BROADBAND PLANAR NEARFIELD ACOUSTIC HOLOGRAPHY BASED ON ONE-THIRD-OCTAVE BAND ANALYSIS

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Planar nearfield acoustic holography is usually based on narrow-band, single frequency analysis, which is time consuming when the source behavior over a broad frequency range is of interest, as is the case with many industrial sources. In the present paper a method, broadband planar nearfield acoustic holography based on one-third-octave band analysis (BPNAH), is developed, of which data concerning the complex band pressure on the hologram is obtained by the combination of the one-third-octave band mean-square pressure and the phase of the pressure corresponding to a single frequency line. A numerical study shows that there is no noticeable difference between the quality of the reconstruction by the BPNAH method and the quality of the conventional reconstruction, which is based on the summation of one frequency by one frequency in the corresponding band, even in the presence of noise. Further analysis shows this procedure can be extended to a source consisting of a set of fully correlated points, of which the primary points are all close together and other relatively secondary points can be far away from the former points. This method is a simple, time-saving and robust technique and thus particularly adapted to industrial studies.

1. Introduction

Nearfield acoustic holography (NAH) was first proposed by Williams and Maynard\(^1\)\(^-\)\(^2\) in the mid-1980s. Its major attraction is its solution to the inverse problem that backtracks the pressure field in space and time towards the sources. More specifically, with the introduction of evanescent waves detected in the nearfield together with propagating waves, not only source details greater but also smaller than the sound wavelength can be retrieved in this procedure. Although during the past decade, NAH has developed in a variety of directions\(^3\)\(^-\)\(^5\), Fourier-based NAH still finds a wide application for its efficient computation ability and it enables the sound field decomposed in wavenumber components, providing physically meaningful information.

As a powerful and fast acoustic imaging technique, Fourier-based planar nearfield acoustic holography (PNAH) reconstructs the surface velocity field of plate radiators from a measurement of the pressure on a parallel plane at a small distance from the plate\(^6\). Until now, researches on PNAH focus on pure tone signals under laboratory conditions, where the excitation of the source can be controlled and external interferences can be eliminated. However, in more practical and industrial situations, most sound and vibration signals are complex broadband signals, which make the single frequency analysis time consuming. Another problem is related to the determination of a representa-
2. Theory

2.1 Principle of BPNHA

Consider an infinite space with a number of sources in it and an infinite plane at \( z = z_s = 0 \), separating the source space \(( z \leq 0)\) from the source-free space \(( z > 0)\). On a plane at \( z = z_h > z_s \) parallel to the source plane, the complex sound pressure \( \tilde{p}(x, y, z_s, \omega) \) is measured and its angular spectra \( \hat{p}(k_x, k_y, z_h, \omega) \) is given by

\[
\hat{p}(k_x, k_y, z_h, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{p}(x, y, z_s, \omega) e^{-i(k_x x + k_y y)} \, dx \, dy,
\]

where \( k_x \) and \( k_y \) represent wavenumbers of the structural waves in the \( x \) - and \( y \) -axis directions.

The key in PNAH, however, is to project acoustic quantities from a measurement surface, named the hologram plane, to a parallel surface in the source-free region. Let \( \tilde{v}(x, y, z_s, \omega) \) be the normal velocity on the source plane, then the mathematics behind PNAH is summarized in the single formula:

\[
\tilde{v}(x, y, z_s, \omega) = \mathcal{F}_y^{-1} \mathcal{F}_x^{-1} \left\{ \mathcal{F}_y \mathcal{F}_x \left[ \tilde{p}(x, y, z_s, \omega) \right] G_{\nu_1}^{-1} \left( k_x, k_y, z_h - z_s \right) \right\},
\]

where the velocity propagator \( G_{\nu_1}^{-1} \left( k_x, k_y, z_h - z_s \right) = \left(k_z / \rho_0 c \right) e^{ik_z \left( z_h - z_s \right)} \).

We can see that the conventional PNAH requires knowledge of amplitude and phase of the complex pressure on the hologram plane corresponding to every single frequency line. When people are interested in the source behavior over a broad frequency range, generally a 1/3-octave band, the complex hologram pressure at each frequency in the band \( \tilde{p}(x, y, z_s, \omega) \) has to be measured and then the corresponding source normal velocity \( \tilde{v}_{rec} (x, y, z_s, \omega) \) is reconstructed, summed together to obtain the band mean-square normal velocity on the source plane, that is,

\[
\left| \tilde{v}_{\text{band}} (x, y, z_s) \right|^2 = \sum_{i=1}^{i_{\text{max}}} \left| \tilde{v}_{rec} (x, y, z_s, \omega_i) \right|^2,
\]

where every frequency in the band is numbered from \( i = 1 \) to \( i_{\text{max}} \).

However, generally a problem appears for practical measurement, especially the phase, which varies more or less randomly. On the other hand, it is definitely time-consuming to measure and reconstruct one frequency by one frequency, especially for high frequency bands. The feasibility of reconstructing a 1/3-octave band mean-square velocity on the source plane, through the combina-
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of the mean-square hologram pressure of the same 1/3-octave band and the phase of the hologram pressure corresponding to a selective single frequency line, gradually becomes a problem worthy of consideration.

To achieve the above objective, a 1/3-octave band mean-square pressure on the hologram plane \( \tilde{p}_{\text{band}}(x, y, z_h) \) is needed, which is also easy to obtain, defined as

\[
\tilde{p}_{\text{band}}(x, y, z_h) = \sqrt{\sum_{\omega_i} |\tilde{p}(x, y, z_h, \omega_i)|^2}.
\]

Next, by expressing the complex pressure in the form \( p = |p|e^{i\phi} \), we get the complex 1/3-octave band pressure on the hologram plane

\[
\tilde{p}_{\text{band}}(x, y, z_h) = |\tilde{p}_{\text{band}}(x, y, z_h)|e^{i\phi_p(x, y, z_h, \omega_k)},
\]

where \( \phi_p(x, y, z_h, \omega_k) \) is the phase of the hologram pressure corresponding to a certain single frequency \( \omega_k \) in the 1/3-octave band.

As stated in Eq. (2), \( \tilde{v}_{\text{band}}(x, y, z_i) \) is obtained by following the general process of PNAH:

\[
|\tilde{v}_{\text{band}}(x, y, z_i)| = \left| \mathcal{F}_x^{-1}\mathcal{F}_y^{-1}\left\{ \mathcal{F}_x\mathcal{F}_y\left[ \tilde{p}_{\text{band}}(x, y, z_h) \right] \right\} \right|.
\]

With a source provided, \( \tilde{p}_{\text{band}}(x, y, z_h) \) is given. Hence, the key to the success of BPNAH, that is, \( |\tilde{v}_{\text{band}}(x, y, z_i)| \) approximates the real band mean-square normal velocity

\[
|\tilde{v}_{\text{band}}(x, y, z_i)| = \sqrt{\sum_{\omega_i} |\tilde{v}(x, y, z_i, \omega_i)|^2},
\]

over the total source plane, is a desirable selection of \( \phi_p(x, y, z_h, \omega_k) \).

2.2 Determination of optimal phase

Firstly, in order to simplify our approach, assume there is only a single point source on the source plane located at \( (x_i, y_i, z_i) \). The relationship between normal velocities on the source plane corresponding to two frequencies in the same band, \( \tilde{v}(x_i, y_i, z_i, \omega_i) \) and \( \tilde{v}(x_i, y_i, z_i, \omega_j) \), is

\[
|\tilde{v}(x_i, y_i, z_i, \omega_i)|/|\tilde{v}(x_i, y_i, z_i, \omega_j)| = |v_i(x_i, y_i, z_i, \omega_i, \omega_j)|.
\]

According to Rayleigh’s first integral, we have

\[
\tilde{p}(x, y, z_h, \omega_j) = -\frac{j\rho ck}{2\pi} \int \tilde{v}(x, y, z_i, \omega_i) e^{jkR_1} dS,
\]

where \( R_1 = \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z_z - z_h)^2} \) and \( \tilde{p}(x, y, z_h, \omega_j) = |\tilde{p}(x, y, z_h, \omega_j)|e^{i\phi_j(x, y, z_h, \omega_j)} \).

Taking the point source into account, \( dS \) can be seen as a very small constant \( A \), then

\[
|\tilde{p}(x, y, z_h, \omega_j)| = \frac{\rho c A k}{2\pi R_1} |\tilde{v}(x_i, y_i, z_i, \omega_i)|.
\]
\[
\tilde{p}(x, y, z_h, \omega_j) = \frac{1}{k_i} \left[ \frac{1}{\nu_i(x_i, y_i, z_s, \omega_j)} \right] \tilde{p}(x, y, z_h, \omega_i). \tag{10}
\]

If we define
\[
|\tilde{p}_{\text{band}}(x, y, z_h)| = |\tilde{p}(x, y, z_h, \omega_i)| \cdot H(x, y, z_h, \omega_i), \tag{11}
\]
and
\[
|\tilde{v}_{\text{band}}(x, y, z_s)| = \left| \frac{1}{\nu_i(x_i, y_i, z_s, \omega_j)} \right| \cdot |\tilde{v}(x_i, y_i, z_s, \omega_j)| \cdot S(x_i, y_i, z_s, \omega_j) \quad (x_i, y_i, z_s),
\]
then both
\[
H(x, y, z_h, \omega_i) = \sqrt{\sum_{i=1}^{n} (k_i \nu_i(x_i, y_i, z_s, \omega_i))^2} = h(\omega_i) \quad \text{and} \quad S(x_i, y_i, z_s, \omega_j) = \sqrt{\sum_{i=1}^{n} (k_i \nu_i(x_i, y_i, z_s, \omega_i))^2} = s(\omega_j)
\]
are constants for a given \(\omega_j\).

With the combination of \(|\tilde{p}_{\text{band}}(x, y, z_h)|\) and \(\phi(x, y, z_h, \omega_j)\), in fact \(h(\omega_j) \tilde{p}(x, y, z_h, \omega_i)\) is used, then in the general PNAH approach \(|\tilde{v}_{\text{band}}(x, y, z_s)| = h(\omega_j) |\tilde{v}(x, y, z_s, \omega_j)|\) will be reconstructed, as illustrated in Eq. (6). Apparently, \(h(\omega_j) = s(\omega_j)\) will lead to
\[
|\tilde{v}_{\text{band}}(x, y, z_s)| = |\tilde{v}_{\text{band}}(x, y, z_s)|
\]
everywhere on the source plane.

Observe that \(|\tilde{p}(x, y, z_h, \omega_i)|\) at a single point \((x, y, z_h)\) can be measured easily in practice. From Eq. (10) we have
\[
\lambda(\omega_j) = \frac{h(\omega_j)}{s(\omega_j)} = \frac{\sum_{i=1}^{n} |\tilde{p}(x, y, z_h, \omega_i)|^2}{\sum_{i=1}^{n} \left| \frac{1}{\nu_i(x, y, z_h, \omega_i)} \right|^2} \frac{\sum_{i=1}^{n} f_i |\tilde{p}(x, y, z_h, \omega_i)|^2}{\sum_{i=1}^{n} f_i |\tilde{p}(x, y, z_h, \omega_i)|^2}.
\tag{13}
\]

As long as \(\lambda(\omega_j) \approx 1\), this \(\omega_j\) can be considered to be optimal, denoted by \(\omega_{opt}\), and the corresponding \(|\tilde{p}_{\text{band}}(x, y, z_s)|\) will approximate \(|\tilde{v}_{\text{band}}(x, y, z_s)|\) as desired. Moreover, it can be shown that this procedure can be extended to a source consisting of a set of fully correlated points, of which the primary points are all close together and other relatively secondary points can be away from formers. With respect to fully correlated, it means two points in phase have an amplitude relationship as \(|\tilde{v}(x_1, y_1, z_s, \omega_j)|/|\tilde{v}(x_2, y_2, z_s, \omega_j)| = \mu_{12}\) and \(\mu_{12}\) is a constant.

3. Numerical simulations

3.1 Overviews of settings

Consider a plane source of radius \(a = 0.1 m\) excited by point force located at \((0.2 m, 0.3 m, 0)\) mounted in a flat rigid baffle of \(1 \times 1 m\). Let the radiating surface of the source move in phase with velocity amplitude decreasing linearly from \(v_a(f_1)\) m/s at the centre to \(0 m/s\) at the edge normal to the baffle. More specifically, suppose a point’s radial distance from the piston centre is \(r\) m, and its normal velocity amplitude \(v_a(f_1, r)\) is equal to \(v_a(f_1) \cdot (1-r/a)\), as shown in Fig. 1(a). Another similar source is described in Fig. 1(b), consisting of two parts. A virtual microphone array of di-
mensions 2×2 m with 200×200 microphones corresponding to a sampling distance \( \Delta x = \Delta y = 1 \text{ cm} \) is placed 2 cm above the source plane.

![Figure 1](image)

(a) Specification of case 1
(b) Specification of case 2

**Figure 1.** Normal velocity distribution and frequency response for the 500-Hz band of case 1 and case 2.

### 3.2 Flow diagram

In order to obtain an accurate reconstruction, a strategy is applied during the simulation procedure, included in Fig. 2. More detailed information is illustrated below.

![Flow diagram](image)

**Figure 2.** Schematic representation of the simulation procedure.

#### Step 1: Rayleigh’s first integral of each frequency sub-band

To facilitate the calculation, in this paper each frequency band was divided into a number of smaller sub-band increments centered at \( f_i \) with a width of \( \Delta f \), listed in Table 1, where the centre frequency is defined as \( f_0 \), the lower frequency \( f_l \) and the upper frequency \( f_u \).

<table>
<thead>
<tr>
<th>( f_0 ) (Hz)</th>
<th>( f_l ) (Hz)</th>
<th>( f_u ) (Hz)</th>
<th>( \Delta f ) (Hz)</th>
<th>( f_i ) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>440</td>
<td>565</td>
<td>5</td>
<td>( 442.5 + (i - 1) \cdot \Delta f ), ( i = 1, 2, \cdots, 25 )</td>
</tr>
<tr>
<td>1000</td>
<td>880</td>
<td>1130</td>
<td>5</td>
<td>( 882.5 + (i - 1) \cdot \Delta f ), ( i = 1, 2, \cdots, 50 )</td>
</tr>
<tr>
<td>2000</td>
<td>1760</td>
<td>2260</td>
<td>5</td>
<td>( 1762.5 + (i - 1) \cdot \Delta f ), ( i = 1, 2, \cdots, 100 )</td>
</tr>
</tbody>
</table>

#### Step 2: Band mean-square pressure calculation

Note that in order to make the simulations more realistic and research the robustness of our proposed method, white Gaussian noise with a signal to noise ratio (SNR) equal to 30 was added to the hologram pressure signal.
Step 3: Spatial window & Step 4: Zero-padding

To reduce the influence of the spatial sampling and the finite aperture, a tapered spatial window and zero-padding should be applied before the discrete spatial Fourier transform is calculated. The spatial window was a 200-point Tukey window defined in the two-dimensional space, making the leakage arising from the effect of truncation reduced. Zeros were added outside the measurement area to reduce wrap-around errors, and after zero-padding the hologram data became a 512×512 matrix. The symbol \( \tilde{\cdot} \) indicates the windowed data and zero-padding is denoted by \( \cdot \).

Step 5: Fourier-based PNAH

Fourier-based PNAH was performed as described above. Because higher wavenumbers have a higher sensitivity to noise, an exponential \( k \)-space low-pass window is needed here\(^3\), defined by

\[
F^{k_r} = \begin{cases} 
1 & \text{for } k_r < k_c (1-\gamma) \\
\frac{1}{2} \frac{1}{2} \cos \left( \frac{k_r - k_c (1-\gamma)}{2k_c \gamma} \pi \right) & \text{for } k_c (1-\gamma) \leq k_r \leq k_c (1+\gamma), \quad (14) \\
0 & \text{for } k_r > k_c (1+\gamma)
\end{cases}
\]

where \( k_c \) is the cut-off wavenumber for \( k \)-space filter, and \( \gamma \) is the taper ratio between 0 and 1. In following sections \( \gamma = 0.3 \) and an optimal \( k_c \) was given by the Generalized Cross-Validation.

Step 6: Quantitative evaluation

For the assessment of the reconstruction accuracy of the BPNAH method, a quantitative measure was calculated in the form of a 2-norm error:

\[
\varepsilon = 100\% \cdot \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} \left| \tilde{v}_{\text{band}} (x_m, y_n, z_s) - \tilde{v}_{\text{band}} (x_m, y_n, z_s) \right|^2}{\sum_{m=1}^{M} \sum_{n=1}^{N} \tilde{v}_{\text{band}} (x_m, y_n, z_s)} \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} \tilde{v}_{\text{band}} (x_m, y_n, z_s)}{2}, \quad (15)
\]

where \( M \) and \( N \) represent the numbers of microphones in the \( x \)- and \( y \)-directions respectively.

4. Results and discussion

Table 2 presents a quantitative comparison between reconstructions by the conventional method based on the summation of every PNAH result corresponding to a sub-frequency and those by our proposed BPNAH method. Notice that no matter what \((x, y, z_n)\) is selected, the obtained \( \omega_{pr} \) in the same case is moderately consistent, as we expect. With other setups identical, there is no significant quantitative difference between the reconstruction by the conventional method and that by our proposed BPNAH method for each case. Although the conventional method usually performs slightly better than BPNAH, it is noted that \( \varepsilon \) by the conventional method in case1a exceeds \( \varepsilon \) determined via BPNAH. The reason is mainly that a fixed \( k_c \) used in the BPNAH method which is given by GCV cannot be optimal for every step of pure tone analysis of the conventional method. However, even with an independent \( k_c \) corresponding to every certain \( \omega \) in the process of the conventional method, the enhancement of reconstruction could prove limited. Moreover, in practical situation it will be a difficult task and sometimes almost impossible to have the specific information, including amplitude and phase, for each frequency in a relatively wide frequency band. Nevertheless, this problem can be significantly improved by the application of the BPNAH method, because the band mean-square pressure used in the BPNAH method can be easily measured and only a single phase is necessary for an acceptable reconstruction, which greatly reduces our load.

Figs. 3-5 further compare the reconstructions by the conventional method and our proposed BPNAH method qualitatively. In order to facilitate the explanation, a region of interest (ROI) closely surrounding the source is selected. In Fig. 3, we can clearly see that compared to \( |\tilde{v}_{\text{rec}}| \) by
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the conventional method, all results obtained by our proposed BPNAH method perform very well. Figs. 4 and 5 show that in addition to properly reconstruct the primary part, i.e., source 2R, the relatively secondary part, i.e., source 2L, has a certain level of reconstruction, which is, however, not as good as the former, e.g., \( |v_{rec}| \) by the BPNAH method in Fig. 4(b).

Besides accuracy considerations, the processing speed is another important factor. Take the 2000-Hz band with a bandwidth of 500 Hz for example, with respect to the conventional pure tone analysis, at least 500 times of reconstructions are necessary, which will definitely take a very long processing time. In the process of the BPNAH method, only a single reconstruction is sufficient to produce a result including the accurate information of interest, as described in Table 2 and Figs. 3 and 4. This advantage seems particularly apparent for higher frequency bands.

**Table 2.** Reconstruction errors of case 1 and case 2.

<table>
<thead>
<tr>
<th>case</th>
<th>( \omega_{opt} )</th>
<th>( k_c ) (rad/m)</th>
<th>( \varepsilon ) by PNAH sum</th>
<th>( \varepsilon ) by BPNAH</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>( \omega_h )</td>
<td>47</td>
<td>6.9%</td>
<td>6.3%</td>
</tr>
<tr>
<td>1b</td>
<td>( \omega_l )</td>
<td>50</td>
<td>5.6%</td>
<td>6.2%</td>
</tr>
<tr>
<td>1c</td>
<td>( \omega_{29} )</td>
<td>58</td>
<td>4.9%</td>
<td>5.1%</td>
</tr>
<tr>
<td>2a</td>
<td>( \omega_h )</td>
<td>32</td>
<td>5.3%</td>
<td>5.6%</td>
</tr>
<tr>
<td>2b</td>
<td>( \omega_l )</td>
<td>39</td>
<td>4.8%</td>
<td>5.9%</td>
</tr>
<tr>
<td>2c</td>
<td>( \omega_{14} )</td>
<td>55</td>
<td>2.3%</td>
<td>2.5%</td>
</tr>
</tbody>
</table>

**Figure 3.** Comparison of the actual and reconstructed results of ROI in case 1.

**Figure 4.** Comparison of the actual and reconstructed results of left ROI in case 2.
5. Conclusions

Numerical simulations produced for certain typical sources demonstrate the validity of the principle of a newly proposed method, based on 1/3-octave band analysis. The so-called BPNAH method thus gives access to source reconstruction techniques, with relatively fewer data to store and to process. Moreover, it has been observed that the quality of reconstructions by the BPNAH method is similar to the quality of conventional reconstructions summed one frequency by one frequency in the corresponding band.

In terms of the reference phase, which is usually difficult to acquire in measurements with the sensor array, the BPNAH method is far better than the conventional method based on pure tone signals because of the fact that only a single selection of the reference phase will be sufficient for the former. In addition, the introduction of noise does not influence our reconstructions evidently.

The present approach is restricted to the radiation from a set of fully correlated sources where those dominant ones are close together. However, it can possibly be expanded to some more complex situations to give quantitative or qualitative results. An experimental study is also necessary to further confirm the validity of our proposed method.

REFERENCES