ACOUSTIC BAND GAP CHARACTERISTICS OF ONE-DIMENSIONAL PIPE PERIODICALLY INSTALLING DOUBLE HELMHOLTZ RESONATORS*

Yafeng Zhang, Dianlong Yu, Huijie Shen, Jihong Wen

Vibration and Acoustics Research Group, Science and Technology on Integrated Logistics Support Laboratory, College of Mechatronic Engineering and Automation, National University of Defense Technology, Changsha, Hunan, P. R. China, 410073

e-mail: willnudt@sina.com

The study of acoustic and elastic wave propagation in artificial periodic structures has received increasing attention in the last few decades. The presence of band gaps in phononic crystals, which forbids elastic/acoustic wave propagation within the band-gap frequency range, supplies a new way to control noise and vibration. Koo and Sorokin study the acoustic and elastic wave propagation in artificial periodic pipe, focus attention into the vibration of the wall, and find the band gaps. The acoustic band gaps in artificial periodic pipe also get some researchers’ attention recently. Acoustic characteristics in the cavity of the pipe periodically installing double Helmholtz resonators are investigated theoretically and numerically in this paper. The results are compared to double Helmholtz resonators with variable parameters including the length and radius of the necks and resonators. The resonance frequency, transmission loss and acoustic band gaps of the pipe system can be further modulated by the parameters of the double Helmholtz resonators based on our study. These numerical results show a good agreement with those of the simulation by FEM software, while showing that one double Helmholtz resonator has some positive acoustic band-gap characteristics. This provides us a possible way to achieve a wider acoustic band gap via using the double Helmholtz resonator with appropriate parameters.

1. Introduction

The propagation of waves in periodic media is studied for many years and covers a large area of problems in optics, acoustics, mechanics, and electromagnetic etc. The photonic crystals, as typical artificial periodic structures, have received increasing attention in the last few decades. Due to the existence of forbidden frequency bands (BG, a so-called band gaps referred to as stop bands) of acoustic and elastic wave propagation, it supplies a new way to control noise and vibration. There are two BG formation mechanisms. One called Bragg-type BG is due to the Bragg reflection when the periodic spacing between the neighboring resonators becomes a multiple of a half-wavelength of the sound wave. As a result, for environmental low-frequency solid elastic wave, gigantic structures are required for the Bragg-type BGs. The other is called locally resonant (LR) BGs. It is due to resonance with the resonator when the frequencies of the sound wave coincide with its eigenmode.

* Supported by the National Natural Science Foundation of China under Grant Nos. 11372346
frequencies. According to the two BG formation mechanisms, not only can the low-frequency acoustic LR gaps be produced by mounting local resonators periodically to the structural waveguide systems, but also a small Lattice constant can obtain the low-frequency acoustic Bragg-type BG.

Some researchers pay attention to the acoustic wave propagation in artificial periodic structure. The one-dimensional (1D) acoustic artificial periodic structure appears in most of the papers. The classical structure is a 1D pipe periodically installing Helmholtz resonators (HRs) by analogy with the spring mass resonators. Anne-Christine and Hladky-Hennion’s study about the acoustic artificial structural materials supports recent observations of acoustic wave band gap in a 1D array of HRs. E H El Boudouti and other researchers study the transmission gaps of a pipe with periodic simple acoustic filter consisting of two slender side tubes. Xinhua Hu and Kai-Ming Ho give a brief report to show the study on acoustic metamaterials consisting of cylindrical and spherical HRs in fluid respectively. The sound absorbing structure in their papers can be seen as the spring-mass resonator. The band-gap materials with composite resonator such as double HR (analogly with the spring-mass-spring-mass resonator) also have been studied by some researchers. However, the relevance between the acoustic characteristics of HR and the band gaps in the pipe cavity still needs further study.

Double HR is taken account in this paper. We mainly use plane wave assumption to analyze the acoustic performances of double HRs, including the transmission loss (TL) and resonance frequency. The cavity energy band expression of the main pipe is given by the transfer matrix method. The simulation work is done to ensure that the analytical and computational predictions are right. We discuss the effect of different parameters on the acoustic characteristics of double HR and a further effect on the BGs. Some suggestions are given to regulate the parameters to get a wider BG.

2. **Analytical approaches**

Figure 1 shows the wave propagation in a piston-driven double HR. The first neck is between the first cavity and the main pipe. The second neck is between the two cavities. In light of the low frequencies of our interest, the geometrical dimensions considered here are significantly smaller than the relatively long wavelengths. The frequency of the wave in the HR is less than the cot-off frequency of the circular duct given by the acoustic waveguide theory. Hence, we ignore the spatial resolution and use the plane wave assumption to study the double HR. Accounting for the higher order wave propagation effects, the neck length has already contains the end correction given by Reference 14 in this paper.

![Figure 1. The wave propagation in a piston-driven double HR.](image)

2.1 **TL of HR**

For 1D wave propagation in a circular duct, the acoustic equation is given as the following ordinary differential equation for $p^{13}$:

$$
\frac{\partial^2 p}{\partial x^2} = \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2}.
$$

The solution in the first neck part (domain I) can be written as

$$
p_A(x) = (A_n e^{-jkx} + A_n e^{jkx}) e^{j(\omega t)}.
$$
According to the relation between sound press and acoustic velocity, \( v(x) = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} dt \), where \( \rho_0 \) is the air density, one obtains the acoustic velocity as

\[
v_A(x) = \frac{1}{\rho_0 c_0} (A_x e^{-jkx} - A_y e^{jkx}) e^{i\omega t}.
\] (3)

According to Eq. (1), the solution in domain II, III and IV, \( p_B(x_2) \), \( p_C(x_3) \) and \( p_D(x_4) \) are the same with Eq. (2) in the form. The corresponding acoustic velocities have the same form of Eq. (3). The pressure continuity conditions between domains I, II, III and IV are \( p_A(x_1 = l_{n1}) = p_B(x_2 = 0) \), \( p_A(x_2 = l_1) = p_C(x_3 = l_{n2}) = p_D(x_4 = 0) \). At \( x_1 = l_{n1} \) or \( x_2 = 0 \), the rigid boundary condition is \( v_B(x_2 = 0) = 0 \). Similarly, at \( x_2 = l_1 \), \( x_4 = 0 \) and \( x_4 = l_{n2} \), the rigid boundary conditions are \( v_B(x_2 = l_1) = 0 \), \( v_D(x_4 = 0) = 0 \) and \( v_D(x_4 = l_{n2}) = 0 \) respectively. The volume velocity continuity at \( x_1 = l_{n1} \) or \( x_2 = 0 \) is \( S_n v_B(x_1 = l_{n1}) = S_1 v_B(x_2 = 0) \), where \( S_{n1} \) is the first neck cross-sectional area and \( S_{c1} \) is the first cavity cross-sectional area. The other two volume velocity continuities are \( S_{c1} v_B(x_2 = l_{c1}) = S_{n2} v_B(x_3 = 0) \) at \( x_1 = l_{n1} \) or \( x_2 = 0 \) and \( S_{n2} v_C(x_3 = l_{n2}) = S_{c2} v_D(x_4 = 0) \) at \( x_3 = l_{n2} \) or \( x_4 = 0 \). In view of these conditions and Eq. (1) to (3), \( A_n^+ \) and \( A_n^- \) can be expressed by \( D_n^+ \). The expressions are given in Appendix A. The acoustic impedance of the HR can be written as

\[
Z_H = \frac{P_A(x_1 = 0)}{S_n v_B(x_1 = 0)} = \frac{\rho_0 c_0}{S_n} \frac{1 + \lambda}{1 - \lambda},
\] (4)

where \( \lambda = A_n^- / A_n^+ \) (see Appendix A). The acoustic impedances \( Z_H \) can yield the TL of the pipe through Reference 15, as

\[
TL = 20 \log_{10} |1 + \frac{\rho_0 c_0}{2 S_p Z_H}|,
\] (5)

where \( S_p \) is the cross-sectional area of the main pipe.

2.2 Acoustic BGs of pipe

Figure 2. The schematic diagram of an acoustic waveguide in the artificial periodic pipe.

21st International Congress on Sound and Vibration (ICSV21), Beijing, China, 13-17 July 2014

ICSV21, Beijing, China, 13-17 July 2014 3
$$\begin{bmatrix} E^+_{n+1} \\ E^-_{n+1} \end{bmatrix} = e^{iqa} \begin{bmatrix} E^+_n \\ E^-_n \end{bmatrix}.$$  \hfill (7)

In which, $q$ is called the Bloch wave vector (wave number). Combining Eq. (6) and (7), one obtains the dispersion relation of the infinite periodic system:

$$\cos(qa) = \cosh(ka) = \cos(ka) + \frac{jZ_{pl}}{2Z_H} \sin(ka).$$  \hfill (8)

Eq. (8) will be used to calculate the acoustic band structure of the infinite systems. While the real part of the Bloch wave vector, $\text{Re}(q)$, is commonly used in the literatures to characterize the phase change of propagating waves in a pass band; the imaginary part, $\text{Im}(q)$, can be employed to describe the amplitude decay of evanescent waves in a BG.

3. Results and discussion

The standard double HR has been fabricated with the parameters in Table 1. The main pipe is built of a square cross-section (5cm×5cm). All the comparisons in the paper require one parameter change in the condition of the other parameter parameters adopting the corresponding data in Table 1. The Lattice constant is fabricated with $a=1\text{m}$ in all the comparisons.

<table>
<thead>
<tr>
<th>Table 1. The standard double HR parameters [cm].</th>
</tr>
</thead>
<tbody>
<tr>
<td>First neck length ($l_{n1}$)</td>
</tr>
<tr>
<td>7.62</td>
</tr>
</tbody>
</table>

The acoustic impedance of the double HR can be given by Eq. (4). Then, the Eq. (8) can give the theoretical band gap results. Figure 3 shows the band gap results. The red curve shows the simulation results obtained via FEM software Comsol Multiphysics. As explained in Section 2.2, $\text{Im}(q)$ can be employed to describe the amplitude decay of evanescent waves in a BG. What’s more, the higher $\text{Im}(q)$ value, the stronger sound attenuation will be. The theoretical results (as the blue curve shows) agree well with the simulation results, which ensure our theory and numerical calculation are correct.

![Figure 3](image.png)

**Figure 3.** The acoustic BGs of the pipe periodically installing double HR.

In the following figures, it gives the results of the acoustic characteristics with varying parameters. Figure 4 shows the acoustic characteristics with varying 1st neck parameters. Figure 4 (a) and (b) shows the TL and BG with $0.5r_{n1}$, $r_{n1}$ and $2r_{n1}$. The increasing neck radius makes the two resonance frequencies increase at the same time. The higher resonance frequency has the TL with a
relatively higher value. The resonance frequency and TL characteristics affect the BG performance. The starting frequencies of the first-two-order Bragg-type BGs given by the Bragg-type BG formation mechanism are 171.5Hz and 343Hz. Obviously, the two resonance frequencies with 0.5 \(r_{n1}\) are all lower than the first starting frequency, it has the Bragg-type BGs with the development towards the high frequency showed in Figure 4 (b). The 2\(^{nd}\) resonance frequency with \(r_{n1}\) or 2\(r_{n1}\) is higher than the first starting frequency. As a result, it should have the Bragg-type BGs with the development towards the low frequency. However, these Bragg-type BGs don’t appear in Figure 4 (b). This phenomenon which confuses us does not observe the law above. We think the two parts of the TL have an opposite effect on the first Bragg-type BG. It seems that the first part with a low resonance frequency plays a greater role. A Bragg-type BG with the development towards the low frequency appears in Figure 4 (d) as the blue curve shows, which validates our conjecture. Figure 4 (c) and (d) shows the TL and BG with 0.5\(l_{n1}\), 1.1\(l_{n1}\) and 2\(l_{n1}\). The increasing neck radius mainly makes the second resonance frequency of the TL fall down. But the corresponding TL value has a little change. The first resonance frequency nearly has no change.

![Figure 4](image-url)  
**Figure 4.** The acoustic characteristics with varying 1\(^{st}\) neck parameters. (a) and (b): TL and IM (q) with varying neck radius; (c) and (d): TL and IM (q) with varying neck length.

Figure 5 shows the acoustic characteristics with varying 1\(^{st}\) cavity parameters. Figure 5 (a) and (b) shows the TL and BG with 0.5\(r_{c1}\), \(r_{c1}\) and 2\(r_{c1}\). Slimly with the effect of the neck radius, the increasing cavity radius makes the two resonance frequencies increase at the same time. But cavity radius effect on the TL value is not such significant as the neck radius effect. The increasing cavity length also makes the two resonance frequencies increase at same time. It plays a smaller role than the cavity radius in changing the resonance frequency and value of the TL.

![Figure 5](image-url)  
**Figure 5.** The acoustic characteristics with varying 1\(^{st}\) cavity parameters. (a) and (b): TL and IM (q) with varying cavity radius; (c) and (d): TL and IM (q) with varying cavity length.

Figure 6 shows the acoustic characteristics with varying length of 2\(^{nd}\) neck and cavity. Figure 6 (a) and (b) shows the TL and BG with 0.5\(l_{n2}\), \(l_{n2}\) and 2\(l_{n2}\). The increasing neck length mainly makes the value of the two TLs increase with little resonance frequency change. The increasing 2\(^{nd}\) cavity length mainly makes the TL move towards to the higher frequency with no value change.
From Figure 4 to 6, we can know TL characteristics influence the BG performance. The two TLs have an opposite effect on the Bragg-type BG which is between the two TLs and close to the second TL. The first TL has a greater effect because it is with a bigger TL value. The linear measurement between the resonance frequency and the starting frequencies of the Bragg-type BGs is another factor. The nearer the resonance frequency closes to the starting frequency, the bigger role the corresponding TL plays. This explains that the Bragg-type BG with the development towards the low frequency appears in Figure 4 (d).

4. Conclusion

Acoustic characteristics in the cavity of the pipe periodically installing double Helmholtz resonators are investigated theoretically and numerically. Double Helmholtz resonators with variable parameters including the length and radius of the neck and resonator have different TL characteristics and have a further influence on the BG performance. The TL with a higher value and a resonance frequency close to the starting frequency might obtain a wider BG. The increasing radius of 1st neck and length of 2nd neck have a positive effect on the TL value. All the parameters can regulate the resonance frequencies. These inspire us to achieve a wider acoustic band gap by using appropriate parameters. The neighbouring Bragg-type BG of the first LR BG should be kept. It’s advisable to adopt a bigger 1st neck radius to get a TL with a higher value. By regulating other parameters, more than two BG starting frequencies should appear between the two resonator frequencies. What’s more, the parameters should make the linear measurement between the first resonance frequency and its neighboring Bragg-type BG starting frequency as small as possible.

Appendix A

From the rigid boundary condition $v_p(x_1 = l_c) = 0$, we get the following equation

$$\alpha = \frac{D_n^-}{D_n^+} = e^{-2jkl_z}$$

According to the pressure continuity conditions and the volume velocity continuity conditions, one obtains

$$\beta = \frac{C_n^-}{C_n^+} = \frac{(1 - \frac{S_{c_2}}{S_{n_2}}) + (1 + \frac{S_{c_2}}{S_{n_2}})\alpha}{(1 + \frac{S_{c_2}}{S_{n_2}}) + (1 - \frac{S_{c_2}}{S_{n_2}})\alpha} e^{-2jkl_n}$$
\[
\gamma = \frac{B_n^-}{B_n^+} = \left(1 - \frac{S_{n2}}{S_{c1}}\right) + \left(1 + \frac{S_{n2}}{S_{c1}}\right) \beta e^{-2 j k l_1}
\]
\[
\lambda = \frac{A_n^-}{A_n^+} = \left(1 - \frac{S_{c1}}{S_{n1}}\right) + \left(1 + \frac{S_{c1}}{S_{n1}}\right) \gamma e^{-2 j k l_{n1}}
\]

The transfer matrix \( T \) is written as
\[
T = \begin{bmatrix}
(1 - \frac{Z_{pi}}{2Z_H})e^{-jka} & -\frac{Z_{pi}}{2Z_H} e^{jka} \\
-\frac{Z_{pi}}{2Z_H} e^{-jka} & (1 + \frac{Z_{pi}}{2Z_H})e^{jka}
\end{bmatrix}
\]
where \( Z_{pi} = \rho_0 c_0 / S_{pi} \) refers to the acoustic impedance of the pipe, \( Z_H \) is the acoustic impedance of the HR neck duct.

**Appendix B**

**List of symbols**

- \( a \) Lattice constant
- \( A_n \) to \( E_n \) Modal amplitudes in domain I, II, III, IV and main pipe
- \( c_0 \) Speed of sound
- \( f \) Frequency
- \( k \) Wave number
- \( l_{n1}, l_{n2} \) First and second neck length
- \( l_{c1}, l_{c2} \) First and second cavity length
- \( p \) Acoustic pressure
- \( q \) Bloch wave vector (wave number)
- \( r_{n1}, r_{n2} \) First and second neck radius
- \( r_{c1}, r_{c2} \) First and second cavity radius
- \( S_{n1}, S_{n2} \) First and second neck cross-sectional area
- \( S_{c1}, S_{c2} \) First and second cavity cross-sectional area
- \( S_{pi} \) Pipe cross-sectional area
- \( T \) Transfer matrix
- \( TL \) Transmission loss
- \( v \) Acoustic velocity
- \( Z_H \) Acoustic impedance of a Helmholtz resonator
- \( \rho_0 \) Density of air

**Superscript**

- \( + \) Travelling in the positive direction
- \( - \) Travelling in the negative direction
Subscript

\( \text{A, I, I to D, IV, } \text{A, I, II, III, IV} \)

Domain I, II, III, IV

\( c \)

Cavity

\( n \)

Neck

\( pi \)

Main pipe

References


