The present paper deals with the identification of noise sources in internal ducted flows. Large Eddy Simulation (LES) and System Identification (SI) are combined in order to characterize simultaneously both the acoustic passive scattering and the active noise generation of an orifice placed in a duct. This system is studied within the plane wave linear acoustic regime, through the so-called multi-ports representation. The acoustic response to a given acoustic perturbation is modeled by the system acoustic scattering matrix, whereas the noise source is represented by a noise vector. In this work, the scattering matrix and the active noise generation are determined introducing a parametric system identification analysis based on a Box-Jenkins model structure. This model structure is an extension of the common correlation analysis approach and includes a dynamic model of the noise sources. The identified passive acoustic scattering is in good agreement with experimental results. The identified noise source shows a nonlinear behavior for different levels of excitation and it is compared with the noise extracted from a non-excited LES.

1. Introduction

Acoustic scattering and identification of noise sources are of fundamental importance in the construction of duct systems. Duct elements like orifices and constrictions are widely used in several engineering applications such HVAC systems or aircraft liners and are often responsible of turbulent flow noise and whistling. The characterization of the latter requires the development of new techniques to describe the dynamics of the noise sources and the flow-acoustic interaction.

The classic approach to determine the acoustic scattering and the noise sources is based on the Multiport and Plane Wave Decomposition method [1]. Thereby, the passive acoustic scattering in a duct system is mathematically represented by the so-called scattering matrix, whereas the active noise generation is modeled through a source vector [2]. The identification of the latter is possible by performing acoustical measurements without acoustic external excitation, once the scattering matrix and the impedance of the boundaries of the domain of interest are known. Therefore, for these methods, the noise is assumed to be uncorrelated with the acoustic excitation. Any correlated component is considered as the response of the acoustic system.

This work aims to introduce a new source identification method in the context of the so-called LES/SI approach [3]. The acoustic data series extracted from an acoustically broadband excited Large Eddy Simulation (LES) are post-processed by mean of System Identification (SI) analysis to
characterize simultaneously the acoustic passive scattering and the active noise generation of an ori-
fice placed inside of a duct. Previous works on LES/SI have been able to correctly characterize the
acoustic scattering of acoustic elements such as a sudden area expansion or an orifice [4, 5]. In order
to identify the noise sources, the classical SI procedures based on correlation analysis and Wiener-
Hopf filter inversion are reviewed in the present study and extended to include a modeling of the noise
terms. Accordingly, a parametric system identification analysis based on Box-Jenkins model structure
[6] is introduced and applied to an aeroacoustic problem.

In section 2, the basic duct acoustic theory is introduced. In section 3, the classical SI method
based on correlation analysis is reviewed and the new identification approach based on prediction error
estimation is introduced. In section 4, the numerical setup adopted for LES is presented. Finally, in
section 5, the results are reported, discussed and compared to experimental results in terms of acoustic
scattering matrix.

2. One Dimensional Acoustics

The acoustic scattering of a duct element in the linear regime, below the plane wave cut-off
frequency and in presence of a turbulence noise source, can be described by the mathematical formul-
ation [2]:

\[
\begin{bmatrix}
  f_d \\
  g_d \\
  f_u \\
  g_u
\end{bmatrix}
= \begin{bmatrix}
  T^+ & R^- \\
  R^+ & T^-
\end{bmatrix}
\begin{bmatrix}
  f_u \\
  g_u \\
  f_d \\
  g_d
\end{bmatrix}
+ \begin{bmatrix}
  f_s \\
  g_s
\end{bmatrix},
\]

(1)

where \( f \) and \( g \) represent the characteristic acoustic waves traveling in the downstream and in the
upstream direction, respectively. Subscript \( u \) indicates the region upstream of the acoustic element
whereas subscript \( d \) indicates the region downstream of the acoustic element (see Figure 1). The
passive acoustic properties of the element are described in terms of transmission coefficients \( T \) and
reflection coefficients \( R \) by the so-called scattering matrix. Here, the symbols + and − indicate the
direction of propagation upstream and downstream of the transmitted (or reflected) acoustic wave,
respectively. The active acoustic properties due to noise sources inside the domain are taken into
account through the source vector in terms of characteristic acoustic waves traveling downstream \( f_s \)
and upstream \( g_s \) the configuration under analysis, respectively. The acoustic waves \( f_s \) and \( g_s \), in the
linear acoustic regime, according to Eq. (1) are independent and uncorrelated with the input wave
perturbations \( f_u \) and \( g_d \).

The characteristic waves can be related to the acoustic velocity fluctuations \( u' \) and to the acoustic
pressure fluctuations \( p' \) by means of
\[ f_{u,d} = \frac{1}{2} \left( \frac{p'}{\rho c} + u' \right)_{u,d} \quad \text{and} \quad g_{u,d} = \frac{1}{2} \left( \frac{p'}{\rho c} - u' \right)_{u,d} \]  
\[ (2) \]

## 3. System Identification

The numerical characterization of the scattering matrix and of the noise source in Eq. (1) is based on the LES/SI method. First a compressible LES, excited at the boundaries by a broadband acoustic signal, is carried out. Subsequently, acoustic data series extracted from the LES are post-processed by means of SI to model the acoustic scattering and the noise terms. In this section, we aim to present the classical SI approach based on correlation analysis and an innovative technique based on the prediction error method, in which a model of the acoustic noise sources is introduced. The analysis is carried out assuming linearity of the acoustic propagation and considering an input-output mathematical description as in Eq. (1). The method is shown for a Single Input Single Output (SISO) system; the extension to a Multiple Input and Multiple Output (MIMO) system, as in case of the scattering matrix representation, is straightforward.

### 3.1 Correlation analysis

The response \( y(t) \) of a generic dynamical system to a given input \( u(t) \) can be characterized in terms of the so-called Finite Impulse Resonse (FIR) disturbed by an unpredictable, Gaussian distributed, white noise \( e(t) \) of variance \( \sigma_e^2 \) \[ (6, 7) \]

\[ y_{FIR}(t) = \sum_{i=0}^{N_b} b_i u(t - i) + e(t), \]  
\[ (3) \]

with unknown coefficients \( \Theta = [b_0 \ldots b_{N_b}]^T \) to be estimated. The simplest way to estimate \( \Theta \) is to assume that the noise disturbance \( e(t) \) in Eq. (3) does not influence the coupling between input and output, and to minimize the error between the estimated output \( \hat{y}_{FIR}(t, \hat{\Theta}) \), where \( \hat{\Theta} = [\hat{b}_0 \ldots \hat{b}_{N_b}]^T \) denotes an estimate of \( \Theta \), and the measured output \( y_{LES}(t) \),  
\[ \hat{y}_{FIR}(t, \hat{\Theta}) = \sum_{i=0}^{N_b} \hat{b}_i u(t - i), \]  
\[ (4) \]

\[ \hat{\Theta} = \arg \min_{\Theta} \left\{ \left( y_{LES}(t) - \hat{y}_{FIR}(t, \hat{\Theta}) \right)^2 \right\}. \]  
\[ (5) \]

The minimization of Eq. (5) leads to the Wiener-Hopf (WH) filter inversion,  
\[ \hat{\Theta} = \left[ \sum_{t=1}^{N} \phi(t) \phi^T(t) \right]^{-1} \phi(t) y_{LES}(t), \]  
\[ (6) \]

with regressor \( \phi(t) = [u(t) \ldots u(t - N_b)] \) and total number of samples \( N \). \( R_{\phi\phi} \) and \( R_{\phi y} \) are recognized in SI as autocorrelation matrix and cross correlation vector, respectively.

The correlation analysis can also be used to identify the acoustic scattering matrix of a given acoustic element if the noise is not white, as long as the system is not influenced by feedback loops due to reflections at the boundaries (cap. 13, \[8\]). Therefore, this approach requires perfectly non reflecting boundaries to yield correct results.

However, within this SI technique, it is not possible to model the dynamics of the noise, as done for the scattering matrix, since the noise terms in Eq. (3) have been considered as an exogenous...
unpredictable disturbance acting directly on the output of the system. Consequently, a different SI method must be adopted in order to introduce a dynamical model of the noise sources. This can be done by means of the prediction error method (PEM).

3.2 Prediction error method

The input-output representation of a linear, time invariant system may be expressed in a Box-Jenkins (BJ) form as [6, 7],

\[ y_{BJ}(t) = \frac{B(q, \Theta)}{F(q, \Theta)} u(t) + \frac{C(q, \Theta)}{D(q, \Theta)} e(t), \]

where \( B, C, D, F \) are polynomials in terms of the time shift operator \( q \) and of the unknown model parameters \( \Theta \). \( u(t) \) and \( y(t) \) are the input and output of the system, and \( e(t) \) is an unpredictable, Gaussian distributed error term (white noise) of variance \( \sigma^2_e \). The time shift operator \( q \) is mathematically expressed as

\[ y(t - 1) = q^{-1} y(t). \]

Note that in the case with \( C = D = F = 1 \), the input-output representation reduces to that one of FIR (Eq. (3)). Since \( e(t) \) is not predictable, to estimate the output \( \hat{y}_{BJ}(t, \Theta) \), Eq. (7) is re-written in the one step prediction form:

\[ \hat{y}_{BJ}(t, \Theta | t - 1) = \frac{D(q, \hat{\Theta}) B(q, \hat{\Theta})}{C(q, \hat{\Theta}) F(q, \hat{\Theta})} u(t) + \frac{C(q, \hat{\Theta}) - D(q, \hat{\Theta})}{C(q, \hat{\Theta})} y_{LES}(t), \]

where \( \hat{\Theta} \) contains the estimations of the parameters \( \Theta \). Eq. (9) corrects the estimated output \( \hat{y}_{BJ}(t, \Theta) \) by means of the past measurements \( y_{LES}(t) \). Therefore, the predicted output is computed using all the measured data up to the present time step to be estimated. The estimation of the parameters \( \hat{\Theta} \) is done by minimizing the prediction error between the one step ahead predicted output \( \hat{y}_{BJ}(t, \Theta | t - 1) \) and the measured output \( y_{LES}(t) \):

\[ \hat{\Theta} = \arg \min_{\Theta} \left\{ (y_{LES}(t) - \hat{y}_{BJ}(t, \Theta | t - 1))^2 \right\}. \]

4. Numerical Setup

The configuration analyzed is a duct in which one orifice has been placed (see Figure 1). The duct diameter is \( D = 3 \times 10^{-2} \) m and the orifice has a diameter \( d = 1.5 \times 10^{-2} \) m and a thickness \( t = 5 \times 10^{-3} \) m. The length of the whole pipe \( L = 0.515 \) m is divided by the orifice into two sections of length \( L_u = 0.15 \) m upstream and \( L_d = 0.36 \) m downstream. The mean flow velocity is \( U_{\text{mean}} = 9 \) m/s and corresponds to a Mach number \( M = 2.6 \times 10^{-2} \) and to a Reynolds number \( Re \approx 18000 \). The operating temperature is \( T_0 = 298.15 \) K and the operating pressure is \( p_0 = 101325 \) Pa. The present configuration for this operating condition has been experimentally analysed in [9].

The CFD solver used in this work is AVBP, developed by CERFACS and IFP-EN\(^1\). This code allows to perform a Large Eddy Simulation (LES) of the three-dimensional compressible Navier-Stokes equations on unstructured meshes. The spatial and the temporal discretization adopted is a second order Lax-Wendroff scheme. The time step has been fixed to be \( 5.6 \times 10^{-8} \) s in order to satisfy a CFL number of 0.7 once a steady flow condition is reached. The large turbulent eddies are resolved, whereas the small scales are modelled with the WALE subgrid-scale model. The boundary conditions adopted in the present work consist of a modified version of the Navier-Stokes Characteristic Boundary Conditions (NSCBCs) [10] based on the Plane Wave Masking method [11]. Thereby,

\(^1\)www.cerfacs.fr/4-26334-The-AVBP-code.php
it is possible to achieve acoustically perfectly non-reflecting boundary conditions in the acoustic plane wave frequency range.

The computational grid consists of a hexahedral mesh of 7.0 millions elements. The grid has been refined in radial direction to have elements whose dimensions are of the order of the Taylor microscale. In order to correctly capture the flow-acoustic interaction and the noise generation, the boundary layers developed at the orifice section must be sufficiently resolved. This requirement is achieved by positioning the first cell next to the walls inside the viscous sublayer of the turbulent boundary layer at \( y^+ = 5 \) or \( y = 2.545 \times 10^{-5} \) m. The grid has been refined in axial direction to have at least 80 grid points per wavelength at the cut-off frequency of the duct (here 6700Hz). Finally a correct prediction of the acoustic scattering requires a further grid refinement in axial direction in proximity of the orifice. Here, the cell size adopted is \( 2.50 \times 10^{-5} \) m.

Two broadband statistically independent wavelet signals are simultaneously applied at the inlet and the outlet of the configuration [3]. This kind of signal has been chosen because it affords two input acoustic perturbations low correlated. Moreover, the adoption of a wavelet-type signal allows to get constant power density in the range of frequency of interest. The acoustic responses to the imposed signals are extracted from the computational domain, at different planes positioned upstream and downstream of the orifice, in term of average pressure and velocity. The acoustic pressure fluctuations \( p' \) and velocity fluctuations \( u' \) are then retrieved applying the characteristic based filter [12] as sketched in Figure (1). This filter allows to separate the acoustic fluctuations from the turbulent fluctuations using the property that acoustic plane waves propagate with the speed of sound whereas turbulent fluctuations are convected with a velocity of the same order of the mean flow velocity. The characteristic wave amplitudes \( f_{u,d} \) and \( g_{u,d} \) are then computed from the acoustic pressure and velocity fluctuations using Eq. (2).

5. Results

The parametric model structure of Box-Jenkins (Eq. (7)) has been used to identify both the scattering matrix and the dynamics of the noise source. The scattering matrix is the \( z \)-transform of the estimated transfer function \( G(q, \Theta) = B(q, \Theta)/F(q, \Theta) \) of Eq. (7), whereas the dynamics of the noise term is represented by the \( z \)-transform of the transfer function \( W(q, \Theta) = C(q, \Theta)/D(q, \Theta) \).

The identification procedure has been applied to three different LES of length \( T = 0.2 \) s. Each simulation has been acoustically excited with an excitation signal of different amplitude to assess possible nonlinear dependencies of the noise sources. The amplitudes considered are 1.7%, 1.1%, 0.8% of the mean flow velocity, respectively. These three signal amplitudes correspond to a mean Signal to Noise Ratio \( SNR = \sigma_{Input}^2/\sigma_{Noise}^2 \), with \( \sigma_{Input}^2 \) variance of the inputs and \( \sigma_{Noise}^2 \) variance of the noise extracted from a non-excited LES, around 11.4, 4.7, 2.52, respectively. The excitations considered have constant amplitudes in the range of frequency between \([0 – 5000]\)Hz in order to not excite higher order modes.

The order of the polynomials of Eq. (7) has been determined by means of residual analysis. Let \( \epsilon(t, \Theta) = (\hat{y}_{LES}(t) – \hat{y}_{BJ}(t, \Theta)|t – 1) \) be the residual of an estimated model. The best estimation of Eq. (7) is the lowest order dynamical model which grants a residual perfectly uncorrelated with itself and with the inputs imposed. Different model structures, with different orders of the polynomials \( B, C, D, F \), of Eq. (7) have been computed. The best estimation of the acoustic scattering assessed through the BJ model structure has the form:

\[
\hat{y}_{BJ}(t, \Theta) = B(q, \Theta)u(t) + \frac{1}{D(q, \Theta)}e(t). \tag{11}
\]

According to Eq. (11), the passive acoustic scattering is described in the time domain by means of
Figure 2. Module of the Acoustic Scattering Matrix: experimental results (dotted-grey) vs. SI results with BJ model at different levels of excitation ([-] blue line 0.8%$U_{\text{mean}}$, [-.-] green line 1.1%$U_{\text{mean}}$, [-] black line 1.7%$U_{\text{mean}}$)

$B(q, \Theta)$ whereas the active noise generation is described by

$$\hat{n}(t, \Theta) = \frac{1}{D(q, \Theta)} e(t),$$

where $\hat{n}(t, \Theta)$ are the estimated noise sources and $W(q, \Theta) = 1/D(q, \Theta)$ is the estimated dynamical noise model. Eq. (12) is known as an autoregressive model structure (AR), since $\hat{n}(t, \Theta)$ at a given time step only depends on previous values taken by $\hat{n}(t, \Theta)$ itself. This is in agreement with a possible acoustic feedback mechanism that may take place inside the orifice. Vortices may grow inside the orifice boundary layer and generate an acoustic pulse at the outlet (of the orifice). This acoustic pulse triggers subsequently the release of a new vortex causing an autoregressive interaction. The results are exposed in the following and discussed in terms of acoustic scattering matrix and noise source model.

5.1 Scattering Matrix

The module of the scattering matrices identified are compared with experimental results carried out by [9] (see Figure 2). The Strouhal number is computed w.r.t. the thickness of the orifice $t$ and the orifice flow velocity $U_d = U(D/d)^2$. Numerical and experimental results are in a good agreement up to $St = 0.6$. The region of potential whistling around $St = 0.25$ is well predicted by the simulations for all the excitation levels considered. However, by reducing the amplitude of the excitation signals the predicted scattering matrix becomes more oscillating between $St = 0.15$ and $St = 0.45$. Decreasing the excitation amplitude results in a lower signal to noise ratio of the measured data and therefore an increase of the uncertainty of the estimated parameters. Hence, to reduce the variance of the estimates with lower excitations signals, one should carry out longer simulations, increasing the computational costs. Nevertheless, the estimation of the scattering matrix has been considered sufficiently accurate for the three investigated cases.

5.2 Noise Sources

In Figure 3, the spectra of the noise model $W(q, \Theta)$ identified with the BJ model are compared to the spectrum of the noise extracted from a non-excited LES. The latter has been computed applying an averaging on a Hanning time window according to Welch’s method. Results are reported up to the
Figure 3. Identified noise spectra: Non-excited simulation (grey) vs. excited simulations with different levels of excitation ([ - ] blue line 0.8% $U_{mean}$, [ - - ] green line 1.1% $U_{mean}$, [- ] black line 1.7% $U_{mean}$)

cut-off frequency at $St = 0.93$. The spectra present two peaks, one around $St = 0.25$, in the whistling frequency range, and a second one around $St = 0.75$, where the numerical approach predicts a second whistling mode [5]. Noise spectra obtained from experiments on the same configuration but for different operating conditions do not present a strong peak around $St = 0.75$ but only a small hump [13] which seems to correspond to an amplification of the first transverse mode. In this case, a possible transversal mode could be amplified in the LES, generating a resonating condition. Indeed, the boundary conditions based on Plane Wave Masking are non reflecting only for plane acoustic waves whereas non-plane acoustic modes are reflected. A second possible explanation is related to the laminar inflow adopted in the simulations which is prone to generate coherent vortices at the orifice outlet, producing tonal noise.

Results show that the identified noise spectra strictly depend on the amplitude of the external excitation imposed. The lower the excitation, the higher is the agreement with the noise spectrum measured through a non-excited LES. On the one hand, the increase of the excitation amplitude increases the level of the noise in the frequency range below $St = 0.25$. On the other hand, the peak around $St = 0.75$ is damped with an increase of the excitation. It is therefore expected that a better agreement between spectra may be obtained by further reducing the excitation amplitudes, i.e. for lower signal to noise ratios. These results stand in contrast with the initial mathematical representation assumed in Eq. (1). There, the noise vector term is assumed to be independent and uncorrelated with the given input signals. However, results displayed in Figure 3 suggest that the noise sources behave nonlinearly to an imposed acoustic field, since different excitation levels lead to different noise estimates. Note, in view of Figure 2, the acoustic scattering still present a linear behavior for the different excitation levels considered. The assumption of nonlinearities in the noise term w.r.t the input acoustic excitation leads to reconsider the assumption of linear acoustic scattering in Eq. (1). On the one hand, by assuming linearity in the input-output coupling, possible nonlinear dynamics may not be captured in the classical scattering matrix representation. On the other hand, the assumption of noise sources independent and uncorrelated with the imposed acoustic perturbations may not take into account possible flow acoustic interactions. Indeed, an external acoustic excitation interacting with the shear layers, developed at the orifice, may cause a nonlinear answer of the latter leading to a modification either of the passive acoustic scattering or of the noise generation mechanism. The turbulent eddies responsible of the active noise scattering, may therefore be affected by an exogenous acoustic field leading to an amplification or a reduction of the emitted noise levels. The physical explanation of these nonlinear phenomena is still an open question, currently under investigation.
6. Conclusions

Advanced SI methods based on a prediction error estimation and a Box-Jenkins model structure are introduced to characterize the passive acoustic scattering and the noise generation mechanism of an orifice configuration. The identified scattering matrix is in a good agreement with previous computational and experimental results. The identified noise model presents an autoregressive behavior index of possible feedback loops in the noise generation mechanism. The identified noise sources are also depending on the level of acoustic external excitation imposed suggesting a possible nonlinear dependency of the noise generation on the acoustic field. The lower the excitation level, the higher is the agreement with the noise generated in a non-excited LES. This leads to reconsider the classical linear mathematical representation which assumes the noise terms independent and uncorrelated with an imposed external perturbation. Therefore, the characterization of the acoustic propagation may be achieved only by identifying the scattering matrix and the noise source simultaneously.

REFERENCES


