REGENERATION OF THE RETARDED TIME VECTOR TO ENHANCE THE ACCURACY OF ACOUSTIC PYROMETRY

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Temperature distribution of a gas field in a cross section can be restored by the acoustic pyrometry based on the relation between speed of sound and temperature. An inverse problem relating a set of sensors and actuators located on the wall can be formulated using the inverse Radon transform. The transfer matrix and its coefficient vectors approximate the speed of sound by using the radial basis function with finite number of deployed interpolation points inside the target field. The temperature is approximated by using interpolation points, which is subdivided into many grids, thus bearing high spatial resolution that allows a successful restoration of a complicated temperature distribution. A large number of retarded time data between sensors and actuators is needed to construct an over-determined system, although it is not easy to obtain due to practical limitations. In this work, regeneration of retarded time data by using initially restored temperature field is suggested to overcome the shortage of the number of sensors and actuators. The number of interpolation points can be increased due to regenerated retarded time data for an arbitrary acoustic path. However, the accuracy of regenerated retarded time strongly depends on the accuracy of initial reconstruction result. One cause of reconstruction error, singularity of transfer matrix, is reduced by selecting the optimal position of interpolation points employing a proper algorithm. As a simulation example, two rectangular duct sections having 1 and 4 local temperature maxima over the target field, respectively, are considered by the suggested regeneration technique and optimally deployed interpolation points using the genetic algorithm. Additionally, the modified Tikhonov regularization scheme is used to prevent divergence of inverse solution and spurious noise effect in actual situation. It is observed that the restoration of complicated temperature field is possible with reasonable accuracy by applying the present regeneration technique.

1. Introduction

In the acoustic pyrometry based on the tomographic algorithm, temperature field is reconstructed by solving the inverse problem. The problem is composed of the measured retarded time vector in between wall-mounted sensors and the transfer matrix which approximates the speed of sound within the medium and its coefficients\(^1\). The spatial interpolation technique is adopted for an approximation bearing high spatial resolution to achieve accurate restoration of target field. However, the high spatial resolution is preceded with large number of interpolation points which is dependent on the number of sensors and actuators. In this study, regeneration of retarded time data by
using initially restored temperature field is proposed to overcome the low spatial resolution induced by the shortage of the number of sensors and actuators.

2. Theoretical background

2.1 Reconstruction of two-dimensional temperature field

Consider a two-dimensional rectangular domain as shown in Fig. 1.

![Figure 1. A two-dimensional measurement section with wall-mounted sensor and actuator pair.](image)

The retarded time, \( t_d \), of a sound pulse between two points can be defined by using the Radon transform relating the speed of sound and the retarded time as\(^2\)

\[
t_d = L \int_0^1 \frac{ds}{c(u,v)} = L \int_0^1 F(u,v) ds .
\]

Here, \( F(u,v) \) is the slowness function \((1/c=1/\sqrt{\gamma RT})\) defined as an inverse of the speed of sound, \( u, v \) are the normalized coordinates of a point in the domain, and \( s, L \) are the normalized and real distance between two points, respectively. The slowness function means that the time required for a sound pulse to travel a unit length. One can express \( F(u,v) \) by an approximation with a finite number of added terms as

\[
F(u,v) = \frac{1}{c(u,v)} \approx \sum_{n=1}^{q} A_n \psi_n ,
\]

where \( \psi_n \) is the selected basis function, \( q \) the number of added term, \( n \) the order of basis function, and \( A_n \) the unknown coefficient. The spatial interpolation method with radial basis function is selected to approximate the distribution of speed of sound in the target field\(^3\). The selected basis function is multiquadric function which assumes that characteristics of target field should be smooth\(^4\). The definition of multiquadric function is as follows:

\[
\psi_n = \sqrt{(u-(u_j)_n)^2 + (v-(v_j)_n)^2 + \sigma^2} .
\]

Here, subscript \( j \) is the coordinates of interpolation points, \( n \) the \( n \)-th interpolation point, and \( \sigma \) the shape factor, which is a weighting on the selected distance\(^5\). The shape factor is determined based on the mean distance between each interpolation points\(^6\). Finally, the approximated retarded time between all possible sensor/actuator pairs can be expressed as follows:

\[
t_d = L \int_0^1 \left( \sum_{n=1}^{q} A_n \psi_n \right) ds = \sum_{n=1}^{q} A_n \Psi_n ,
\]

\[
\Psi_n = L \int_0^1 \sqrt{(u-(u_j)_n)^2 + (v-(v_j)_n)^2 + \sigma^2} ds .
\]
If there are multiple paths, \( p \), due to multiple sensor/actuator pairs at the wall, a matrix equation can be formulated as
\[
\{t_d\}_{p \times 1} = [\Psi]_{p \times q} \{A_n\}_{q \times 1}.
\] (5)

Here, \( \Psi \) is the transfer matrix which expresses the distribution of speed of sound on the field from the information of sensor position and acoustic path. The temperature field of the target plane is given by
\[
T(u, v) = \frac{1}{\gamma R} \left( \sum_{n=1}^{q} \{\Psi^n \} \{t_d\} \{y_n\}^{-2} \right).
\] (6)

The symbol ‘\(^\dagger\)’ denotes the Moore-Penrose generalized inverse.

### 2.2 Regeneration of retarded time vector

The spatial resolution of reconstructed field by using the acoustic pyrometry mainly depends on the number of added terms in the basis function. It means that the number of deployed interpolation points in the field. If one can employ the large number of added terms, the large number of interpolation points can be deployed in the field to ensure the high spatial resolution. However, the number of added terms is usually limited due to number of acoustic paths as expressed in the Eq. (5). Also, the number of acoustic paths depends on the number of sensors and actuators which cannot be easily increased due to practical limitations. In this study, to overcome the shortage of number of sensors, regeneration technique proposed in the acoustic holography is adopted for a calculation of retarded time.

It is possible to calculate the retarded time, \( t_{d2} \), for an arbitrary acoustic path by using the information of initially calculated temperature field with solution of Eq. (5). It can be expressed as
\[
t_{d2} = \int \sum_{n=1}^{q} A_n \sqrt{(u - (u_j)_n)^2 + (v - (v_j)_n)^2 + \sigma^2} ds,
\] (7)

where \( A_n \) is initially calculated coefficient vector, subscript \( j \) the coordinates of interpolation points, \( n \) the \( n \)-th interpolation point. Hence, the matrix equation in Eq. (5) can be reformulated by using the regenerated retarded time, \( t_{d2} \), as follows:
\[
\begin{bmatrix} t_d \\ t_{d2} \end{bmatrix} = \begin{bmatrix} \Psi_n \\ \Psi_{n2} \end{bmatrix} \begin{bmatrix} A_n \\ A_{n2} \end{bmatrix}.
\] (8)

Here, subscript 2 expresses the regenerated elements of matrix equation. The new distribution of temperature field can be obtained by using the solution of Eq. (8) and (6). By adding the regenerated retarded time into the original equation, it is possible to deploy the large number of interpolation points. However, the accuracy of restored field is mainly dependent on the accuracy of initially calculated temperature field. Therefore, it is important to get an accurate initial reconstruction result.

### 3. Simulation

#### 3.1 Calculation condition

A numerical example is prepared to simulate the reconstruction of the complicated temperature field by using the proposed regeneration technique. Two reference temperature fields having 1 (Case I) and 4 (Case II) local maxima are shown in Fig. 2. The retarded time between sensors is calculated by using the given reference field. Total 12 sensors are deployed in an equidistant manner for the calculation of initial temperature field as depicted in Fig. 3(a). Gaussian noise with 20
dB SNR is added to retarded time data to include the effect of noise. Total 16 interpolation points are selected by using the genetic algorithm to reduce the singularity of transfer matrix which can be a major source of reconstruction error. The condition number of transfer matrix is selected as a constraint for the genetic algorithm to minimize the singularity.

After calculation of initial temperature field, regeneration technique is applied to the sensor deployment as shown in Fig. 3(b). The retarded times in between real sensor and virtual sensor are calculated by using Eq. (7). Finally, the reconstructed temperature field with regeneration technique can be obtained by using Eq. (8).

![Figure 2](image-url)  
**Figure 2.** Reference temperature fields. (a) Symmetric field with 1 local maximum (Case I), (b) symmetric field with 4 local maxima (Case II).

![Figure 3](image-url)  
**Figure 3.** Layout of the deployed real sensors (●) and virtual sensors (▲).

### 3.2 Reconstruction results

Reconstruction result with and without the regeneration of retarded time is summarized in Table 1. For an initial calculation of temperature field, 12 sensors and 16 interpolation points are used and the modified Tikhonov regularization method is adopted to suppress the spurious effect of noise which induces the divergence of inverse solution. The regularization parameters are calculated by using the generalized cross validation (GCV) function. For a simple temperature field as Case I which is shown in Fig. 4(a) and 5(a), the reconstruction result between original and regenerated case shows little difference in the view point of RMS error and reconstructed pattern. Since the reference temperature field is very simple, the large number of interpolation points is not needed to obtain the reasonable reconstruction result. However, the difference between original and regenerated reconstruction result can be clearly seen for a complicated temperature field as Case II which is shown in Fig. 4(b) and 5(b). Although the difference of RMS error between original and regenerated case is not large, the multiple hot-spots are not clearly distinguishable for an original case as depicted in Fig. 4(b) due to the low spatial resolution induced by the small number of interpolation points in the field. It is not possible to deploy the enough number of interpolation points for an original case due to small number of acoustic paths related with number of sensors and actuators. However, a large number of interpolation points can be deployed in the field because of increased number of acoustic paths by applying the regeneration technique. Therefore, the multiple hot-spots can be clearly distinguishable as shown in Fig. 5(b) and RMS error is also decreased due to the high
spatial resolution. One can conclude that the proposed regeneration technique enables to obtain the high spatial resolution not adding the real sensors and actuators but regenerating the retarded time in between virtual sensors by using initially calculated temperature field.

Table 1. Results of temperature reconstruction.

<table>
<thead>
<tr>
<th>Tested temperature fields</th>
<th>E_{RMS} (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original</td>
</tr>
<tr>
<td>Symmetric field with 1 local maxima (Case I)</td>
<td>41</td>
</tr>
<tr>
<td>Symmetric field with 4 local maxima (Case II)</td>
<td>101</td>
</tr>
</tbody>
</table>

Figure 4. Reconstructed temperature without regeneration technique. (a) Case I, (b) Case II.

Figure 5. Reconstructed temperature result with regeneration technique. (a) Case I, (b) Case II.

4. Conclusions

The regeneration technique to obtain extra retarded time to increase the input information for an acoustic pyrometry has been studied. The retarded time is regenerated by using the initially calculated temperature field. As a numerical example, symmetric temperature fields having 1 and 4 local maxima are simulated. The results reveal that application of regeneration technique can increase the spatial resolution of reconstruction result of acoustic pyrometry. Therefore, a complicated temperature field can be successfully restored with limited number of sensors and actuators. A further study is needed to find out the relation between the number of deployed interpolation points and spatial resolution.

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