PREDICTION OF ACOUSTIC SURFACE IMPEDANCE OF BULK REACTING LINING WITH GRAZING FLOW

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A transfer matrix methodology to determine the acoustic properties of multi-layered absorbers in different environments is proposed in this paper. The methodology allows inclusion of grazing flow and the boundary layer effects on the surface properties, avoiding the need of several complementary methods to obtain the surface properties of a sound reducing material in a specific environment. The predicted surface properties are given as a function of angle of sound incidence, allowing for arbitrary sound fields to be simulated. This is a useful tool in for example automotive applications such as engine bays where multi-layered bulk reacting sound absorbing materials are exposed to flow and complex sound fields. Correct prediction of the acoustic performance of absorbing material where flow is present enables optimization of the noise reducing components for which conflicting requirements such as weight and space constraints are present as well.

1. Introduction

A large variety of sound absorbing materials are used in automotive applications to reduce noise levels from different vehicle sources. The noise reduction obtained from the material is not an intrinsic material property since it depends both on the material itself, the sound field\(^1\) and the environment where it is positioned\(^2\,\,^5\). The noise reducing performance of a material in a specific environment can be described in several ways, either by the reflection or absorption coefficient or the surface impedance. The surface impedance carries more information in its phase and amplitude representation as opposed to the energy based absorption coefficient. Due to this, the surface impedance is preferably used in simulations where detailed information of the influence of the material on individual sound waves from arbitrary incidence angles is needed.

The noise reducing performance of a material in different sound fields can be determined from measurements\(^6\,\,^7\) or from either of the numerous simulation techniques available today: for example, finite element methods (FEM)\(^8\) or transfer matrix methods (TMM)\(^9\). The choice of simulation method of course depends on material complexity, on the environment and on the calculation time in relation to required accuracy. The advantages of the TMM are the calculation speed and simplicity to find the properties of a multi-layered structure while the properties of each layer of the structure is known\(^9\). One limitation of this method is the forced homogeneity of each layer. Numer-
ous publications on solutions for inhomogeneous layers exist\textsuperscript{10,11} and are used in some fields of research. In this paper, an alternative method is proposed, which is numerically straightforward.

The influence of environmental factors on the surface impedance, such as boundary layers induced by flow above the surface, can also be addressed in the TMM. In duct and aircraft applications, the impact of the boundary layer developed at the surface is included via a modified surface impedance at the duct walls\textsuperscript{12-16}. There, the absorbing linings are often locally reacting and in this kind of modeling the surface impedance without flow needs to be determined by a separate numerical, analytical or experimental method. The flow speeds in these applications are high (ex. Mach number around 0.5) and the sound is grazing the surface, diverging significantly from the case for sound absorbers in many automotive applications. Inside the engine bay of a passenger car or a truck, the absorbing materials are exposed to flow and complex sound field. The flow speeds are typically of order Mach number < 0.2, the absorbers are often thin bulk reacting material and the sound waves impinge the absorber from various angles of incidence. The flow has significant effect on the absorption properties even at these relatively low flow speeds\textsuperscript{2} why this effect is relevant to include. By continuing the use of the transfer matrix for the boundary layer together with the transfer matrix for the material, a total description of the performance of the lining is obtained.

This paper describes a general method to predict the surface impedance of a bulk reacting lining for different angles of incidence and environmental impacts, such as grazing flow. The method is based on the transfer matrix approach determining the surface impedance for multi-layered structures taking the bulk reaction of the material and the environmental factors in account. The method is easy to implement and to adapt to the specific environment.

The classic transfer matrix method is described in the next section, followed by the methodology to model a boundary layer with the same approach. The methodology is applied to a simple absorber, placed in three different environments to show the simplicity to alter the method for the environmental characteristics. Finally, some comments on the results and the method itself are given.

2. Transfer matrix method

The technique of relating properties in two positions by a matrix is a common way to handle problems in many areas of mechanics and physics. In acoustics, this is for example used to couple the acoustic state at the in- and outlet of a duct element\textsuperscript{17}, the dynamic response of a structure\textsuperscript{18} and to determine sound transmission through a structure. The latter of these examples is called the transfer matrix method (TMM)\textsuperscript{8}. This is a useful frequency domain method determining transmission and absorption properties of multilayered structures in plane acoustic wave fields. The incorporated layers are normally translationally invariant and can be of arbitrary nature: solids, porous material, fluids etc. By reducing the propagation in the structure to a matrix, the surface impedance, reflection and transmission coefficients are easily obtained given the boundary conditions. A general case of plane wave incidence at an angle $\theta$ at a multilayered structure is shown in Fig. 1 where the angle of incidence and the transmission media are arbitrary

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure.png}
\caption{General case of plane sound wave incidence at angle $\theta$ at a multi-layered structure.}
\end{figure}
By using the state vector representation (the so called Stroh formalism\textsuperscript{19}) the vector \( S \) represents the acoustic properties in each position and are related as

\[
[S]_2 = [T][S]_1, \quad (1)
\]

by the transfer matrix \( T \) between point 1 and 2. The transfer matrix between point \( l \) and \( n \) is simply the product of the transfer matrices of each layer if the layers are of the same type (if they are of different type, this is solved by using coupling matrices, see ref \textsuperscript{9}). In this paper, all layers are represented as fluids where the acoustic state is determined by the pressure and normal particle velocity \( (S) = [p; v_z] \). The classic TMM is applied on homogeneous layers, and the transfer matrix for such structures is given in Section 2.1. When layers in the structure has depth dependent variables an extension to the classic method is used and this extension is described in Section 2.2.

### 2.1 Layers with constant properties

In a fluid layer the acoustic state is described by the pressure and normal particle velocity. The sound field is assumed to consist of plane harmonic waves on the form

\[
p(x, z, t) = Pe^{i\omega t} e^{-ik_xx - ik_zz}, \quad (2)
\]

where \( P \) is a wave of amplitude \( P \) in the positive \( x \)- and \( z \)-direction, \( k_x \) and \( k_z \) are the wave numbers and \( \omega \) is the angular frequency. The transfer matrix for a fluid layer of thickness \( d \) is derived by writing the pressure (according to (2)) and the particle velocity in two points and it can be found in any textbook\textsuperscript{20} and is given by

\[
[T] = \begin{bmatrix}
\cos(k_zd) & i\frac{\omega P}{k_z} \sin(k_zd) \\
-i\frac{k_z}{\omega P} \sin(k_zd) & \cos(k_zd)
\end{bmatrix}, \quad (3)
\]

where \( \rho \) is the density of the fluid.

### 2.2 Inhomogeneous layers

For a parameter changing in the \( z \)-direction in a layer a constant matrix for the layer can not directly be stated on the form of Eq. (3). There are several approaches to find the transfer matrix for inhomogeneous layers\textsuperscript{10,11}, however, in this paper another approach is used on the case of boundary layer flow where the varying factor in the layer is the flow speed, \( V(z) \). The governing equations in the boundary layer flow are the linearized Navier-Stokes equations for the continuity of mass and momentum. This system of equations can be rearranged on the form

\[
\frac{d}{dz} [S] = [A][S] \quad (4)
\]

with the adiabatic pressure relation, and the matrix \([A]\) given as\textsuperscript{21}

\[
[A] = \begin{bmatrix}
0 & -i\rho(V(k_x - \omega)) \\
-i\frac{\rho c_0^2 V(k_x - \omega)^2}{\rho c_0^2(k_x - \omega)} & \frac{k_x}{\omega - \nu k_x} \frac{dV}{dz}
\end{bmatrix}. \quad (5)
\]

The solution to the system is obtained from the matrix exponential as
\[ [S]_n = \expm([A]d)[S]_0 \]  

when \([A]\) is independent of \(z\) on the distance \(d\) between \(z_0\) and \(z_n\). Here, \([A]\) is indeed a function of \(z\) imposing the need for an approximation. The value of the matrix at the midpoint of the layer is assumed to be the \([A]\) matrix representing the entire layer\(^1\). For this approximation to be valid for thick layers with steep gradients, discretization of the layer is performed with stepwise constant parameters and gradients. The transfer for each sub layer of thickness \(d_i\) is hence

\[ [T]_i = \expm([A(z_i)]d_i). \]  

This method is of second order, although only a few sub layers are needed for high accuracy. Higher order methods can also be applied to find \(T\). Then the information in several points in the layer is included\(^2\) and an even smaller number of sub layers are needed.

### 3. Application of the methodology

The method proposed in this paper can be applied to general problems involving layered structures. To give an example of the usefulness of the method, an absorbing porous material of thickness \(D\) in front of a rigid wall is exposed to three different ambient states, i.e., different external factors affecting the obtained performance of the absorber. The three applications are shown in Fig. 2: 1) quiescent air 2) grazing air flow above the surface developing a boundary layer on the surface 3) grazing air flow above the surface developing both a boundary layer on the surface and a flow inside the absorber.

The porous material is modelled as an equivalent fluid according to Delay-Bazley\(^2\) with flow resistivity \(\sigma_0 = 5430 \text{ Ns/m}^4\). The flow resistivity of the porous material is chosen according to reference 3 where data for the change in flow resistivity due to internal flow is given.

![Figure 2](image)

Figure 2. Three examples of different ambient conditions for the same absorber. The incident medium (Inc.) is air, the layers (L1-L3) are described in Table 1-3 and S1-S4 are the state vectors.

The three examples in Fig. 2 is hence the same material placed in three different environments, all solvable with the transfer matrix methodology with different number of layers. The layers in the three examples are described in Table 1 – 3 below. The first case consists of one layer with a known transfer matrix \(T\) (Eq. (3)) with parameters according to Table 1.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Type</th>
<th>Mach no</th>
<th>Thickness [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>porous</td>
<td>0</td>
<td>0.02</td>
</tr>
<tr>
<td>Incidence</td>
<td>air</td>
<td>0</td>
<td>Semi infinite</td>
</tr>
</tbody>
</table>

The absorber in case 1 is exposed to grazing flow in the second case, i.e. a boundary layer is developed above the absorber surface. This case is modelled by two layers described in Table 2.
Layer 2 is discretized in sub layers with the method in Section 2.2. The boundary layer profile is chosen to be linear for simplicity since the shape of the boundary layer profile has small effect on the impact of the boundary layer on the surface properties. The third case incorporates flow inside the absorbing material, yielding three layers in the transfer matrix calculations, see Table 3.

Table 3. Variables for each layer in the problem in example 3.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Type</th>
<th>Mach no</th>
<th>Thickness [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>porous</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
<td>porous</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>3</td>
<td>air</td>
<td>0.02-0.2</td>
<td>0.02</td>
</tr>
<tr>
<td>Incidence</td>
<td>air</td>
<td>0</td>
<td>Semi infinite</td>
</tr>
</tbody>
</table>

Layer 3 is treated as layer 2 in case 2, with the velocity 0.715 m/s (M = 0.02 with sound speed c₀ = 343 m/s) at the surface. According to measurements the flow resistivity normal to the internal flow direction is increased as

\[ \sigma_{\text{flow}} = \sigma_0 + \sigma_i |v| \]

where v is the flow speed and \( \sigma_i \) is determined experimentally as 3070 Ns²/m⁵ i.e., \( \sigma_{\text{flow}} = 7625 \) Ns/m⁵ at the given flow speed. The specific impedance determined for this material with the equivalent fluid model is verified to measurements for the given flow speed above.

In this paper, the surface impedance, \( Z_s \), and the absorption coefficient

\[ \alpha(\theta) = 1 - |R|^2, \]  

versus the angle of incidence, \( \theta \), are of determined. The absorption coefficient in Eq. (8) is related through the reflection coefficient to the surface impedance as

\[ R(\theta) = \frac{Z_s \cos \theta}{Z_s + \cos \theta}. \]  

The boundary condition in all examples is a rigid backing, i.e., zero normal velocity at the backing (\( v_0 = 0 \)). This gives a simple expression for the surface impedance as

\[ Z_s(\theta) = \frac{p_s}{v_5} = \frac{T_{11} p_0 + T_{12} v_0}{T_{21} p_0 + T_{22} v_0} = \frac{T_{11}}{T_{21}}. \]  

Here, \( T_{11} \) and \( T_{21} \) are elements in the transfer matrix, \( p_0 \) is the pressure at the rigid backing and the index \( s \) represents surface parameters.
4. Results and discussion

The absorption coefficient and surface impedance for example 1 – 3 are shown in Fig. 3 and Fig. 4 respectively.

**Figure 3.** Predicted absorption coefficient for an absorber in front of a rigid wall in the example cases 1–3.

**Figure 4.** Predicted surface impedance for an absorber in front of a rigid wall in the example cases 1–3.

The absorption is seen to change substantially at certain ranges of the angle of incidence for both example 2 and 3 compared to the quiescent case in example 1. The effect of the boundary layer
at the surface is seen to increase the absorption for incidence angles $140 - 170^\circ$. The absorption is on the other hand decreased due to the boundary layer at angles between $0$ and $40^\circ$. Flow inside the absorber is seen to increase the flow resistivity, and the absorption is shown to increase in a large range of incidence angles. The effect of the boundary layer is seen to be more significant than the flow inside the absorber, although both effects are relevant to include.

This clearly shows the importance of environmental aspects for the obtained absorption coefficient and surface impedance for a specific absorber. All calculations are made with the same method, only varying the number and properties of the layers in the program, according to case 1, 2 and 3.

5. Conclusions

A method to reduce multi-layered structures to an angular dependent surface impedance has been described in this paper. The method is based on the classic transfer matrix method, now also including a module to include inhomogeneous layers. The method is easy to implement and to use, and is general for 2D multi-layered structures. It can easily be adapted to the specific case, for example to include boundary layer effects. The advantage of this method is that it enables prediction of the total acoustic surface properties including both material and external factors, avoiding the use of several methods to obtain this.

REFERENCES


