A VARIABLE STEP-SIZE FREQUENCY MISMATCH COMPENSATION ALGORITHM FOR NARROWBAND ACTIVE NOISE CONTROL

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In this paper, a variable step-size filtered-X LMS (VSS-FXLMS) algorithm is first introduced to replace the filtered-X LMS (FXLMS) algorithm in the main controller of conventional narrowband ANC system, and extensive simulations reveal that this ANC system may also suffer from significant performance degradation in the presence of Frequency mismatch (FM), although it acts better in the key characteristics on steady-state performance, convergence speed, and tracking ability. Second, a compensation block updated by LMS algorithm is incorporated to mitigate the influence of the FM. Third, a variable step-size compensation algorithm is derived to improve the efficiency of tracking the FM. Numerous simulations for stationary and nonstationary scenarios are conducted to demonstrate the superior performance of the modified compensation algorithm.

1. Introduction

Generated by rotating machines, noise signals such as fans on the airplane, engines in the car, large-scale cutting machine, etc. may usually be modelled as sinusoidal ones in many places. Narrowband active noise control is an attractive way to remove these undesirable acoustic noise, which may widely exist in our life and do harm to our health. Since 1970s, active noise control (ANC) system has been researched and carried out to solve the problem which was puzzled people a long period ¹,².

An ANC system is in charge of importing an equal-but-opposite phase source to restrain the unwanted noise in an acoustic region. To track the noise source in real time, a reference sensor is placed, and from the measurement a new sound signal was sent to the region where the noise needed restrained with an adjustable filter structure. In order to control the filter properties, an additional sensor is placed to measure an error feedback. In conventional ANC systems, a filtered-X least mean square (FXLMS) algorithm is used to update the controllers for its good applicability in terms of both noise reduction performance and implementation cost in real-life application ³,⁴. Other algorithms such as the filtered-X recursive least squares (FXRLS) and Kalman filtering based ones may also be applied²,⁹. Recently, a variable step-size filtered-X LMS (VSS-FXLMS) algorithms intro-
duced to cope with system in nonstationary noise environments, and the VSS ones acts better in the key characteristics on steady-state performance, convergence speed, and tracking ability.

In a narrowband ANC system, the signal frequencies from the reference sensor may be different from the true ones, which is referred to as frequency mismatch (FM), may significantly deteriorate the performance of system. All of the above-mentioned adaptive algorithms, including the newly proposed VSS-FXLMS algorithms based systems will suffer from the FM. The ill-effect of which has been simulated and analysed by many researchers, that even 1% FM could render the entire FXLMS ANC system totally useless. To mitigate the influence of the FM, many solutions are developed. Xiao proposed a series of modified ANC systems with a compensate block based on AR model updated by an LMS-like or RLS algorithm, which shows good performance but can only solve small FM in stationary noise environment. Jeon developed a frequency estimator based on a minimum variance distortionless response (MVDR) spectrum, which has advantages of accuracy and fast convergence at a sacrifice of computational burden.

In this paper, we propose a new narrowband ANC system to make up the performance degradations due to the existence of FM. A variable step-sized (VSS) based LMS-like algorithm is introduced into the compensation part so as VSS-FXLMS put into the controller part are adapted simultaneously for each channel in the new system. Results demonstrate that it has a brilliant performance in compensating an FM of even as much as 10%.

The rest of this paper is organized as follows. Section II introduces the conventional narrowband ANC system and uncovers its performance limitations in the presence of FM. The conventional and Proposed FM compensation blocks are depicted in Section III. Extensive simulation results are provided to show the effectiveness of the proposed systems in the presence of FM. Section IV concludes the paper.

2. ANC algorithm

A conventional ANC system is depicted in Fig. 1. The primary noise is expressed by

\[ p(n) = \sum_{i=1}^{q} a_i \cos(\omega_i n) + b_i \sin(\omega_i n) + v_p(n) \]  

(1)

Where \( q \) is the number of frequency components residing in the primary noise, \( \omega_i \) is the \( i \)th component, \( v_p(n) \) is a zero-mean additive noise with variance \( \sigma_p^2 \). The reference cosine and sine waves denoted by

\[ x^c_i(n) = \cos(\omega_i n), \quad x^s_i(n) = \sin(\omega_i n) \]  

(2)

Which are generated based on a sync signal captured by a non-acoustical sensor like a tachometer. Namely, the frequencies coefficients \( [a_i, b_i]_{i=1}^q \) of the reference waves may be identified in real time by using a linear regressive relationship between the sync signal, e.g., rotational speed, and the noise fundamental frequency. Coefficients \( [\hat{a}_i(n), \hat{b}_i(n)]_{i=1}^q \) are discrete Fourier coefficients (DFC) of the noise components. Here we reproduce two typical adaptive algorithms that are frequently used by Fig. 1.

2.1 Filtered-X LMS (FXLMS) algorithm

Among numerous adaptive algorithms proposed for various ANC systems, the FXLMS algorithm has been the most popular one, as it requires relatively fewer computational resources and enjoys practically good performance. The FXLMS algorithm for the above mentioned narrowband ANC system (\( i \)th channel) is given as follows

\[ \hat{a}_i(n+1) = \hat{a}_i(n) + \mu_{\alpha} e(n) \hat{x}_i(n) \]  

(3)

\[ \hat{b}_i(n+1) = \hat{b}_i(n) + \mu_{\beta} e(n) \hat{x}_i(n) \]  

(4)
Parameter $\mu_a, \mu_b$ are small positive numbers determined by user and called step sizes, the values of which directly influenced the convergence properties or dynamics as while as the steady-state performance of the whole narrowband ANC system. $\hat{x}_a(n), \hat{x}_b(n)$ are the reference signals filtered by the secondary-path estimate $\hat{S}(z)$, which effectively make up for the system variance brought about by the real secondary path. The parameters of the secondary-path estimate $\hat{S}(z)$ specified in advance by some parameter identification technique such as the LMS algorithm, wiener filter and so on.

$$\hat{x}_a(n) = \sum_{j=0}^{M-1} \hat{s}_j x_a(n - j)$$

$$\hat{x}_b(n) = \sum_{j=0}^{M-1} \hat{s}_j x_b(n - j)$$

Where $\hat{M}$ and $(\hat{s}_j)_{j=0}^{M-1}$ are the length and FIR coefficients of the secondary-path estimates $\hat{S}(z)$, respectively. Signal $e(n)$ is the residual noise, given by

$$e(n) = p(n) - \sum_{j=0}^{M-1} s_j \left[ \sum_{i=1}^{q} y_i(n - j) \right]$$

Where $M$ and $s_j, (j=0)_{j=0}^{M-1}$ are the length and FIR coefficients of the true secondary path $S(z)$, respectively.

![Fig. 1 The conventional NANC system (i th channel)](image)

### 2.2 VSS-FXLMS algorithm

VSS-FXLMS algorithms are proposed to ensure a relatively small steady-state error, convergence properties, and the ability of the coping with nonstationary. By employing $\mu_a(n)$ and $\mu_b(n)$, instead of fixed ones $\mu_a$ and $\mu_b$, for the $i$ th channel, the conventional FXLMS algorithm may be reproduced as follow

$$\hat{a}_i(n+1) = \hat{a}_i(n) + \mu_a(n) e(n) \hat{x}_a(n)$$

$$\hat{b}_i(n+1) = \hat{b}_i(n) + \mu_b(n) e(n) \hat{x}_b(n)$$

The cost function $J(n) = \frac{1}{2} e^2(n)$, then we can have the step size updating sequence:

$$\mu_a(n) = \xi_a \mu_a(n-1) + \eta_a e(n) \hat{x}_a(n) e(n-1) \hat{x}_a(n-1)$$

$$\mu_b(n) = \xi_b \mu_b(n-1) + \eta_b e(n) \hat{x}_b(n) e(n-1) \hat{x}_b(n-1)$$
\[
\mu_i(n) = \xi_a \mu_i(n-1) + \eta_a e(n) \hat{x}_i(n) e(n-1) \hat{x}_i(n-1)
\]  

(11)

Where \( \eta_a \) and \( \eta_b \) are both very small positive values, \( \xi_a \) and \( \xi_b \) will be close to but smaller than unit.

3. Compensation for FM

FM may significantly deteriorate the performance of systems mentioned above, and none of them is able to suppress primary noise ideally, making compensation for FM necessary. The robustness nature of adaptive algorithm recommend itself to be a promising means to cope with FM.

3.1 Conventional FM compensation algorithm (C-FMC)

In his analysis on FIR type ANC, Xiao proposed a second-order AR model to pre-process reference signal, enabling the whole system to cope with FM to some extent. Fig. 2 shows the structure of the narrowband ANC system with conventional FM compensation part. Using first-order FIR filter, or frequency-phase controller, reference signal of \( i \)th channel can be expressed as follow:

\[
x_a(n) = -c_i(n)x_a(n-1) - x_a(n-2), \quad n \geq 2
\]

(12)

\[
x_h(n) = -c_i(n)x_h(n-1) - x_h(n-2), \quad n \geq 2
\]

(13)

Where frequency-related parameter \( c_i(n) \) is obtained though iteration of a LMS-like algorithm.

\[
c_i(n+1) = c_i(n) - \mu_i e(n) \left[ \hat{a}_i(n) \hat{x}_i(n-1) + \hat{b}_i(n) \hat{x}_i(n-1) \right]
\]

(14)

Where \( \mu_i \) is the related step size of \( c_i(n) \), the essence of the above equation is to minimize target function \( e^2(n) \) by employing steepest descent principle. The original value of \( c_i(n) \) is set to \(-2\cos(\omega_i)\). Frequency Mismatch is defined as blow:

\[
\Delta \omega_i = \frac{\omega_i - \omega_{p,i}}{\omega_{p,i}} \times 100(\%)
\]

(15)

\[\text{Sync Signal} \]

\[\text{FXLMS} \]

\[\text{Secondary Path} \]

\[\text{LMS} \]

\[\text{ANC system based on FXLMS algorithm with conventional FM compensation}\]

*Fig. 2 ANC system based on FXLMS algorithm with conventional FM compensation*. 

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The compensation method described above is able to ameliorate the effect FM imposed on the system, meanwhile unable to cope with nonstationary signal, e.g., mutation of signal amplitude, but that does not forbid us from integrating it into VSS-FXLSM algorithm to solve the FM problem. Nonetheless, the combination of a second-order AR model pre-process with VSS-FXLSM controller is insufficient to solve nonstationary FM, especially when FM exceed 2%, the convergence speed will be too slow to track and compensate the system, naturally makes a faster compensation method to improve the overall system performance in need.

### 3.2 Variable step-size FM compensation algorithm (VSS-FMC)

Inspired by the usefulness of variable step-size means, we integrate it into FM compensation process, in other words, substitute the fixed step-sizes of the AR model’s updating equation into variable ones, then we receive a variable step-size FM compensation algorithm (VSS-FMC). And the new cost function is blew

$$J_c(n) = \frac{1}{2} e^2(n)$$

The relevant gradient can be expressed as such

$$\nabla_c J_c(n) = -e(n) \frac{s_c \partial y_c(n)}{\partial c_c(n)}$$

$$= -s_e e(n) \left[ \frac{\partial y_c(n)}{\partial x_c(n)} \frac{\partial x_c(n)}{\partial c_c(n)} + \frac{\partial y_c(n)}{\partial \hat{y}_c(n)} \frac{\partial \hat{y}_c(n)}{\partial c_c(n)} \right]$$

$$= s_e e(n) [\hat{a}(n)x_\alpha(n-1) + \hat{b}(n)x_\beta(n-1)]$$

Resorting to steepest descent, one yield the iterative equation of $c_c(n)$

$$c_c(n) = c_c(n-1) - \mu_c(n) \nabla_c J_c(n)$$

$$= c_c(n-1) - \mu_c(n)e(n)[\hat{a}(n)x_\alpha(n-1) + \hat{b}(n)x_\beta(n-1)]$$

Similarly, update equation of $\mu_c(n)$ can be expressed as

$$\mu_c(n) = \xi_c \mu_c(n-1) - \eta_c \nabla_{\mu_c} J_c(n)$$

$$= \xi_c \mu_c(n-1) - \eta_c e^2(n)[\hat{a}(n)x_\alpha(n-1) + \hat{b}(n)x_\beta(n-1)]$$

The new algorithm inhabits fast convergence speed for the introduction of variable step sizes means, whose integration with main controller will not debase the steady-state performance of the whole system.

### 4. Simulation

In this section, VSS-FMC is integrated with FXLMS, FXRLS and VSS-FXLSM algorithm respectively, and compared with the conventional FMC based ones in the case of 1%, 5%, and 10% FM.

Secondary path is generated by MATLAB with a cut-off frequency of 0.4π, and the estimate of secondary path based on AR model is obtained by off-line LMS algorithm, whose coefficients are the mean of the last 2000 points of iteration adaptation process.

In the middle of the iteration, abrupt changes are intentionally caused to happen with the DFCs of the primary sinusoidal noise. The frequencies of reference signal are kept steady all the time to simulate the time delay of physical sensor, namely, the inevitable frequency mismatch (FM) of sensor measurement.

In Fig. 3, performance comparisons in the case of 1% FM are presented, with a), c), e) denoting convergence performances of FXLMS, FXRLS and VSS-FXLSM integrated with C-FMC means respectively, while b), d), f) present the their performance in combination with the new VSS-FMC.
method. Similarly, Fig. 4 presents the comparisons of coefficient convergence trend of the different conditions mentioned in Fig. 3, Fig. 5 exhibits performances of VSS-FXLMS algorithm in combination with C-FMC and VSS-FMC in the case of 5% FM. Fig. 6 shows the brilliant performance of the proposed system in the case of 10% FM.

Fig.3 Comparisons between algorithms for FM 1% with DFC jumps
As we can see from the simulation results:

- Main controllers of FXLMS, FXRLS and VSS-FXLSM work very well in the case of 1% FM, namely, a stationary noise condition. But in condition of nonstationary, or 5% FM, though FXRLS and VSS-FXLSM algorithm work as usual, convergence of FXLMS algorithm is too tardy to track and control the primary noise, as showed in Fig. 3.
- As can be seen from Fig. 3, the steady-state errors of FXLMS and VSS-FXLSM in the condition of both stationary and nonstationary are reduced when they are working with VSS-FMC.
• In the condition of nonstationary (FM=5%) , all the three algorithms with C-FMC are too slow to convergence; when VSS-FMC is employed, VSS-FXLMS shows the much lower steady-state error and faster convergence, as showed in Fig. 5. Numerous simulations reveal that the integration of VSS-FMC and VSS-FXLMS still able to work stably even FM approaching to 10% as showed in Fig. 6.

• Pay attention to Fig. 4 and Fig. 5, and we can see that the VSS-FMC makes contribution to the improvement of the performance of the whole system lies in its cooperation with VSS-FXLMS. Their cooperation not only leads to a faster convergence, but also empower the main controller step size more control over the entire system.

• The combination of VSS-FMC and VSS-FXLMS processes steady-state performance and convergence speed equal to that of FXRLS, but a better tracking ability, and much less computational complexity.

5. Conclusion

A frequency mismatch compensation method is proposed for narrowband ANC. This VSS-FMC block combined with the VSS-FXLMS based main controller is capable of suppressing the influence of FM in both stationary and nonstationary environment, and works considerably better than the conventional FXLMS or FXRLS based robust ANC systems. Extensive simulations are conducted to demonstrate that the proposed system performs much better tracking ability and faster convergence in the suppressing of FM as much as 10%.

REFERENCES