STRUCTURAL DISCONTINUITY LOCATING BASED ON THE REFLECTION OF GUIDED WAVES

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As some guided waves may propagate along the structures in a long distance and occur reflection and transmission at discontinuities, the discontinuities may be located by positioning guided-wave probes along the structure and measuring the reflected and transmitted waves. In this inspection method, the locating accuracy is highly related to the dispersive characteristics of the guided waves. However, although the dispersive characteristics of waves, which is can be estimated by theoretical and numerical methods, some particular types of waves are difficult to excite and measure in experiments, and consequently their dispersive characteristics are difficult to obtain. This paper explores a discontinuity monitoring system for infinite structures using measured wavenumbers. The propagating features of waves, as well as the reflection properties near a discontinuity, were firstly derived to relate the discontinuity location and the wave amplitudes. Meanwhile, the measured displacements, which are acquired from the probes, are introduced to represent the wave dispersive characteristics to provide a robust discontinuity locating. An experiment on a cracked beam was designed to validate the effectiveness of this method. It's expected that this method can provide an accurate discontinuity detection with the locating error less than 0.2%.

1. Introduction

The discontinuity detection problem has been extensively investigated due to its practical importance on structure health monitoring. Recently, the guided wave method has become one of the most popular methods to detect discontinuities in large waveguides, such as rail and pipes, since this method enables a long-distance and large-area detection of the discontinuities from a fixed transducer position\textsuperscript{1,2,3}.

The guided wave method is usually done in the pulse-echo mode\textsuperscript{4}. The successful operation of the pulse-echo method requires low-dispersive detecting waves to prevent the spreading of the detecting waves in space and time as they propagate\textsuperscript{5}. Therefore, only waves in a narrow frequency bandwidth with low dispersion can be employed in the pulse-echo method.

As some propagating waves may be partially reflected by discontinuities in the waveguide, the reflection and transmission characteristics associated with the presence of the discontinuity may be used to give some indication of both the location and size of the discontinuity\textsuperscript{6,7}. This paper concentrates on discontinuity locating on 1-D structures. First, we analyzed the propagation characteristics of guided waves in a cracked structure. The relationship was derived between the discontinuity
location and wave amplitudes as well as phases. A discontinuity locating method was then developed to identify the location of the possible discontinuities by measured displacements. Experiments on a 6 m beam have shown that the location error is less than 0.2%.

2. Relation between discontinuity location and wave characteristics

2.1 Propagating waves in a cracked structure

An infinite 1-D structure with a discontinuity $\mathcal{D}$ lies along x-axis, as shown in Fig. 1, where only the propagating waves are considered.

![Figure 1. Propagating waves in a structure with the in-going waves from $x_{\infty}$.](image)

When an in-going wave from the negative infinite is incident upon $\mathcal{D}$, a reflected and a transmitted propagating waves may be produced at $\mathcal{D}$. The amplitudes of these waves at $M_{1,2}$ and $\mathcal{D}$ can be related by

$$A_{p1}^{+} = A_{1}^{+} e^{-j\beta L_1} \quad \text{and} \quad A_{p1}^{-} = A_{1}^{-} e^{j\beta L_1}, \quad (1)$$

where $A_{D1,D2}^{\pm}$ are the wave amplitudes just at two sides of $\mathcal{D}$; $A_{1,2}^{\pm}$ are the wave amplitudes at $M_{1,2}$; the superscripts + and − denote the positive- and negative-going waves; the real wavenumber $k$ represents the spatial phase change velocity; $L_{1,2}$ denote the distances from $M_{1,2}$ to $\mathcal{D}$.

The reflection coefficient $r_1$ is defined as

$$r_1 \triangleq \frac{A_{D1}^{-}}{A_{D1}^{+}}, \quad (2)$$

On the other hand, when an in-going wave from the positive infinite is incident upon $\mathcal{D}$, reflection and transmission may occur. Therefore, employing $B_{1,2,D1,D2}^{\pm}$ to denote these wave amplitudes, the relations between these wave amplitudes can be represented in the form similar to Eq. (1), which are

$$B_{D2}^{-} = B_{2}^{-} e^{-j\beta L_2} \quad \text{and} \quad B_{D2}^{+} = B_{2}^{+} e^{j\beta L_2}. \quad (3)$$

And the reflection coefficient $r_2$ is defined as

$$r_2 \triangleq \frac{B_{D2}^{-}}{B_{D2}^{+}}, \quad (4)$$

By substituting Eq. (1) into Eq. (2), and Eq. (3) into Eq. (4), the reflection coefficients can be represented by the wave amplitudes at $M_{1,2}$. The reflection coefficients $r_1$ and $r_2$ can be rewritten into the form of magnitude and phase as

$$r_1 = \frac{A_{1}^{-}}{A_{1}^{+}} e^{j(\psi_{1} + 2\beta L_1)} \quad \text{and} \quad r_2 = \frac{B_{2}^{-}}{B_{2}^{+}} e^{j(\psi_{2} + 2\beta L_2)}, \quad (5)$$
where $\varphi_{1,2}$ are the phase difference between the positive- and negative-going waves at $M_{1,2}$, respectively.

### 2.2 Discontinuity location and measured wave phases

As to a symmetrical discontinuity, in general, $r_1 = r_2$, the phases in Eq. (5) can then be equalized

$$\varphi_1 + 2kL_1 = \varphi_2 + 2kL_2 + 2\pi n. \quad (6)$$

Set the distance between $M_1$ and $M_2$ to be $L = L_1 + L_2$, and the phase difference $\varphi = \varphi_2 - \varphi_1, \varphi \in [0, 2\pi)$, Eq. (6) can be rewritten as

$$k(4L_1 - 2L) = \varphi + 2\pi n, \quad (7)$$

where $n$ is an undetermined integer. In general, $n$ keeps constant in a narrow frequency band. Therefore, the differentiation of Eq. (7), after some arrangement, can be expressed as

$$L_1 = \frac{\frac{\partial \varphi}{\partial \omega}}{\frac{\partial k}{\partial \omega}} + \frac{L}{2}. \quad (8)$$

Theoretically, the location $L_1$ can be estimated, according to Eq. (8), by the measured phase difference $\varphi$ and the dispersive characteristics of the cracked structure.

### 2.3 Reflection coefficients using measured dispersive characteristics

As is shown in Eq. (8), the locating error is sensitive to the dispersive characteristics, e.g. the wavenumber $k$, of the 1-D structure. Unfortunately, the dispersive characteristics of some complex structures are difficult to calculate analytically. However, as for an infinite structure, they are easy to acquire via displacement measurements.

![Figure 2. Propagating waves and displacements with the in-going waves from $x_{-\infty}$.](image)

When an in-going wave from the negative infinite is incident upon $D$, as shown in Fig. 2, the wave amplitudes on the left side can be expressed by the measured displacements as

$$A_1^+= \frac{W_1e^{jk\Delta/2} - W_2e^{-jk\Delta/2}}{2jsin k\Delta}, \quad A_1^- = \frac{W_2e^{jk\Delta/2} - W_1e^{-jk\Delta/2}}{2jsin k\Delta}. \quad (9)$$

where $W_1$-$W_4$ are the displacements at the 4 measured points. On condition that the structure is infinite and $\Delta$ is much less than the wave length, the wavenumber and the displacements on the right side can be related as follows

$$k = \frac{1}{j\Delta}(\ln W_3 - \ln W_4). \quad (10)$$

The reflection coefficient $r_1$, by substituting Eqs. (9) and (10) into Eq. (5), can be rewritten as
\[ r_1 = \frac{W_2 V_2 - W_2 V_1}{W_2 V_2 - W_1 V_2} \left( \frac{W_1}{V_1} \right)^{\frac{2L_1}{\Delta}}. \]  
(11)

Correspondingly, employing \( V_1 - V_4 \) for the displacements when a negative-going wave propagates from the positive infinite, the reflection coefficient can be rewritten as

\[ r_2 = \frac{W_3 V_1 - W_4 V_4}{W_4 V_4 - W_3 V_4} \left( \frac{V_4}{V_1} \right)^{\frac{2L_2}{\Delta}}. \]  
(12)

Given \( L = L_1 + L_2 \), the discontinuity location \( L_1 \) can then be identified by setting \( r_1 = r_2 \).

3. **Discontinuity locating using curve fitting strategies**

Define

\[ r_M = \sqrt{r_1 r_2} \quad \text{and} \quad r_p(x) = \frac{r_1 + r_2}{2}. \]  
(13)

The modulus difference \( \psi \) between amplitudes of \( r_p \) and \( r_M \) can be represented by

\[ \psi = \sqrt{\sum_{j=1}^{S} \left[ |r_M(j)| - |r_p(j)| \right]^2}, \]  
(14)

where \(|\cdot|\) denotes the absolute value and \( S \) denotes the length of the reflection coefficient vector \( r_M \). The value of \( \psi \) varies as the predicted discontinuity location \( x \) moves from \( x = 0 \) to \( x = L \). The predicted discontinuity location vector \( x = [x_1, x_2, \ldots, x_N] \) represents \( N \) different locations distributing evenly along the beam, where \( x(1) = 0 \) and \( x(N) = L \). Substitution \( x(i) \) into Eqs. (13) and (14) as a predicted location yields the corresponding \( \psi(i) \).

In order to locate the discontinuity, a dimensionless discontinuity location factor is proposed to describe the probability of the identified discontinuity location. The vector of the location factor \( C \) can then be defined as

\[ \frac{1}{C} = \frac{\psi}{\sum_{i=1}^{N} \psi(i)/N}, \]  
(15)

where \( \psi = [\psi_1, \psi_2, \ldots, \psi_N] \). The predicted discontinuity location \( x(i) \), corresponding to the global maximum of \( C \), is the identified discontinuity location \( L_1 \).

4. **Experimental results and discussion**

4.1 **Experimental setup**

The experimental rig for detecting the crack is shown in Fig. 3. The dimension and material properties of the beam are listed in Table 1. The beam was suspended by three strings evenly along its length. Two sandboxes were placed at each end of the beam to approximate the infinite boundary conditions. The excitation for the beam was provided by a PCB K2004E01 electro-dynamic shaker. The excitation and response signals were collected by a PCB force sensor (208C01) and four PCB accelerometers (352C22), respectively. The accelerometers were mounted to the beam with bee wax. The signals were acquired and analyzed by the dynamic signal analyzer (B&K Pulse 3560) with the ensemble average of 200. The frequency responses considered here are up to 10 kHz with a frequency resolution of 1.56 Hz.
The detection region is between \( x = 0 \) and \( x = 4 \) m, so the predicted discontinuity location vector \( x = [0: 0.001: 4] \) m. The crack location is randomly selected at \( L_1 = 2.5 \) m. The depth of the crack is 3 mm, corresponding to a depth ratio of 0.5. The excitation points are set at -0.6 m and 4.6 m, so the effects of the near-field waves generated at the excitation points above 50 Hz can be ignored at the measurement positions.

### Table 1. Material properties and dimensions of the beam.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ( \rho )</td>
<td>7800 kg/m(^3)</td>
</tr>
<tr>
<td>Young’s modulus ( E )</td>
<td>( 2 \times 10^{11} ) Pa</td>
</tr>
<tr>
<td>Poisson’s ratio ( \mu )</td>
<td>0.3</td>
</tr>
<tr>
<td>Width ( h )</td>
<td>0.05 m</td>
</tr>
<tr>
<td>Thickness ( b )</td>
<td>0.006 m</td>
</tr>
</tbody>
</table>

**Figure 3.** Experimental rig for identifying the location of the crack on the infinite beam.

### 4.2 Locating the crack

The measured reflection coefficient curve is shown in Fig. 4. This curve exhibits a strong modulation and is less accurate at low frequencies. Because the ‘infinite’ boundaries of the beam are not ideal, the waves could not attenuate completely by the sandboxes placed on each end of the beam. The waves reflected by the boundaries lead to the poor conditions of the reflection coefficient curve. Poor conditions of the reflection coefficient curve, especially in the low frequencies, are also caused by the near-field waves generated at the beam boundaries and the crack.

**Figure 4.** Magnitudes of the reflection coefficients. ---, measured; ---, predicted.

By using these measured reflection coefficients, the location factors were obtained as shown in Fig. 5. The identified location of the crack is at \( x = 2.501 \) m, with a location error of 0.04%. The location error might be caused by the progress in measuring the location of the crack.
Figure 5. Discontinuity location factors of the crack with a depth ratio of 0.5.

The locating results for others cracks are listed in Table 2. All the cracks with depth ratios more than 0.1 can be located accurately, with location errors less than the location resolution 1 mm, by using the reflection coefficients up to 10 kHz. The small cracks with depth ratios less than 0.1 might be detected by using the waves in higher frequency range.

Table 2. Identification results of the crack.

<table>
<thead>
<tr>
<th>Depth ratio</th>
<th>Location (m)</th>
<th>Error (%)</th>
<th>Depth ratio</th>
<th>Location (m)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>2.501</td>
<td>0.04%</td>
<td>0.60</td>
<td>2.500</td>
<td>0.00%</td>
</tr>
<tr>
<td>0.20</td>
<td>2.500</td>
<td>0.00%</td>
<td>0.70</td>
<td>2.500</td>
<td>0.00%</td>
</tr>
<tr>
<td>0.30</td>
<td>2.499</td>
<td>-0.04%</td>
<td>0.80</td>
<td>2.500</td>
<td>0.00%</td>
</tr>
<tr>
<td>0.40</td>
<td>2.500</td>
<td>0.00%</td>
<td>0.90</td>
<td>2.505</td>
<td>0.20%</td>
</tr>
<tr>
<td>0.50</td>
<td>2.501</td>
<td>0.04%</td>
<td>0.95</td>
<td>2.503</td>
<td>0.12%</td>
</tr>
</tbody>
</table>

5. Conclusions

An analytical discontinuity locating method, based on the interaction of propagating waves and the discontinuity in a 1-D structure, is derived firstly to theoretically estimate the location of the discontinuity in the beam using reflection coefficients of guided waves. Furthermore, a structure monitoring system is proposed for infinite 1-D structures, via introducing measured displacements to represent the wave dispersive characteristics and curve fitting strategies to improve the locating accuracy. Experimental results show that discontinuities in the beam can be identified accurately by using low-frequency flexible waves with wavelengths dozens of times greater than the size of discontinuities along the direction of wave propagating. This reflection-based discontinuity detection method can also be extended to other waveguide structures by choosing a proper type of waves.

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