DETERMINING THE TRANSMISSION LOSS OF VARIABLE CROSS-SECTION APERTURES

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Minimizing the transmission of sound through leaks is central to enclosure and barrier design. In the current work, a transfer matrix method is used to predict the transmission loss of leaks assuming that the cross-sectional dimensions of the leak are small compared to an acoustic wavelength. The approach used can be applied to leaks of any shape and special consideration is given to apertures with varying cross-section. Reactive strategies like Helmholtz resonators and abrupt area changes are considered to reduce the sound transmission. For larger openings, an acoustic finite element strategy is suggested. The boundary condition on the source side is a diffuse field applied by determining the cross-spectral force matrix of the excitation. At the termination, the radiation impedance is calculated using a wavelet algorithm. Simulation results using the transfer matrix and finite element approaches agree with published results and demonstrate good agreement with one other.

1. Introduction

Enclosures or booths are commonly used for sound insulation and act as a barrier between the source and the receiver. If machinery inside an enclosure is structurally isolated, the primary path for sound propagation is through small holes or leaks in the enclosure walls. Apertures and leaks are normally unavoidable for a number of reasons. Some are purposeful to allow air circulation or room for wiring. Others are inevitable such as slits between doors and walls.

The metric most commonly used to define the properties of an aperture is the acoustic transmission loss. Transmission loss is the difference between the incident and transmitted power in decibels. The transmission loss of openings can be found using the two reverberation room approach standardized in ISO 140. Results are often used in statistical energy analysis (SEA) models for rooms and partial enclosures.

If the thickness of the wall is small compared to an acoustic wavelength, formulas are available for determining the transmission loss of apertures. However, apertures are often lengthened to improve the transmission loss. Gomperts, Gomperts and Kihlman, Wilson and Soroka, and Sauter Jr. and Soroka developed expressions for circular and rectangular apertures where the length of the opening was considered. Mechel contributed by including the effects of both sealing and absorption in the leak and also considered muffler elements in the aperture. In each of the aforementioned papers, plane wave propagation was assumed in the aperture. Sgard et al. summarized the prior work...
and developed equations for larger apertures with uniform cross-section including transverse wave behavior.

Since transfer matrix theory is widely used for designing mufflers in the plane wave regime\cite{10}, it seems reasonable to employ a similar approach to apertures which have variable cross-section. Ouchi and Matsui\cite{11} successfully used transfer matrix theory to determine the acoustic behavior of variable cross-section apertures and Mechel\cite{8} suggested a similar network approach.

There are advantages to a transfer matrix approach especially when analyzing apertures of varying cross-section. Transfer matrices are readily available in the literature for seals and absorptive plugs, lined ducts, cones, and resonators\cite{10,12}, and commercial muffler software is in place including most of these elements. Using the work of Ouchi and Matsui\cite{11} and Mechel\cite{7,8} as a starting point, this paper derives the transmission loss equations for a diffuse acoustic field on the incident side in terms of transfer matrices. Results are compared to a finite element approach detailed in References 13-16 and implemented in the ESI VA-One software\cite{17}.

2. Transfer matrix approach

The development that follows is based primarily on Mechel\cite{7} and is similar to the work by Ouchi and Matsui\cite{11} with a few minor differences. A schematic showing an aperture and the component acoustic waves is shown in Figure 1. The oblique incident and reflected waves at the inlet are $p_i$ and $p_r$ respectively. The waves radiating from the entrance and exit sides of the hole are $p_s$ and $p_t$. Inside the aperture, the incident and reflected waves are $p_{i1}$ and $p_{i2}$. $p_1$ and $p_2$ are the sound pressure at the orifices on the entrance and exit sides. Similarly, $u_1$ and $u_2$ are the particle velocities at the respective orifices. Plane wave propagation is assumed in the aperture so the procedure is valid below the cutoff frequency. Figure 1 shows the assumed directions for the sound waves and impedances.

Assuming harmonic motion and the $e^{+j\omega t}$ sign convention, the oblique incident and reflected sound pressures at the entry can be expressed as

$$p_i = \hat{p}_i e^{-j k_0 (z \cos \theta_i + y \sin \theta_i)}$$

and

$$p_r = \hat{p}_r e^{-j k_0 (-z \cos \theta_i + y \sin \theta_i)}$$

![Figure 1. Schematic of an aperture with the entry on the left and exit on the right.](image-url)
The sound field at the entry to the orifice is the superposition of the sound for a blocked opening plus the sound radiating from the opening. For the blocked case, the velocity at the entrance plane is zero so
\[ \hat{p}_r = \hat{p}_i \] (3)
For the sound radiating from the entrance, the complex magnitude of the sound pressure can be expressed in terms of the particle velocity at the entrance and the radiation impedance as
\[ \left( p_s \right)_{z=0} = -Z_{r1}S_1u_1 \] (4)
where \( Z_{r1} \) is the specific radiation impedance at the entrance. Superimposing the two sound fields, the sound pressure at the entrance can be expressed as
\[ p_1 = (p_i + p_r + p_s)_{z=0} \] (5)
Inserting Eqns. (3) and (4) into Eqn. (5), the incident sound pressure can be expressed as
\[ p_i = \frac{1}{2} (p_1 + Z_{r1}S_1u_1) \] (6)
At the exit surface, the sound pressure \( p_2 \) can be expressed in terms of the outlet radiation impedance \( Z_{r2} \) as
\[ p_2 = (p_e)_{z=a} = Z_{r2}S_2u_2 \] (7)
Assuming plane wave behavior in the aperture, the relationship between the sound pressure and particle velocity at the inlet and outlet to the aperture can be expressed as
\[ \begin{bmatrix} p_1 \\ S_1u_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} p_2 \\ S_2u_2 \end{bmatrix} \] (8)
where \( A, B, C, D \) are the four pole parameters which define the transfer matrix for the aperture.

After Mechel, the sound power for the incident (entrance) and radiating (exit) waves can be written as
\[ W_i = \frac{1}{2Z_0} \cos \theta_i |\hat{p}_i|^2 \] (9)
and
\[ W_r = \frac{1}{2} S_2^2 Re(Z_{r2}) |u_2|^2 \] (10)
where \( Z_0 \) is the characteristic impedance divided by the cross-sectional area \( (\rho c / S_1) \).

The transmission coefficient can be expressed as
\[ \tau(\theta_i) = \frac{W_r}{W_i} = \frac{1}{2} S_2^2 Re(Z_{r2}) \left| u_2 \right|^2 = \frac{1}{2Z_0} \cos \theta_i |\hat{p}_i|^2 \] (11)
and can be expressed in terms of the four pole parameters as
\[ \tau(\theta_i) = \frac{W_r}{W_i} = 4 \frac{1}{\cos \theta_i} \frac{Z_0 Re(Z_{r2})}{AZ_{r2} + B + (CZ_{r2} + D)Z_{r1}} \left| u_2 \right|^2 \] (12)
after inserting Eqns. (6), (7), and (8) into (11). The transmission loss for oblique incidence waves is
\[ TL(\theta_i) = -10 \log(\tau(\theta_i)) = 10 \log \left( \frac{1}{4Z_0 Re(Z_{r2})} \left| AZ_{r2} + B + (CZ_{r2} + D)Z_{r1} \right|^2 \right) \] (13)

The diffuse field transmission coefficient \( \tau_d \) is defined by integrating \( \tau(\theta_i) \) over all incident angles. From Sgard et al.,
\[ \tau_d = \frac{2(1 - \cos \theta_{lim})}{\sin^2 \theta_{lim}} \tau(0^\circ) \] (14)
where \( \theta_{lim} \) is the limit angle for diffuse sound waves and \( \tau(0^\circ) \) is the normal incident transmission coefficient. An angle of 90° is used for a diffuse field but an angle of 78° (called field incident) is
often used since it agrees better with measurement. The transmission loss for field incidence can be expressed as

\[ TL_d = 10 \log \left( \frac{\sin^2 \theta_{lim}}{8(1 - \cos \theta_{lim})} \frac{1}{Z_0 \Re(Z_{r2})} \left| AZ_{r2} + B + (CZ_{r2} + D)Z_{r1} \right|^2 \right) \] (15)

Eqn. (15) is similar to Ouchi and Matsui\(^{11}\) and Mechel\(^{8}\). However, diffuse field or field incident loading is included. Sgard et al. assumed an angle of 78° for the limit angle.

In the results that follow, the four-pole transfer matrix expressions from Reference 12 were used. In addition, expressions from Reference 18 were used for the radiation impedances (\(Z_{r1}\) and \(Z_{r2}\)).

### 3. Acoustic finite element approach

Sgard et al.\(^{9}\) used a mixed acoustic boundary (BEM) and finite element (FEM) model to simulate apertures and compared the results to an analytical solution. The simulation strategy presented in the discussion which follows is similar, but is a finite element approach which is implemented in the ESI VA-One software\(^{17}\). The primary difference is that a diffuse field acoustic loading is applied directly whereas Reference 9 used a set of plane wave sources. Additionally, the impedance boundary conditions at the source and termination are determined using a wavelet approach that, though approximate, is much faster than BEM.

The modeling approach is illustrated in Figure 2 and is parallel to the approach described in prior work by the authors for modeling plenums\(^{16}\). A FEM model is used to simulate the aperture and the model is solved in modal coordinates. The loading is defined as a cross-spectral force matrix using a reciprocity relationship\(^{13-14}\) between the direct field radiation and diffuse field reverberant loading. The radiation impedance on both sides of the aperture is computed using a wavelet approach developed in Reference 15 in which Jinc functions were selected as the wavelet basis.

![Figure 2. Acoustic finite element model with boundary conditions indicated.](image)

### 4. Examples

Several example apertures were considered in this study. The transfer matrix and acoustic FEM approaches are compared with good agreement. After which, the model was used to better understand
the effect of varying the cross-sectional area of an aperture in order to suggest design strategies for improving the transmission loss.

4.1 Circular Aperture

Both approaches were first validated for a cylindrical aperture which is well documented in the literature. Results were identical with those from Mechel\textsuperscript{7} and were then compared to Reference 9. The diameter and length of the aperture are 11.28 mm and 10 cm respectively. The transmission loss comparison is shown in Figure 3. Notice that the transfer matrix approach and acoustic FEM compare well with Reference 9. It was also noted that changing the shape of the cross-section had a minimal effect on the transmission loss of the aperture.

![Figure 3. Transmission loss of a circular aperture with radius of 5.64 mm and length of 10 cm.](image)

4.2 Aperture with abrupt contraction

An aperture with abrupt contraction (Figure 4) is considered next. The effect of changing the radius of the entry cylinder ($R$) was examined. The radius of the exit cylinder ($r$) was held constant at 5.64 mm. The abrupt contraction was assumed to be at the midpoint of the aperture length (i.e., $l_1 = l_2$). The total length of the aperture was 10 cm.

Figure 5 shows transmission loss results. A radius ratio is defined as $R/r$ where $r$ is held constant. It can be seen that adding the abrupt contraction significantly increases the transmission loss at both low and high frequencies. For instance, the transmission loss will increase by roughly 5 and 10 dB at low and high frequencies respectively if the radius ratio is quadrupled. The resonance frequencies of the aperture are only slightly decreased.

4.3 Aperture with Helmholtz Resonator

For the next case, a Helmholtz resonator was added as a side branch. Figure 6 shows a schematic representation of the Helmholtz resonator including dimensions. The transfer matrix result is compared with acoustic FEM in Figure 7 with good agreement. As shown in Figure 7, the addition of a Helmholtz resonator allows the designer to target a particular frequency for high transmission loss. The dimensions of the Helmholtz resonator are $l_n = 10$ mm, $R_n = 5.64$ mm, $V_c = 12$ cm$^3$. 
Figure 4. Schematic showing an aperture with abrupt contraction. $l_1 = l_2 = 50$ mm and $r = 5.64$ mm.

Figure 5. Effect of radius ratio ($R/r$) on the transmission loss for an aperture shown in Figure 4.

Figure 6. Schematic showing aperture with Helmholtz resonator.
4.4 Transmission Loss Above the Plane Wave Cutoff

Quite often, large openings are used in enclosures. The acoustic FEM approach described is compared to published results by Trompette et al. in Figure 8 with good agreement for an opening with a 6 cm × 13 cm cross-section and length of 30 cm. The results demonstrate that the transmission loss above the plane wave cutoff (approximately 1310 Hz) of the aperture can be determined using the acoustic FEM approach described.

5. Conclusions

In this paper, a transfer matrix method, similar to that developed by Ouchi and Matsui, was used to determine the transmission loss of apertures. The procedure is applicable for diffuse acoustic
fields applied on the entry side where the limit angle of the sound waves can be varied. The model was checked by comparing the results to acoustic FEM using appropriate radiation impedance boundary conditions at the entry and exit of the aperture and a diffuse acoustic field loading.

Apertures including abrupt contractions and Helmholtz resonators were examined. Further studies are recommended using the transfer matrix approach detailed in this paper to investigate other reactive element combinations. Most importantly, the results have shown that aperture transmission loss can be increased over 10 dB without restricting the flow using reactive strategies.

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REFERENCES