**NON-SYMMETRICAL SEMI-ACTIVE VIBRATION CONTROL BASED ON SYNCHRONIZED SWITCHING DAMPING**

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An unsymmetrical switch circuit is designed for semi-active control method based on synchronized switching damping principle of piezoelectric actuators. A bypass capacitor and an additional switch are used to realize unsymmetrical bipolar voltage. The control logic of the switches is introduced in detail and the switched voltages, which directly influence the control performance, are derived as functions of the vibration amplitude and the outputs of the voltage sources. Experiments were carried out to verify the designed circuit and the theoretical results of the switched voltage. For simplification of experimental conditions the beam is fully clamped and the voltage is fully generated by the voltage source. The influence of the voltage source and bypass capacitor on the actuator was investigated. Experimental results show that the circuit leakage is a critical factor to the amplification of actuator voltage. The voltage ratio increases with increasing bypass capacitance, but its increasing rate decreases.

1. **Introduction**

Structural vibration control has been an active research area since early 1990s\(^1\). Several control methods including active, passive and semi-active have been proposed\(^2\). Recently, a type of semi-active control, which is called Synchronized Switch Damping (SSD) techniques has been proposed to overcome the disadvantages of active and passive methods\(^3,4\). It consists in a fast inversion of voltage on the piezoelement using a few basics electronics. The voltage on the piezoelectric element is switched at the strain extrema or displacement extrema of vibration.

In the original SSD technique, the control performance mainly depends on the value of the voltage on piezoelectric element. Several SSD methods including SSDI and SSDV (SSDV stands for synchronized switch damping on voltage) have been proposed to improve the control performance\(^5\). Due to the inherent properties of piezoelectric materials, the applicable bipolar voltage of a
piezoelectric actuator is unsymmetrical. That is, the voltage applicable in the positive direction (poled direction) is much higher than that in the negative direction. Otherwise, the actuator is depoled. The typical electric field applicable in the positive direction is higher than 2000 V/mm, but that in the negative direction is only about 500 V/mm. As an example of applicable voltage on an actuator, the working voltage range of MFC actuator is from 500 V to 1500 V. If symmetrical switching circuit is used, the voltage range will be from 500 V to 500 V so that a half of the actuator capability will be wasted. In this paper, an unsymmetrical switch circuit is designed for semi-active control method based on synchronized switching damping principle of piezoelectric actuators.

2. Principle of Unsymmetrical SSD Methods

2.1 Principle of unsymmetrical methods

The schematic diagram of the designed unsymmetrical SSDV control system are shown in Fig. 1. The polarity of the PZT actuator is label by “+” and “-” in the figure. The schematic diagram of the switched voltage is shown in Fig. 1(b). The positive voltages just after and before switching action are expressed as \( V_{mp} \) and \( V_{Mp} \), and the absolute values of negative voltages just after and before switching action are denoted by \( V_{mn} \) and \( V_{Mn} \). In order to obtain unsymmetrical actuator voltage, the switching logic shown in Fig. 2 is used to control the three switchers. In one cycle of vibration, the voltage is inverted twice. The variation of voltage on the PZT actuator can be divided into six phases.

\[
V_a = V_{mp} + \alpha u_t \sin(\omega_m t - \pi/2) + C_p \quad (0 \leq t \leq \pi/\omega_m) \tag{1}
\]

where \( \omega_m \) is the angular resonance frequency of the mechanical system. After half a cycle of mechanical vibration, the displacement reaches its maximum and the voltage on the PZT actuator reaches \( V_{Mp} \). Hence, there exists

\[
V_{sh} = V_{mp} + 2\alpha u_t/C_p \tag{2}
\]

2.1.1 Displacement increasing from minimum to maximum (Phase 1)

After the switching action is finished at the displacement minimum, the voltage on the PZT actuator is positive. The voltage on the PZT actuator is

\[
\begin{align*}
L \frac{d^2 \hat{V}_a}{dt^2} + RC_p \frac{d \hat{V}_a}{dt} + V_a - V_{cc} &= 0 \\
V_a(0) &= V_{mp} \\
\hat{V}_a(0) &= 0
\end{align*}
\]

The voltage can be expressed in the following form:
where $\omega_c = 1/\sqrt{LC}$ is the angular resonance frequency of the shunt circuit, $Q_c = \sqrt{C_c/R}$ is the quality factor of the shunt circuit, mainly contributed by the inductor and the transistors in the circuit. $\zeta_e = 1/(2Q_e)$ is the damping ratio. The electric current flowing through the inductor can be expressed as

$$I(t) = C_p V_e = -C_p \frac{\omega_e}{2Q_e} (V_{mp} + V^n_e) e^{\frac{i\omega_e}{2Q_e} t} \cos \sqrt{1 - \zeta_e^2 \omega_e^2} - C_p \frac{\omega_e}{\sqrt{C_c}} (V_{mp} + V^n_e) e^{\frac{i\omega_e}{2Q_e} t} \sin \sqrt{1 - \zeta_e^2 \omega_e^2}$$

Here it is assumed that $\zeta_e \ll 1$ and $\sqrt{1 - \zeta_e^2} \approx 1$.

The boundary between Phase 2 and Phase 3 is the moment at which $V_e = 0$. If $V^n_e = 0$, boundary is $t_{2,3} = \pi/(2\omega_e)$, at which there exit

$$V_e = 0 \quad \text{and} \quad I = -C_p V^n_e \gamma^{1/2} \omega_e$$

where $\gamma = e^{-i\omega_c t}$ is the inversion coefficient.

If $V^n_e = 0$, $t_{2,3} = \pi/(2\omega_e)$, but its expression will be very complicated. Hence, for convenience it is assumed that $t_{2,3} = \pi/(2\omega_e)$ even when $V^n_e \neq 0$. The error due to this assumption is acceptable because $V^n_e \ll V_{mp}$. The voltage and current at this moment is

$$V_e = -V^n_e \quad \text{and} \quad I = -C_p (V_{mp} + V^n_e) \gamma^{1/2} \omega_e$$

### 2.1.3 Voltage inversion at displacement maximum with bypass capacitor closed (Phase 3)

In Phase 3 the voltage on PZT actuator becomes negative and the current will flow through both the PZT actuator and the bypass capacitor, as shown by the arrows in Fig. 2(c).

$$L(C_p + C_b) \dot{V}_e + R(C_p + C_b) \dot{V}_e + V_e + V^n_e = 0$$

The voltage can be expressed in the following form:

$$V_e(t') = (V_{mp} + V^n_e) e^{\frac{i\omega_e}{2Q_e} t'} \cos \sqrt{1 - \zeta_e^2 \omega_e^2} - V^n_e \quad (\pi/2 \leq \omega_e t' \leq \pi)$$

where $\omega'_e = 1/\sqrt{L(C_p + C_b)}$ is the angular resonance frequency of the shunt circuit when the bypass capacitor is in parallel with the PZT actuator, $Q'_e = \sqrt{L/(C_p + C_b)}$ is the frequency factor of the shunt circuit in the same condition, and $\zeta'_e = 1/(2Q'_e)$ is the damping ratio. The voltage on the PZT is denoted by $V'_e$ in order to distinguish it from that in Eq. (5). The new time $t'$, which is the shift of time $t$ and satisfies $\omega_e t' = \pi/2$ when $\omega_e t = \pi/2$, is used for convenience. Using the condition that the voltage and current at the beginning of Phase 3 are the same as those at the end of Phase 2, shown in Eq. (7) for $V^n_e = 0$ and in Eq. (8) for $V^n_e \neq 0$, the following relationship can be obtained:

$$V_{mp} + V^n_e = \frac{C_p}{C_p + C_b} \frac{\gamma}{\gamma'}^{1/2} \frac{\omega_e}{\omega'_{e}} (V_{mp} + V^n_e) = \left[ \frac{C_p}{C_p + C_b} \frac{\gamma}{\gamma'} \right]^{1/2} \left( \frac{V_{mp} + V^n_e}{V_{mp} + V^n_e} \right)$$

where $\gamma' = e^{-i\omega_c t}$ is the inversion coefficient when the $C_b$ is in parallel with the PZT actuator.

At the end of Phase 3, the voltage is $-V^n_{ma}$ and the current is zero. Hence the following holds

$$V_{ma} = \gamma' \left[ \frac{C_p}{C_p + C_b} \right]^{1/2} \left( \frac{\gamma}{\gamma'} \right)^{1/2} \left( V_{mp} + V^n_e \right) + V^n_e = \left[ \frac{C_p}{C_p + C_b} \right]^{1/2} \left( \gamma^{1/2} \right) V_{mp} + \left[ 1 + \left( \frac{C_p}{C_p + C_b} \right) \right]^{1/2} (V^n_{ma})$$
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actuator is negative. The actuator voltage is

\[ V_e = -V_{m0} - \alpha u_{m0} \left( 1 + \sin(\omega_m t - \pi/2) \right) \left( C_p + C_s \right) \quad (0 \leq t \leq \pi / \omega_m) \]  

(13)

After half a cycle of mechanical vibration, the displacement reaches its minimum and the voltage on the PZT actuator reaches \( V_{M0} \). Hence, there exists

\[ V_{M0} = V_{m0} + 2 \alpha u_{m0} \left( C_p + C_s \right) \]  

(14)

### 2.1.5 Voltage inversion at displacement minimum with bypass capacitor closed (Phase 5)

In Phase 5, both SW2 and SW3 are closed so that the PZT actuator and the bypass capacitor are discharged simultaneously, as shown in Fig. 2(e). The voltage on the PZT actuator satisfies

\[ L(C_p + C_b)\dot{V}_e + R(C_p + C_s)\dot{V}_e + V_e = 0 \]  

where \( V_e^p \) is the output of the positive voltage source. The initial conditions are

\[ V_e(0) = -V_{M0}, \quad \dot{V}_e(0) = 0 \]  

(16)

The voltage can be expressed in the following form:

\[ V_e(t) = -(V_{m0} + V_e^p) e^{\frac{-\alpha}{\omega_c} t} \cos \sqrt{1 - \frac{\omega_c^2}{\omega^2} \omega \omega' + V_e^p \omega} \quad (0 \leq \omega' t \leq \pi/2) \]  

(17)

If \( V_e = 0 \), the time at the end of Phase 5 is \( t_{5,6} = \pi / (2 \omega) \) and the voltage and current are

\[ V_e = 0 \quad \text{and} \quad I = -(C_p + C_b)(V_{m0} + V_e^p)^{\frac{1}{2}} \omega V_{M0} \]  

(18)

If \( V_e > 0 \), \( t_{5,6} < \pi / (2 \omega) \). It is assumed that \( t_{5,6} = \pi / (2 \omega) \) even when \( V_e \neq 0 \). The voltage and current at this moment are

\[ V_a = V_e \quad \text{and} \quad I = -(C_p + C_b)(V_{m0} + V_e^p)^{\frac{1}{2}} \omega V_{M0} \]  

(19)

### 2.1.6 Voltage inversion at displacement minimum with bypass capacitor open (Phase 6)

In Phase 6, Switcher SW2 is kept in the closed state, but SW3 is opened. Hence the electric current flows only through the loop as shown in Fig. 2(f) and the voltage on the PZT actuator is

\[ L(C_p + C_b)\dot{V}_a + R(C_p + C_s)\dot{V}_a + V_a + V_e^p = 0 \]  

(20)

The voltage can be expressed in the following form:

\[ V_e(t) = -(V_{m0} + V_e^p) e^{\frac{-\alpha}{\omega_c} t} \cos \sqrt{1 - \frac{\omega_c^2}{\omega^2} \omega \omega' + V_e^p \omega} \quad (\pi / 2 \leq \omega' t \leq \pi) \]  

(21)

Using the condition that the voltage and current at the beginning of Phase 6 are the same as those at the end of Phase 5, the following relationship can be obtained:

\[ V_{M0} + V_e^p = \left( \frac{C_p + C_b}{C_p} \right)^{\frac{1}{2}} \left( \frac{\omega'}{\gamma} \right)^{\frac{1}{2}} \left( \frac{\omega}{\gamma} \right)^{\frac{1}{2}} \left( V_{M0} + V_e^p \right) \]  

(22)

At the end of Phase 6, the voltage is \( V_{mp} \) and the current is zero. Hence the following holds

\[ V_{mp} = \gamma \left( \frac{C_p + C_b}{C_p} \right)^{\frac{1}{2}} \left( \frac{\omega'}{\gamma} \right)^{\frac{1}{2}} \left( V_{M0} + V_e^p \right) + V_e^p = \left( \frac{C_p + C_b}{C_p} \right)^{\frac{1}{2}} \left( \frac{\omega'}{\gamma} \right)^{\frac{1}{2}} V_{M0} + \left[ 1 + \left( \frac{C_p + C_b}{C_p} \right)^{\frac{1}{2}} \left( \frac{\omega'}{\gamma} \right)^{\frac{1}{2}} \right] V_e^p \]  

(23)

![Diagram](attachment:image.png)

(a) Phase 1: Voltage decreasing as displacement change from maximum to minimum  
(b) Phase 2: Voltage inverted from positive to negative with bypass capacitor open
2.2 Voltages Before and After Switching

Substitution of Eqs. (23), (14) and (12) into (2), Eq. (2) yields

\[
V_{ss} = \gamma V_{sh} + \left[ 1 + \left( \frac{C_p}{C_p+C_b} \right)^{1/2} (\gamma')^{1/2} \right] \frac{2a_{in} + \left( \frac{C_p+C_b}{C_p} \right)^{1/2}}{C_p} \left( \gamma' \right)^{1/2} V_{in} + \frac{1}{1-\gamma'} \left[ \left( \frac{C_p}{C_p+C_b} \right)^{1/2} \right] \left( \gamma' \right)^{1/2} V_{in}^{1/2} \]  

(24)

Solution of \( V_{ss} \) from Eq. (24) gives

\[
V_{ss} = \frac{1}{1-\gamma'} \left[ 1 + \left( \frac{C_p}{C_p+C_b} \right)^{1/2} (\gamma')^{1/2} \right] \frac{2a_{in} + \left( \frac{C_p+C_b}{C_p} \right)^{1/2}}{C_p} \left( \gamma' \right)^{1/2} V_{in} + \frac{1}{1-\gamma'} \left[ \left( \frac{C_p}{C_p+C_b} \right)^{1/2} \right] \left( \gamma' \right)^{1/2} V_{in}^{1/2} \]  

(25)

Using the same method, \( V_{Mn} \) can be obtained:

\[
V_{Mn} = \frac{1}{1-\gamma'} \left[ 1 + \left( \frac{C_p}{C_p+C_b} \right)^{1/2} (\gamma')^{1/2} \right] \frac{2a_{in} + \left( \frac{C_p+C_b}{C_p} \right)^{1/2}}{C_p} \left( \gamma' \right)^{1/2} V_{in} + \frac{1}{1-\gamma'} \left[ \left( \frac{C_p}{C_p+C_b} \right)^{1/2} \right] \left( \gamma' \right)^{1/2} V_{in}^{1/2} \]  

(26)

The control effect depends on the switch voltages \( V_{sw}^{p} \) and \( V_{sw}^{n} \), which can be expressed as

\[
V_{sw}^{p} = \frac{1}{1-\gamma'} \left[ 1 + \left( \frac{C_p}{C_p+C_b} \right)^{1/2} (\gamma')^{1/2} + \gamma' \right] a_{in} \frac{1}{C_p} \left( \gamma' \right)^{1/2} + \frac{1}{1-\gamma'} \left[ \left( \frac{C_p}{C_p+C_b} \right)^{1/2} \right] \left( \gamma' \right)^{1/2} V_{in}^{1/2} \]  

(27)

and

\[
V_{sw}^{n} = \frac{1}{1-\gamma'} \left[ 1 + \left( \frac{C_p}{C_p+C_b} \right)^{1/2} (\gamma')^{1/2} + \gamma' \right] a_{in} \frac{1}{C_p} \left( \gamma' \right)^{1/2} + \frac{1}{1-\gamma'} \left[ \left( \frac{C_p}{C_p+C_b} \right)^{1/2} \right] \left( \gamma' \right)^{1/2} V_{in}^{1/2} \]  

(28)

The unsymmetrical voltage ratio is defined as

\[
\beta = \frac{V_{sw}^{p}}{V_{sw}^{n}} \]  

(29)

Equations (27), (28) and (29) indicate that voltage ratio \( \beta \) depends mainly on the bypass capacitor \( C_b \). If \( C_b = 0 \), then \( \gamma' = \gamma \).

3. Verification Experiments and Simulation

3.1 Experimental and Simulation Conditions

A composite beam was used in the experiments. It has two embedded piezoelectric elements, the capacitance of which is 141 nF. In order to simplify the verification of the circuit, the beam is...
fully clamped so that no bending deformation was generated in the beam, that is, $u_m=0$. The voltage on the piezoelectric element was generated by the voltage source. Because the vibration of the beam was fully suppressed, the voltage is switched at a hypothetical frequency, 2 Hz, which is different from the natural frequency.

A Simulink model of the system including the structure, the piezoelectric actuator and the switching circuit was established. The parameters used in the simulation are shown in Table 1. The quality factor is $Q_e = 9.6$ and the inversion coefficient is $\gamma = 0.85$.

<table>
<thead>
<tr>
<th>Table 1 Parameter of the system</th>
</tr>
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<tbody>
<tr>
<td>Switching frequency</td>
</tr>
<tr>
<td>Capacitance of PZT</td>
</tr>
<tr>
<td>Inductance</td>
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<tr>
<td>Resistance of the circuit</td>
</tr>
<tr>
<td>Quality factor of shunt circuit</td>
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3.2 Experimental and Simulation Results

3.2.1 Unsymmetrical switching

In unsymmetrical switching, a parallel capacitor of 1.4 $\mu$F was connected to reduce the influence of leakage. The waveform of the actuator voltage is shown in Fig. 3 when the bypass capacitor is 4 $\mu$F and voltage source is 5 V. Obvious unsymmetrical voltage was obtained on the actuator.

The theoretical and experimental results of maximum positive and negative voltages are shown in Fig. 4 for the bypass capacitance from 0 $\mu$F to 8 $\mu$F at an interval of 1 $\mu$F and the voltage source is set to $V_{cc} = 5$ V. The theoretical result indicates that both the maximum positive and negative voltages decrease as the bypass capacitance increases. However the maximum positive voltage increases with increasing bypass capacitance in experiments. This is because a larger bypass capacitance is more effective in reducing the influence of circuit leakage. The maximum negative voltage in experiments is larger than that in simulation due to the circuit leakage. Figure 5 shows the variation of absolute ratio of the maximum voltages with the bypass capacitance. The simulation and experimental results have the similar trend.

4. Analysis of Leakage and Improving Formulation

Since the leakage has considerable influence on the maximum voltage on the piezoelectric actuators, it is important to consider the leakage phenomenon in deducting the switched voltage.

4.1 Improving Formulation of Switched Voltage

Because the maximum actuator voltages and the switched voltages calculated Eqs (25)-(29) are very different from the experimental results due to leakage of the switch circuit, these equation should be modified to include the voltage drops in the open states. When the voltage drop is considered, Eq. (24) becomes
\[ V_{w} = \gamma V_{b} + \left[ 1 + \left( \frac{C_{b} + C_{c}}{C_{b} + C_{p}} \right)^{1/2} \right] \frac{2mV_{p}}{C_{p}} + \left[ \frac{C_{b} + C_{c}}{C_{b} + C_{p}} \right] (\gamma')^{1/2} V_{c} + \left[ 1 + \left( \frac{C_{b} + C_{c}}{C_{b} + C_{p}} \right)^{1/2} \right] (\gamma')^{1/2} V_{p} - \left( \frac{C_{b} + C_{c}}{C_{b} + C_{p}} \right) (\gamma')^{1/2} V_{w} - V_{p} \]  

(30)

The control effect depends on the switch voltages \( V_{w}^{p} \) and \( V_{w}^{n} \), which can be expressed as

\[ V_{w}^{p} = \frac{(V_{b} + V_{mp})}{2} = \frac{1}{1-\gamma'} \left[ 1 + 2 \left( \frac{C_{b} + C_{c}}{C_{b} + C_{p}} \right)^{1/2} (\gamma')^{1/2} + \gamma' \right] \frac{a\mu_{b}}{C_{p}} + \frac{1}{1-\gamma'} \left[ \left( \frac{C_{b} + C_{c}}{C_{b} + C_{p}} \right)^{1/2} (\gamma')^{1/2} \right] V_{e}^{p} \]

\[ + \frac{1}{1-\gamma'} \left[ \frac{C_{b}}{C_{b} + C_{p}} \right] (\gamma')^{1/2} V_{w}^{0} + \frac{1}{1-\gamma'} \left[ \frac{C_{b}}{C_{b} + C_{p}} \right] (\gamma')^{1/2} V_{w}^{0} \]

\[ V_{w}^{n} = \frac{1}{1-\gamma'} \left[ 1 + 2 \left( \frac{C_{b} + C_{c}}{C_{b} + C_{p}} \right)^{1/2} (\gamma')^{1/2} + \gamma' \right] \frac{a\mu_{b}}{C_{p}} + \frac{1}{1-\gamma'} \left[ \left( \frac{C_{b} + C_{c}}{C_{b} + C_{p}} \right)^{1/2} (\gamma')^{1/2} \right] V_{e}^{p} \]

\[ + \frac{1}{1-\gamma'} \left[ \frac{C_{b}}{C_{b} + C_{p}} \right] (\gamma')^{1/2} V_{w}^{0} - \frac{1}{1-\gamma'} \left[ \frac{C_{b}}{C_{b} + C_{p}} \right] (\gamma')^{1/2} V_{w}^{0} + V_{p} \]

(31)  

(32)

For the unsymmetrical switching with a bypass capacitor of 4 \( \mu \)F, the experimental and theoretical results are shown in Fig. 6. Unsymmetrical voltages are obtained on the actuator, and the voltage ratio is about 1.8, which is almost the same for experimental and theoretical results.

Figure 7 shows the variation of the theoretical and experimental voltages, \( V_{mp} \) and \( -V_{mn} \), with different bypass capacitances in the unsymmetrical switching circuit, in which the voltage source is \( V_{cc} = 5 \) V. In the calculation, the same voltage drop was used approximately for different bypass capacitances. In experiments, it was found that the voltage drop changes with the bypass capacitor because it influences the actuator voltage though the variation is smaller than that in the case of varying voltage source. Compared with Fig. 8, the theoretical and experimental voltages is in better agreement. The theoretical and experimental results of voltage ratio of \( V_{mp} \) to \( V_{mn} \) with different bypass capacitances and a voltage source are shown in Fig. 8. The relative errors of the voltage ratio calculated from the original equations and that from the modified equations are almost the same, but their direction of deviation are different. The error in the results from the modified equation can be attributed to the approximation of voltage drop used in the calculation.
5. Conclusion

An unsymmetrical switch circuit is designed for semi-active control method based on SSD principle of piezoelectric actuators. A bypass capacitor and an additional switch are used to realize unsymmetrical bipolar voltage. The control logic of the switches is introduced in detail and the switched voltages, which directly influence the control performance, are derived as functions of the vibration amplitude and the outputs of the voltage sources. Experiments were carried out to verify the designed circuit and the theoretical results of the switched voltage. Experimental results show that the circuit leakage is a critical factor to the amplification of actuator voltage. The voltage ratio increases with increasing bypass capacitance, but its increasing rate decreases.

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