STRUCTURE-BORNE SOUND TRANSMISSION ACROSS PERIODIC BOX-LIKE STRUCTURES AFFECTED BY SPATIAL FILTERING

Carl Hopkins and David Wilson
Acoustics Research Unit, School of Architecture, University of Liverpool, Liverpool, UK

e-mail: carl.hopkins@liv.ac.uk

Repeating box-like structures formed from many connected plates can be found in many engineering structures such as buildings or ships. This paper concerns bending wave transmission across such structures in the audio-frequency range. Using an idealised building as an example, ray tracing is used to demonstrate that the angular distribution of power incident on the plate junctions differs significantly from the diffuse field assumption that is commonly made in SEA. Therefore to tackle the prediction of bending wave transmission, computationally efficient ray tracing has been developed for inclusion in Advanced SEA (ASEA) models to account for the spatial filtering that occurs across successive junctions of coupled plates. Comparison of ASEA with finite element models shows that ASEA can give significant improvements over SEA when there are large numbers of structural junctions between the source and receiver plates.

1. Introduction

Many engineering structures such as buildings or ships are formed from many connected plates which result in a periodic box-like structure. This paper is concerned with the low- and mid-frequency ranges in which the plates only support local bending modes. Propagation across successive, nominally identical, junctions causes spatial filtering of the wave field. This results in non-diffuse vibration fields on plates that are not directly connected to the source plate.

To account for spatial filtering and the existence of non-diffuse vibration fields, Langley\(^1\) proposed an alternative to SEA for the prediction of high-frequency vibration, Wave Intensity Analysis (WIA). This used a finite Fourier series to represent the directional dependency of the wave intensity. Heron\(^2\) proposed an alternative approach to prediction using a ray tracing approach, which was referred to as Advanced Statistical Energy Analysis (ASEA). This was primarily developed to allow the inclusion of tunnelling mechanisms between indirectly-connected subsystems but as with WIA it also accounts for spatial filtering, non-diffuse vibration fields and propagation losses. ASEA and WIA both converge on the same result. Heron noted that implementation of ASEA for coupled plates ‘could well turn out to be computationally expensive’ compared with classical SEA (i.e. using wave theory to calculate the coupling loss factors) due to the ray tracing requirement. For this reason an alternative approach, referred to as ‘beam tracing’ is introduced in this paper to reduce computation times.
2. Advanced SEA

The main principles of ASEA were introduced by Heron\(^2\). In ASEA a distinction is made between available power per unit modal energy, \( P \), and unavailable power per unit modal energy, \( Q \). Classical SEA purely considers available power and it is assumed that all power transfer occurs between available power per unit modal energy in one subsystem and available power per unit modal energy in the same or another subsystem. Unavailable power describes power losses within subsystems which will not be available for further transmission; this occurs due to internal losses as energy propagates across a subsystem. Similarly, the modal energy is divided into available modal energy, \( e \) and unavailable modal energy, \( d \). Although the latter is referred to as modal energy it actually refers to energy dissipated as the waves propagate across the subsystem and does not involve the subsystem modes. The available modal energy corresponds to the stored modal energy in a subsystem that is considered in classical SEA. The modal energies are related to the input power through the equations

\[
Ae + Me = P \tag{1}
\]

\[
Be + Md = Q \tag{2}
\]

where matrix \( B \) contains the free power to fixed power transfers per unit modal energy, and the elements of \( A \) contain the free power to free power transfers per unit modal energy. In the absence of free to fixed power transfers the ASEA equations are identical to classical SEA. For rain-on-the-roof excitation which is considered in this paper, \( Q = 0 \); hence

\[
(M + A)(M - B)^{-1}M(e + d) = P \tag{3}
\]

The main procedural aspects needed to implement ASEA are given by Heron\(^2\) and Yin and Hopkins\(^3\). Previous work\(^3\) using ASEA on two coupled plates tracked the power flow using a ray tracing procedure; these were rectangular plates for which each plate had only a single junction line and three perfectly reflecting boundaries. However, applying ray tracing to a large number of coupled plates would result in excessive computation times. For example even with just a few coupled plates, each time the rays intersect the junction line, the number of rays doubles for an L-junction (two plates), and trebles for a T-junction (three plates). To reduce the calculation time, an alternative method is introduced here which is referred to as ‘beam tracing’. Rather than divide the perimeter into small segments of width, \( dL \), and carry out numerical integration around the perimeter of the source plate, it is more efficient to divide the source plate into its constituent edges and use a beam tracing approach to determine the integrals. Hence the tracing of individual rays at each segment \( dL \) along a single edge of the source plate is replaced by a pair of rays at each end of the edge which defines a ‘beam’ of rays. However there will still be an exponential growth in the number of beams with increasing ASEA iteration number. This occurs because transmitted and reflected beams are generated each time a beam ‘illuminates’ the junction line. The ASEA solution converges when the number of ASEA iterations approaches the total number of subsystems in the model, hence, for large numbers of coupled plates there is a need to reduce the growth in the number of beams with ASEA iteration number otherwise the total number of beams will rapidly become unmanageable. This is achieved by combining beams that are incident on a junction line with the same angle of incidence into a single beam.

When a beam travels across a subsystem, different sections of the beam can ‘illuminate’ different edges and they will generally travel different distances before reaching the perimeter of the plate. As an example, consider the assembly of rectangular plates shown in Fig. 1. The two rays comprising the beam travel from points a and c of one edge, and intersect the perimeter again at points f and d respectively. The ray travelling from point a to point f intersects edge 1 and the ray travelling from point c to point d intersects edge 4. Point e is the intersection point between the lines
representing edge 1 and edge 4. Point b is the intersection point between the line representing edge 3 and a line with the same slope as the ray that passes through the point e. When all points have been determined the beam is divided into two sections. The fraction of the initial power entering each section is determined by the ratio $L_1/(L_1+L_2)$. The fraction of power lost in each section is given by

$$D = \frac{\exp\left(-\frac{\omega t}{c_{gi}} d_{\min}\right) - \exp\left(-\frac{\omega t}{c_{gi}} d_{\max}\right)}{\frac{\omega t}{c_{gi}} d_{\max} - \frac{\omega t}{c_{gi}} d_{\min}}$$

where $d_{\min}$ is the minimum distance and $d_{\max}$ is the maximum distance between the initial edge and the intersected edge in each section. The fraction of power lost in each section is added to matrix element $B_{ji}$. The remaining power is available for further tracking.

**Figure 1.** Assembly of three rectangular plate subsystems showing sections of the beam.

ASEA calculations are carried out at one-third octave band centre frequencies with an angular resolution of $1^\circ$. A convergence criterion is defined here as resulting in less than a 0.1dB difference between ASEAN and ASEA($N-1$) for the energy level difference from any combination of source and receiver subsystems.

### 3. Finite Element Methods

Finite element modelling is carried out using Abaqus software v6.10 on a high-performance computing cluster at the University of Liverpool. STRI3 elements are used which are three node triangular elements based on thin plate theory. The element size is assigned such that there are at least 10 nodes per free bending wavelength at the upper frequency of interest which corresponds to the upper frequency of the 1kHz one-third octave band. The nodes along the plate junction line are constrained with all displacements set to zero such that there were only rotational degrees of freedom. Each plate has a prescribed internal loss factor which is related to a critical damping ratio and approximated using Rayleigh damping using curve fitting is used to determine the Rayleigh coefficients for each third octave band. Rain-on-the-roof (ROTR) excitation is applied to the surface of the source plate using point forces with unity magnitude and random phase that are applied in a direction normal to the surface to the unconstrained nodes of the source plate.
4. Box-like periodic structure

A test structure is now considered to assess the implications for the modelling of sound transmission in buildings as shown in Fig.2. All the plates that form this structure represent heavy-weight walls and floors that define rooms with a volume of 33.6\(\text{m}^3\). Details are given in Table 1. Note that the ground floors, frequency-dependent internal losses are required to realistically represent high radiation losses into the soil\(^1\).

![Box-like periodic structure](image)

**Figure 2.** Box-like periodic structure

<table>
<thead>
<tr>
<th></th>
<th>(L_x) (m)</th>
<th>(L_y) (m)</th>
<th>Thickness (h) (m)</th>
<th>Quasi-longitudinal wavespeed (c_L) (m/s)</th>
<th>Density (kg/m(^3))</th>
<th>ILF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower Floor</td>
<td>3.5</td>
<td>4</td>
<td>0.15</td>
<td>3800</td>
<td>2200</td>
<td>(f^{0.5})</td>
</tr>
<tr>
<td>Upper Floor</td>
<td>3.5</td>
<td>4</td>
<td>0.15</td>
<td>3800</td>
<td>2200</td>
<td>0.005</td>
</tr>
<tr>
<td>North Wall</td>
<td>2.4</td>
<td>4</td>
<td>0.215</td>
<td>3200</td>
<td>2000</td>
<td>0.01</td>
</tr>
<tr>
<td>East Wall</td>
<td>3.5</td>
<td>2.4</td>
<td>0.1</td>
<td>3200</td>
<td>2000</td>
<td>0.01</td>
</tr>
<tr>
<td>South Wall</td>
<td>2.4</td>
<td>4</td>
<td>0.215</td>
<td>3200</td>
<td>2000</td>
<td>0.01</td>
</tr>
<tr>
<td>West Wall</td>
<td>3.5</td>
<td>2.4</td>
<td>0.1</td>
<td>3200</td>
<td>2000</td>
<td>0.01</td>
</tr>
</tbody>
</table>

**Table 1.** Material properties and dimensions of the plates.

5. Results

To assess the extent of spatial filtering, Fig.3 shows normalized incident power in 10\(^\circ\) bands that are referenced to the total power incident on different junction lines. Diffuse field excitation is applied to the upper floor of room 1 by allocating equal intensity in all directions such that the projection of the intensity onto the plate boundaries is related to the angle of incidence, \(\theta\), by \(\cos \theta\). The beam tracing method is used to trace this initial power around the entire structure. The power incident per unit angle on a particular plate edge divided by the integrated total power incident at all angles is determined with a resolution of 10\(^\circ\). The number of beams incident on each edge is between 50,000 and 120,000.
Figure 3. Incident power per unit angle divided by the total incident power for different junction lines with 10° resolution. Angles are defined relative to the perpendicular line on each junction. Each cluster of arrows indicates the direction of the incident power that is considered upon the junction line.
The highest transmission coefficients occur at normal incidence and many have cut-off frequencies between 43° and 90° such that there is no transmission at oblique angles close to grazing incidence. Across the upper floors of the linear structure, this results in the power ratio becoming progressively weighted towards normal incidence over successive junctions that are further away from the excited plate. For junctions with plates that are orientated perpendicular to the excited plate (i.e. the side walls), the power ratio rapidly decreases towards zero near normal incidence and increases to higher values than the ideal diffuse field at oblique angles.

On the basis that there are non-diffuse vibration fields, FEM data are now used as a benchmark against which to compare SEA and ASEA models. FEM, SEA and ASEA are each used to predict the energy level difference between the source and receiver plates in terms of \(10\log_{10}(E_{\text{source}}/E_{\text{receiver}})\). Although each structure has a large number of subsystems, the general trends can be assessed on Fig. 4. For this specific source plate and all possible receiver plates these graphs show the difference between the energy level difference predicted using FEM and the energy level difference predicted using SEA or ASEA. These results show that in comparison to SEA, ASEA provides better estimates of the FEM prediction at frequencies at and above 200Hz.

![Figure 4. Difference between predicted energy level differences between the source plate and all receiver plates (left) FEM minus SEA (right) FEM minus ASEA.](image)

Fig. 5 allows comparison of the energy level difference across the upper floors that are predicted using FEM, SEA and ASEA. The general trend is that ASEA predicts increasingly lower energy level differences than SEA as the receiver subsystem becomes increasingly distant from the source subsystem. As noted previously, spatial filtering of the vibration field by successive junctions weights the transmitted power towards normal incidence. Therefore transmission across each junction becomes significantly larger than predicted by angular-average wave theory used in SEA. This results in ASEA showing closer agreement with FEM than SEA due to the fact that ASEA accounts for spatial filtering.

Fig. 6 shows the energy level differences from the upper floor (source) to the side walls. Above 200Hz, both SEA and ASEA overestimate transmission from the upper floor (source) to the adjacent side wall (wall 2) by approximately 2.5dB. Hence whilst ASEA gives closer agreement with FEM than SEA for walls 3, 4, 5 and 6, there is the possibility that this could be partly due to ASEA overestimating the energy in wall 2.
6. Conclusions

An investigation has been carried out on bending wave transmission across periodic box-like structures formed from plates, such as a building. The effects of spatial filtering rapidly become apparent after only a few structural junctions. This results in vibration fields that can no longer be considered as diffuse; hence errors occur when using SEA. However, Advanced Statistical Energy Analysis (ASEA) using ray tracing can significantly improve predictions. To increase the computational efficiency when using ASEA on large systems, beam tracing has been introduced which groups together all rays with the same heading into a single beam.

**Figure 5.** Energy level differences between upper floors - comparison of FEM with (left) SEA and (right) ASEA.

**Figure 6.** Energy level differences between the upper floor (source) and east walls (receivers) - comparison of FEM with (left) SEA and (right) ASEA.

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**REFERENCES**