A STUDY OF VIBRATION CHARACTERISTICS OF PLANETARY GEAR TRAINS

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Epicyclic gear trains are widely used in aircrafts, automotive transmission, and agricultural equipment because of their high torque capability but in compact sizes. Various designs can be obtained such as gear ratios more than 100:1 and differential drives. However, unlike fixed-axis gears, the geometry and kinematics of planetary gears are quite complicated. The vibration analysis and condition monitoring of planetary gearboxes are not fully developed. In this research, three theoretical models for faulted sun, planet, and ring gears are analyzed and the signature frequencies of the three cases are derived, respectively. Simulations in the time domain were conducted for each model to illustrate the impacts caused by faulted gears. The sidebands around gear mesh frequency due to the fault signature frequencies are also discussed accordingly. The existence of each signature frequency can be used to detect mechanical defects and prevent catastrophic consequences. Based on these theoretical models, experiments were carried out on a Drivetrain Diagnostics Simulator which is equipped with a one-stage planetary gearbox of 4.571:1 gear ratio with four planets. A sun gear with chipped tooth was tested. Vibration signal was acquired using accelerometer. Signal analyses in both the time and frequency domains validate the theoretical models.

1. Introduction

Gear trains can be classified into three categories: fixed-axis, epicyclic, and compound gears. Epicyclic gear trains can be further categorized as differential (2 DOF) and planetary (1 DOF). A planetary gear train consists of four components: sun gears whose axes are fixed, a carrier (also called arm) which rotates in space about the fixed axis, planet gears which rotate about the carrier, and frame and bearings. A planetary gear train has one and only one carrier which supports one or more planets. All sun gears and the carrier rotate about the same axis. Large speed reduction/torque increase can be obtained in a compact design. So they are widely used in industries, especially for heavy duty machines.

1.1 Transmission ratio of planetary gearbox

The calculation of transmission ratios can be found in any dynamics of machinery textbooks.\(^1\),\(^2\) Figure 1 illustrates the planetary gear train under study. The input is a sun gear whichmeshes with planet gears supported by the carrier. The output is the floating carrier. The planets also mesh with a fixed ring gears ($\omega_3 = 0$). Since the planets mesh with both the sun and ring gears, all gears must have the same module and pressure angle. For such a configuration, the rotating speeds of carrier ($\omega_c$) and planets ($\omega_2$) are given by

$$\omega_c = \frac{\omega_1 N_1}{N_1 + N_3}, \text{ and } \omega_2 = -\frac{\omega_1 N_1}{2N_2}$$ (1)
where $\omega_1$ is the input angular speed, $N_1, N_2, N_3$ are the numbers of teeth of the sun, planet, and ring gears, respectively. From these formulas it can be seen that the carrier rotates in the same direction as the input sun gear with a slower speed. On the other hand, the planet rotates in the opposite direction of the sun gear and the carrier.

![Planetary gear configuration under study.](image)

**Figure 1.** Planetary gear configuration under study.

### 1.2 Mesh frequency and sidebands

Gear mesh frequency is defined as the number of teeth that enters mesh per unit time. For a pair of fixed-axis gears, as all the speeds are relative to zero, the mesh frequency is simply $\omega_m = N_1 \omega_1 = N_2 \omega_2$. For the planetary gear train shown in Figure 1, however, the carrier’s speed is the reference. As explained above, the carrier is in the same direction as the sun gear, while the planet is in the opposite direction. Therefore, considering the relative speed, the mesh frequency between the sun and planet gears is

$$\omega_m = N_1 (\omega_1 - \omega_c) = N_2 |\omega_2 - \omega_c| = N_3 |\omega_3 - \omega_c| = \frac{N_1}{N_1 + N_3} N_1 \omega_1 = N_3 \omega_c. \quad (2)$$

Notice that the absolute values are used in order to keep the result positive.

The shaft rotation signal causes amplitude modulation to the meshing frequency component, resulting in sidebands on both sides of the gear mesh frequency in the spectrum. Since the sidebands are associated with the shaft rotation, they carry great information for the fault diagnosis purpose. McFadden and Smith explained the asymmetry of the sidebands in planetary gear vibration. Vicuna also derived the same conclusion using the Fourier analysis. However, these authors only considered the cases where all gears are healthy. In this research, theoretical models of three cases have been derived and experiments were conducted to validate the models.

### 2. Theoretical models of faulted gears

Gear meshing causes vibration due to the forces interacting between two engaged teeth. However, if the teeth are healthy, the vibration under normal operational condition is relatively low. On the other hand, a faulted tooth is less rigid than healthy teeth. So the meshing of the faulted tooth with another gear will cause impact-like high vibration, more severe the fault, higher the vibration response. For the fault diagnosis purpose, the time interval between impacts and their signature frequencies are needed. These frequency components identify the existence of faulted gears and their amplitudes indicate the severity of the fault.
2.1 Signature frequency of a faulted sun gear

As shown in Figure 1, the planets are evenly distributed over $2\pi$. If there are $K$ planets, the angular distance between two planets is $\Phi = 2\pi/K$. Since the sun gear rotates faster than the carrier, the time interval for the faulted tooth to travel between two planets is $\Delta T_{FS} = \Phi / (\omega_1 - \omega_c)$, where the subscript stands for “faulted sun.” The associated signature frequency is then

$$\omega_{FS} = K \left( \omega_1 - \omega_c \right) = \frac{N_1 \omega_1 - N_3 \omega_c}{N_1 + N_3} \quad (3)$$

The impact trains occur to all the planets in a cycle. If we assume $K = 3$, the impact trains can be illustrated in Figure 2. The horizontal axis is relative time normalized by the period of the carrier $T_c$. The time interval between two adjacent impacts (occurred to two adjacent planets) is $\Delta T_{FS}$. So after $K$ impacts, the next impact will occur back to the first planet. The number of impacts to a specific planet per $T_c$ depends on the values of $N_1$ and $N_3$. The height of the impact indicates the severity of the fault. In the illustration shown below, the amplitude is assumed to be unity for healthy tooth between impacts.

![Figure 2. Illustration of impact modulation to each planet.](image)

In addition to the impact modulation, as the carrier rotates, the distance between the vibration source and the fixed-mounted transducer varies for each planet which also causes an amplitude modulation once per revolution of the carrier. By considering both the amplitude modulations, each planet’s vibration signal measured by the accelerometer can be illustrated in Figure 3. The numbers are the occurrence order of impacts. Figure 4 shows the combined signal measured by transducer. As it can be seen, the two amplitude modulations affect each other. Not all the impacts are sufficiently captured by the transducer. Only those happened close to the transducer are strong enough due to short transmission path. In the combined signal, the apparent period is some integer multiple of $\Delta T_{FS}$ although the multiplier is related to $\Phi$. This multiplier may or may not be $K$, depending upon the phase relationship between these impacts and $\Phi$.

The signature frequency in Eq. (3) is typically not an integer multiple of $\omega_c$. From the diagnosis perspective, we just need to monitor if new frequency components at $\omega_{FS}$ as well as the sidebands associated with $\omega_{FS}$ appear in the spectrum. The experimental results are shown in Section 4 below.
2.2 Signature frequency of a faulted ring gear

If the ring gear has a faulted tooth, an impact will be generated every time when a planet passes it. Since the planets are evenly distributed on the carrier, the time interval between two planets to hit the faulted tooth on the ring gear is $T_{FR} = \frac{\Phi}{\omega_c}$, where the subscript stands for “faulted ring.” The corresponding signature frequency is then

$$\omega_{FR} = K \omega_s = \frac{KN_1 \omega_1}{N_1 + N_3} \tag{4}$$

Unlike the impacts due to a faulted sun gear, the impact happens to a specific planet only once per $T_c$ shown in Figure 5. Also, the transmission path between the faulted tooth and the fixed-mounted transducer is constant. Therefore, the $K$ impact modulations have the same phase difference between the normal vibration signals caused by planet-ring meshing. For the same reason, they follow the same asymmetric pattern discussed by McFadden and Smith. However, we need to pay attention to the increase in amplitudes of sidebands.
2.3 Signature frequency of a faulted planet gear

Refer to Figure 6 for the kinematics analysis for a faulted planet gear. At time \( t \), assume the faulted tooth on a planet gear is at point \( A \) engaged with the ring gear which causes an impact and the contact point between the sun and planet is \( B \). The next impact will then happen at \( t + \Delta T_{fp} \) when the faulted tooth hits the sun gear at point \( A' \). In the meantime, point \( B \) moves to \( B' \).

The arc length that both \( A \) and \( B \) travel is half the circumference of the planet gear, or \( \overline{AB} = \pi r_2 \). As the planet gear rolls without slipping on the ring gear, the arc length \( \overline{AB'} \) on the ring gear is also \( \pi r_2 \). The corresponding angle is easy to obtain: \( \Psi = \frac{\pi r_2}{r_3} = \frac{\pi N_2}{N_3} \). The time interval between two impacts, which occur on the ring and sun gears alternatively, is \( \Delta T_{fp} = \Psi/\omega_c \), where the subscript stands for “faulted planet.” The associated signature frequency is then

\[
\omega_{fp} = \frac{2N_1 \omega_c}{N_2} = \frac{N_1 N_3 \omega_c}{N_2 (N_1 + N_2)} \tag{5}
\]

Figure 6. Kinematics analysis for faulted planet gear.

Eq. (5) is not an integer multiple of \( \omega_c \), so it is a new frequency component that we need to monitor in the spectrum. Also, the impact modulation only applies to the faulted planet gear as
shown in Figure 7. Red arrows mark the impacts. The time interval between two impacts is \( \Delta T_{FP} \) calculated above.

![Figure 7. Illustration of impact modulated signal of a faulted planet gear.](image)

3. Experimental setup

Experiments were conducted on the Drivetrain Dynamics Simulator (DDS). It consists of one-stage planetary gear and a two-stage spur gearbox. The DDS, as shown in Figure 8, is driven by a 3-phase 3 HP induction motor controlled by a VFD (variable frequency drive). The planetary gearbox has a 28-tooth sun gear, four 36-tooth planets, a 100-tooth ring gear (all being standard module 1 gears), and a floating carrier which is the output. The gear ratio calculated using Eq. (1) is

\[
r = \frac{\omega_1}{\omega_c} = \frac{N_1 + N_3}{N_1} = \frac{28 + 100}{28} = 4.571
\]

The gear mesh frequency is

\[
\omega_m = \omega_1 \frac{N_1 N_3}{N_1 + N_3} = 21.875\omega_1
\]

It is also easy to check that the concentric, homogeneity, and neighbour conditions are satisfied. Figure 9 shows the assembly view of the planetary gearbox. The carrier shaft is used to drive the two-stage spur gearbox. The module is 1.5, and the two stage gear ratios are 100:29 and 90:36, respectively. The two mesh frequencies of the spur gear transmission are 6.344\(\omega_1\) and 2.284\(\omega_1\), respectively.

![Figure 8. Drivetrain Dynamics Simulator.](image)
A PCB U352 C68 accelerometer was mounted on the top of the planetary gearbox to measure the vibration. Data acquisition was performed using LabVIEW with the data acquisition card NI PCI-4472. The data acquisition device provides IEPE power supply to the accelerometer. The acceleration signals are sampled with AC coupling which removes the DC offset. The sun gear with a chipped tooth was installed to the input shaft. The VFD was set to 112.56 RPM (19.876 Hz).

4. Results and Discussion

Figure 10 illustrates the time waveform of the acceleration signal. As expected in Section 2.1, some impacts can be clearly seen. In addition, similar to the simulation in Figure 4, only impacts close to the fix-mounted accelerometer are clearly shown in the combined signal due to the amplitude modulation of transmission path. As the input shaft’s rotation frequency is 19.876 Hz, the time interval between two impacts is

$$\Delta T_{FS} = \frac{1}{4(19.876 - 0.2188 \times 19.876)} = 0.016 \text{ s}.$$  

The five impacts marked by the red cursors occur at 0.1049 s, 0.1688 s, 0.2658 s, 0.4086 s, and 0.5041 s. The time intervals between two impacts are labelled in Figure 10. If we relate these time intervals to $\Delta T_{FS}$, they are $4\Delta T_{FS}$, $6\Delta T_{FS}$, $9\Delta T_{FS}$, and $6\Delta T_{FS}$, respectively. Agreed with the theoretical analysis in Section 2.1, the apparent time interval is some integer multiple of $\Delta T_{FS}$ while the multiplier is related to the phase relationship between these impacts and $\Phi$.

Analyses were also conducted in the frequency domain to further validate the theoretical model. In order to show the spectrum clearly, two frequency ranges of the same power spectrum are plotted in Figure 11 separately. In Figure 11 (a), the red cursor indicates the input shaft frequency at 19.876 Hz, while the blue cursor at 62.104 Hz is the signature frequency of the chipped tooth. According to Eq. (3),

$$f_{FS} = \frac{4 \times 100 \times 19.876}{28 + 100} = 62.113 \text{ Hz}.$$  

The component at 59.64 Hz next
to the blue cursor is the third harmonic of the shaft frequency. The high component at 126.13 Hz is the mesh frequency of the 1st stage spur gear (6.344 × 19.876 = 126.09 Hz). In Figure 11 (b), the high component at 434.81 Hz (red cursor) is the mesh frequency of the planetary gearbox (21.875 × 19.876 = 434.79 Hz). The cluster of close sidebands is due to the carrier’s speed. New sideband components, marked by blue cursors, are all 62 Hz apart, which represent the signature frequency of faulted sun gear. For comparison purpose, it is worth mentioning that the frequency components at 62.113 Hz or the sidebands 62 Hz from the mesh frequency are not shown in the spectrum of healthy gears. Therefore, both the time waveform and the spectral analysis prove the theory developed in Section 2.

![Power spectrum](image)

**Figure 11.** Power spectrum of vibration signal with a faulted sun gear, (a) frequency range from 0 to 200 Hz, (b) frequency range from 200 to 600 Hz.

5. Conclusions

Three theoretical models for faulted sun, ring, and planet gears were developed. As the meshing of a faulted tooth with a healthy gear causes impacts, the time interval and signature frequency of each case were calculated. In vibration spectra, components at these signature frequencies identify the existence of faulted gears and their amplitudes indicate the severity of the fault. Experiment was carried out using a sun gear with a chipped tooth. The time waveform depicts impacts and the time intervals between them agree with the theoretical model. The frequency domain analysis also verifies the signature frequency. Due to amplitude modulation, new sidebands associated with these signature frequencies also appear around the gear mesh frequency.

REFERENCES