers. Once incorporated in the mathematical model, these simple input data allow to compute the bending stiffness of the beams, the critical frequency of the panel, the internal and the radiation losses and, finally, the panel transmission loss. The radiation losses of the panel are estimated according to Maidanik’s work. A software has been developed to implement the mathematical model and several experimental measurements have been performed in order to validate it. One series of tests has been carried out on some gypsum panels, testing them in a laboratory using sound transmission rooms and the previously described model. Finally, a panel placed between the two rooms and moved by a shaker was used to measure the sound intensity and the vibration velocity levels through a probe and a set of 9 accelerometers. The purpose of the test was to make an estimation of the sound radiation ratio. Predicted and measured results are compared. The agreement is fair.

1. Introduction

Gypsum panels can be somehow treated as sandwich structures, since they are made of two thin paper sheets attached to a gypsum core which is reinforced with fibreglass.

The flexural vibrations of sandwich structures have been discussed in several papers and publications\(^1\),\(^2\),\(^3\). However, gypsum panels can be considered homogeneous, and then their bending stiffness almost constant in the frequency range of interest. For this reason the theory developed by Cremer for determining the transmission coefficient can be applied, once the apparent bending stiffness is determined through some simple tests performed on beams cut out from a panel or from point mobility tests carried out directly on the structure under investigation.
2. Wavenumbers and apparent bending stiffness

Timoshenko\textsuperscript{4} extended the Euler-Bernoulli theory to thick homogeneous beams, for which the wavelength approaches the thickness value. In this case rotation as well as shear effects cannot be neglected.

Keeping valid the Euler-Bernoulli hypothesis and starting from the balance of the forces acting on a beam element, the equation describing the bending motion of a thick beam, not subject to external forces, is

\[
\frac{D_b}{\mu I} \frac{\partial^4 w}{\partial x^4} + \frac{\partial^2 w}{\partial t^2} \left( \frac{D_b}{G_s} \frac{I}{\mu} \right) \frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{I}{G_s} \frac{\partial^4 w}{\partial t^4} = 0
\]  

(1)

Where $S$ is the cross section area.

If compared to the classical theory, Timoshenko introduced an “effective” shear modulus $G_e$ equal to $T_b G$. The coefficient $T_b$ is a function of the Poisson’s ratio thus it takes into account the geometry of the beam and the non-uniformity of shear over its cross section. The term $G_e$ considers the effects of transverse shear on the lateral deformation of a beam and allows to approximate a non uniform shear distribution over the cross section area to a uniform one.

Another term was added into the equation: the mass moment of inertia $I_\rho$ equal to $\rho I$. It considers the rotation of a segment of the beam during bending, in fact, for thick structures the cross section does not remain normal to the neutral axis during motion, but it undergoes an angular displacement $\beta$ caused by bending and another angular displacement $\gamma$ caused by transverse shear, as shown in Figure 1.

\[ \beta + \gamma \]

\[ \beta \]

\[ \gamma \]

\[ w \]

Figure 1: Deformation of the cross section of a thick beam under transverse shear effects

The displacement solution for this 4\textsuperscript{th} order equation of motion can be expressed in the same way as for Euler-Bernoulli theory:

\[ w(x,t) = A \cdot \exp[i(\omega t - k_x x)] \]  

(2)

And the wavenumber $k_x$ which allows to satisfy Timoshenko’s equation is the solution to

\[
k_x^4 - k_x^2 \left( k_x^2 + \frac{k_x^2}{T} \right) - \left( k_x^4 - \frac{k_x^2 k_x^2}{T} \right) = 0
\]  

(3)

Where $k_x$: wavenumber for longitudinal waves $= \omega \sqrt{\rho / E}$

$k_x$: wavenumber for transverse waves $= \omega \sqrt{\rho / G} = \omega \sqrt{2 \rho (1 + \nu) / E}$
The wavenumber can be written as $k_s^4 = \omega^2 D_s$, where $D_s$ is the apparent bending stiffness of the structure. Consequently $D_s$ is defined as the bending stiffness of a simple homogeneous beam as discussed for example in\(^3 \)\). For the kind of structures discussed in this paper, these assumptions hold for frequencies below 5 kHz\(^5 \). Above this frequency the paper attached to the two different surfaces of the gypsum panel no longer move in phase.

The experimental determination of the apparent bending stiffness can be carried out using some samples of the panel for which the transmission loss have to be determined. For a beam with free ends, the bending stiffness can be determined by means of some simple measurements. The apparent bending stiffness $D_{sn}$ for mode $n$ having the eigenfrequency $f_n$ is for a beam, length $L$ and mass per unit area $\mu$, given by

$$D_{sn} = 4\pi^2 f_n^2 \mu / \alpha_n^4 \quad \text{(4)}$$

The values assumed by $\alpha_n$ are given in Table 1 as a function of the $n$-th mode for a beam with free ends.

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>$\geq 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_n$</td>
<td>4.73</td>
<td>7.85</td>
<td>11</td>
<td>14.14</td>
<td>$n\pi + \pi/2$</td>
</tr>
</tbody>
</table>

For non-metallic materials Young’s modulus could exhibit a frequency dependency. For a homogeneous beam, the bending stiffness can be written in a general way as

$$D_s = A - B \cdot f \quad \text{(5)}$$

Thus bending stiffness of a homogeneous panel is slightly frequency dependent as given by equation (5).

Using the measured natural frequencies derived by an FRF measurement carried out on a beam, the parameters $A$ and $B$ can be determined by means of the least square method.

### 3. Measurements on beams

Test beams are cut from a large plate. The ratio between the two largest dimensions should be at least 1:10 so that it is possible to avoid torsional effects. The beam is suspended by strings to simulate free-free boundary conditions. The test procedure is described in\(^5 \). Usually the natural frequencies related to bending modes could be easily identified (Figure 2).

The length of the test beams should be in the order of 1.2 m. The width of the beam could be 5 to 10 cm. For anisotropic structures, measurements are performed on beams representing the two main in-plane $x$ and $y$ directions of the plate. The frequency response function is determined to give the natural frequencies for each beam and then the two bending stiffness $D_x$ and $D_y$ are computed. The overall bending stiffness of the plate is determined by using the following relation:

$$D_{tot}(\theta) = \left[D_x^{1/2} \cos^2 \theta + D_y^{1/2} \sin^2 \theta\right]^2 \quad \text{(6)}$$

where $\theta$ is defined in Figure 3.
Based on the half bandwidth method applied to the frequency response function, also the internal loss factors for the two directions can be determined. The total loss factor is

\[
\eta_i = \left[ \eta_x \sqrt{\frac{D_x}{D_{tot}(\theta)}} \cos^2 \theta + \eta_y \sqrt{\frac{D_y}{D_{tot}(\theta)}} \sin^2 \theta \right]^2
\]  

(7)

The radiation losses \( \eta_{rad} \) need to be added to the internal losses \( \eta_i \) in order to obtain the total losses \( \eta_{tot} \) of the specimen. The radiation losses can be computed according to \( 6 \).

### 4. Sound transmission loss

The forced response of a structure can, when predicted in 1/3rd octave bands, be estimated fairly accurately by using the apparent bending stiffness of the structure in combination with the Bernoulli-Euler theory for beams or the Kirchhoff theory for plates\(^3\). These results imply that the expressions defining sound transmission loss for homogeneous panels can be used by allowing the bending stiffness to be frequency dependent\(^8\). The sound transmission loss for single leaf panels is
discussed for example in\(^9\). The sound transmission loss of a single leaf panel depends on the bending stiffness of the plate and on a number of other parameters. For homogeneous single leaf panels it is convenient to introduce the critical frequency \(f_c\) for which the trace matching between flexural waves on the plate and waves in the surrounding medium can occur. The frequency \(f_c\) for which \(k_{\text{plate}} = k_{\text{air}}\) is given by

\[
f_c = \frac{c}{2\pi} k_{\text{plate}}/k_{\text{air}} = \frac{c^2}{2\pi} \sqrt{\mu D_{\text{tot}}}
\]

where \(c\) is the speed of sound in air and \(\mu\) the mass per unit area of the plate. For a thin single-leaf panel \(f_c\) is constant.

Cremer\(^7,10\) derived the sound transmission coefficient \(\tau(\phi)\) for a thin homogeneous plate as a function of the angle of incidence \(\phi\) of the acoustic wave, see Figure 4. The expression is a function of the critical frequency \(f_c\) of the plate. By introducing the definition of \(f_c\), as given by equation (12), in this expression, the transmission coefficient \(\tau(\phi)\) is obtained as

\[
\tau(\phi) = \left[1 + \frac{\mu\omega}{2\rho c} \cos \phi \left(\frac{f}{(c^2/2\pi) \sqrt{\mu D_{\text{tot}}}}\right)^2 \sin^4 \eta_{\text{tot}}\right]^2 + \left[\frac{\mu\omega}{2\rho c} \cos \phi \left(\frac{f}{(c^2/2\pi) \sqrt{\mu D_{\text{tot}}}}\right)^2 \sin^4 \phi - 1\right]^{-1}
\]

The parameters in equation (9) are: \(\mu\) total mass per unit area of plate, \(f\) frequency, \(\omega\) angular frequency, \(\phi\) angle of incidence of acoustic wave, \(\rho c\) wave impedance at room temperature (415 \(\text{kg}/(\text{m}^2 \cdot \text{s})\) for air), \(\eta_{\text{tot}}\) loss factor of the structure. The bending stiffness \(D_{\text{tot}}\) is a function of frequency and obtained from equation (6).

Since the computed bending stiffness is no longer constant but a function of frequency. As a first approximation \(k_{\text{plate}}\) can be set to equal \(\kappa_1\) or the wavenumber for the first propagating mode of flexural waves in the plate. Alternatively the critical frequency \(f_c\) can be computed by replacing \(D_{\text{tot}}\) with the different apparent bending stiffness values of the plate in equation (8).

**Figure 4:** Definition of the sound angle of incidence.

The sound transmission loss \(R\) in decibels for a plate is \(-10 \log \tau_d\), where \(\tau_d\) is the sound transmission coefficient for diffuse incidence, defined as

\[
\tau_d = 2 \int_0^{\pi/2} \tau(\phi) \cos \phi \sin \phi \, d\phi
\]
A numerical integration of equation (10) gives the sound transmission coefficient $\tau_d$ and thus the sound transmission loss $R$ for a sandwich panel assuming a diffuse incident field. The sound transmission loss for a gypsum plate was predicted according to equation (10) and compared to measurements.

5. **Gypsum board**

The gypsum board used during the tests was 1.7 m long and 1.1 m high. Two beams in the $x$– and in the $y$– direction have been cut in order to determine the bending stiffness based on the eigenfrequencies of the normal modes. The results of the measurements are shown in Figure 5.

![Figure 5: Bending stiffness obtained from the FRF method for x and y directions.](image)

The sound transmission losses predicted using the FRF method has been compared with the transmission loss determined in sound transmission rooms. The comparison is shown in Figure 6. The data agree very well even if the simulations slightly underestimate the measured TL.

![Figure 6: Transmission loss obtained from the FRF method.](image)
6. Conclusions

In this paper a method to determine the transmission loss of gypsum panels is proposed. The method is based on experiments carried out on beams cut from the main in plate directions.

Theoretical models have been implemented in a software, which allows the determination of the sound transmission loss of sandwich plates.

The code was validated through laboratory tests carried out in double transmission rooms.

The agreement between calculations and experimental results is fairly good, except a slight difference in the low frequency region due to the finite dimensions of the panel if compared to the dimensions of the mounting wall (also known as baffle effect) and another difference in the high frequency region, which is due to the different values of the losses caused by the mounting of the panel in the frame used during the tests carried out in the sound transmission rooms.

7. Acknowledgment

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REFERENCES