APPLICATION OF SEA AND ADVANCED SEA TO STRUCTURE-BORNE SOUND TRANSMISSION ON A RECTANGULAR BEAM FRAMEWORK

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This paper concerns the prediction of vibration transmission across networks of coupled beams using a bending wave only model and a bending and longitudinal wave model. Statistical Energy Analysis (SEA) is used with coupling loss factors determined from wave theory and is compared with numerical experiments from Finite Element Methods (FEM) for simple beam junctions as well as a rectangular beam framework. The results show that SEA can accurately predict vibration transmission across an isolated L-junction of beams. However, for the rectangular beam framework there is only good agreement between SEA and FEM for beams that are physically connected. Discrepancies occur at mid- and high-frequencies for beams that are not physically connected to the source beam with SEA overestimating the vibration levels for distant beam subsystems. However, Advanced SEA (ASEA) is shown to provide good agreement with FEM because it is able to account for propagation losses within a framework that includes unavailable modal energy.

1. Introduction

Statistical Energy Analysis (SEA) has been shown to be successful in solving many vibro-acoustic problems in engineering\(^1\). However, there are instances where errors occur with some types of structural assembly\(^2-5\). You et al\(^6\) compared random energy flow analysis with SEA to investigate structural vibration power flow in planar beam frameworks. The energy levels on subsystems that are distant from the excited subsystem showed that large differences exist between the two methods indicating that SEA was less reliable. Generally SEA assumes that there is no coupling between physically disconnected subsystems. However, in some situations there is significant indirect coupling, i.e. a ‘tunnelling mechanism’\(^3,5\). To incorporate indirect coupling within a statistical framework of analysis, Heron\(^6\) developed Advanced Statistical Energy Analysis (ASEA) and validated the approach with longitudinal excitation of a series of six one-dimensional rods. The converged result agreed well with the exact response, whereas SEA overestimated the vibration response for subsystems that were physically disconnected from the source subsystem. ASEA has recently been successfully applied to a plate junction incorporating a periodic ribbed plate\(^7\).

Previous applications of ASEA have only considered one wave type. In this paper, ASEA is applied to beam frameworks where there are (a) only bending waves and (b) bending and longitudinal waves. To assess the application of SEA and ASEA to frameworks of beams, an isolated L-junction of beams junction is initially considered before moving on to a rectangular beam frame-
work. In this work, both SEA and ASEA are used to predict the power transmission properties, and are validated against FEM.

2. Theory

The SEA matrix is given in Lyon and DeJong\textsuperscript{1}. ASEA theory is described by Heron\textsuperscript{6}, and is implemented using the more detailed algorithm described by Yin and Hopkins\textsuperscript{7}. The main difference between SEA and ASEA is that in ASEA the total modal energy is split into two parts, available power per unit modal energy and unavailable power per unit modal energy which is tracked as it propagates across the beams. The accuracy of ASEA depends on the ASEA level which describes the times that initial power is tracked in the source subsystem. In this paper, the levels numbers used are chosen by the number of total subsystems plus two.

The theory used to calculate the L-junction transmission coefficients for bending only model are taken from Craik\textsuperscript{8}, and the bending and longitudinal wave model under two wave type excitations on L junction\textsuperscript{9-11}. All the transmission coefficients are checked with energy conservation relationship in which the total transmission coefficient equals zero perfectly.

The finite element modelling in this paper is carried out with Abaqus version 6.12. B33 elements (2-node cubic) are used which implement Euler-Bernoulli theory that does not allow for transverse shear deformation. Ten sets of rain-on-the-roof excitation are applied along the length of the source beam which allows calculation of a mean response with 95\% confidence intervals.

3. Beam constructions

Two models are considered: (a) an L-junction of two beams (Fig.1 (a)) and (b) a two-dimensional rectangular beam network (Fig.1 (b)) which is made of two of these L-junctions. Beams 1 and 2 in the L type junction have the same cross-sectional dimensions, width 0.02m \times thickness 0.01m, but different lengths, 1.3 m and 1.0 m respectively. The material properties correspond to Perspex with Young’s modulus $E=6.9\times10^9$ N/m\textsuperscript{2}, density $\rho=1250$ kg/m\textsuperscript{2} and internal loss factor $\eta_{int}=0.06$.

![Figure 1](image_url)

**Figure 1.** (a) Isolated L-junction (bending only model), (b) Rectangular beam framework (bending only model), (c) Isolated L-junction (bending and longitudinal model), (b) Rectangular beam framework (bending and longitudinal model).

For the bending only model, all junction nodes are pinned and only have rotational degrees of freedom, so that only bending wave motion is considered as indicated in Fig.1 (a) and (b). To allow generation of both longitudinal and bending waves at the beam junctions, the junction nodes are unpinned as shown in Fig.1 (c) and (d).

The frequency range considered for the FEM, SEA and ASEA models covers the one-third octave bands from 10 Hz to 20 kHz.
4. Results and discussion

4.1 Mode count of the 1.0 m and 1.3 m beams

Low mode counts tend to occur in beam systems, particularly for longitudinal wave motion in the low-frequency range. Fig.2 shows the local mode counts for bending and longitudinal modes for the two isolated beams (lengths, 1.3m and 1.0 m) for one-third octave bands over the frequency range from 10 Hz to 20 kHz. These are calculated assuming a free-free boundary condition at each end. The results are indicative of the local mode counts of single beams in both the L-junction and the rectangular beam framework, although in practice there will be global longitudinal modes with lower frequencies. Note that below 2 kHz, there is only one longitudinal local mode, in the 1000 Hz band for the 1.3m beam and in the 1250 Hz band for the 1.0 m beam.

![Figure 2](image_url)

Figure 2. Bending and longitudinal mode counts in one-third octave bands for the two beam lengths.

4.2 Coupling of bending and longitudinal modes in the L-junction

The driving-point mobility at the end of beam 1 in the L-junction (bending and longitudinal model) is calculated using FEM and compared with the analytical result assuming a free-free beam. The driving-point mobility is determined with a transverse force to excite bending wave motion, denoted $Y_B$, and with an axial force to excite longitudinal wave motion, denoted $Y_L$.

![Figure 3](image_url)

Figure 3. Driving-point mobility at the free end of beam 1 in the L-junction and for an isolated beam.
For transverse excitation, $Y_B$ calculated using FEM and the analytical model are similar but not identical because global modes of the L-junction are excited with the former rather than local modes of the isolated beam with the latter.

Above 800 Hz, values of $Y_L$ calculated using FEM and with the analytical model show close agreement. However, below 800 Hz where there are no local longitudinal modes on the isolated beam, $Y_L$ calculated using FEM has significantly higher values. The peaks that occur in $Y_L$ from FEM below 800 Hz correspond to the peaks in $Y_B$ from FEM. This is due to the axial force exciting longitudinal waves on beam 1 that are converted back into bending waves on beam 1. The bending wave motion has an in-plane velocity component that subsequently appears in $Y_L$.

This has important implications when comparing energy level differences from FEM, SEA and ASEA because calculation of the in-plane energy with FEM is unable to distinguish between in-plane energy associated with longitudinal waves and that associated with bending wave motion.

### 4.3 Vibration transmission on the L-junction

For the L-junction, two models are considered: a bending wave only model (B model), and bending-longitudinal model (BL model), depending on the boundary conditions. Energy level differences calculated from the ratio of source subsystem energy to receiving subsystem energy are shown in Fig.4. The notation used in these figures indicates that, for example, $E_{B1}/E_{L2}$ has a source subsystem for bending waves corresponding to beam 1, and the receiving subsystem is the longitudinal wave subsystem corresponding to beam 2. On all figures, SEA and ASEA are compared against the results from the FEM model. Figure 4 (a) allows comparison of the B with BL models using SEA/ASEA and FEM under excitation of bending modes in beam 1. Figure4 (b) to (f) shows the results from the BL model.

Figure 4 (a) allows comparison of $E_{B1}/E_{B2}$ for the B and BL models. Between 10 Hz and 100 Hz the bending mode count is sufficiently low (approximately one mode per band) that there are large fluctuations in the FEM data; however, both SEA and ASEA give a good estimate of the mean value over this low-frequency range. Above 100 Hz there is good agreement between FEM, SEA and ASEA for the B and BL models. Above 1000 Hz, the energy level difference for the BL model is larger than with the B model because less power is transmitted to bending subsystem B2 when longitudinal waves are generated at the junction. Note that for this directly connected receiver subsystem there is no advantage in using ASEA instead of SEA because indirect coupling is negligible and the propagation losses are small.

Figure 4 (b) shows $E_{B1}/E_{L1}$ and Fig.4 (c) shows $E_{B1}/E_{L2}$ with excitation of bending modes in beam 1 and receiving subsystems which consider the longitudinal mode energy. There are only local longitudinal modes (free-free) for beams 1 and 2 at 890 Hz and 1167 Hz respectively. For this reason there are large differences between FEM and SEA/ASEA below 800 Hz in Fig.4 (b) and below 1 kHz in Fig.4 (c). It is not appropriate to compare FEM with SEA/ASEA when bending modes cause in-plane motion below the fundamental longitudinal mode. Above 1.25 kHz there is closer agreement between FEM and ASEA rather than with SEA in Fig.4 (b). However, in Fig.4 (c) there is no significant difference between SEA and ASEA.

Figure 4 (d) shows $E_{L1}/E_{L2}$, Fig.4 (e) shows $E_{L1}/E_{B1}$ and Fig.4 (f) shows $E_{L1}/E_{B2}$ with excitation of longitudinal modes in beam 1 and receiving subsystems which consider either bending or longitudinal energy. Below the fundamental longitudinal mode, FEM overestimates the longitudinal modal energy due to in-plane motion from bending modes as discussed in Section 4.1. Above the fundamental longitudinal mode, FEM shows better agreement with ASEA than SEA, particularly above 5 kHz for $E_{L1}/E_{L2}$ and above 1 kHz for $E_{L1}/E_{B1}$. Above 1 kHz, the biggest difference between SEA and ASEA is about 3 dB in $E_{L1}/E_{L2}$, although ASEA tends to be closer to the mean value of FEM than SEA.

In conclusion, only small improvements are gained by using ASEA on a small system such as an L-junction at high-frequencies. This example has also highlighted the problems in validating the
BL model below the fundamental longitudinal mode. Above the frequency at which successive one-third octave bands have at least one local mode on each beam subsystem, there is good agreement between FEM and statistical models based on SEA or ASEA.

Figure 4. Vibration transmission on the L-junction.

4.4 Vibration transmission on the rectangular beam framework

Excitation is applied on beam 1 in the rectangular beam framework to excite (a) bending modes or (b) longitudinal modes. The bending only model (B model) is compared with bending and longitudinal wave model (BL model) under bending excitation in Fig.5 (a) and Fig.5 (b). Fig.5 (a) to (e) show results under bending excitation and Fig.5 (f) to (j) show results for the BL model under longitudinal excitation. Because of the symmetry of the structure, energy levels for the subsystems representing beam 3 are not shown here.
Referring to Fig. 5 (a), the BL model predicts lower bending vibration levels than the B model does for bending subsystem, B2; this becomes more apparent with increasing frequency. When longitudinal waves are generated at the junction, the total energy is distributed between bending and longitudinal energy; hence less energy is transmitted into the bending subsystems which are physically connected to the source subsystem, B1. However, in Fig. 5 (b), bending subsystem B4 has a higher vibration level in the BL model than that in the B model. This is due to additional transmission paths in the BL model (compared to the B model) between B1 and B4 where energy is carried by in-plane wave energy in the intermediate subsystems and then converted back to bending wave energy in the receiver subsystem.
Figure 5. Vibration transmission on the rectangular beam framework.

In Fig.5 (b), discrepancies occur between SEA and FEM above 5 kHz for the B model and above 10 kHz for the BL model. For both B and BL models, SEA overestimates the vibration level for bending subsystem B4. For the B model, the difference between SEA and FEM increases with frequency up to ≈13 dB at 20 kHz. Discrepancies between SEA and FEM also occur for the BL model with $E_{B1}/E_{L4}$ in Fig.5 (e), $E_{L1}/E_{B4}$ in Fig.5 (h) and $E_{L1}/E_{L4}$ in Fig.5 (j).

For both the B and BL models, receiver subsystems corresponding to beams 2 and 3 which are physically connected to the source subsystem representing beam 1, always give good agreements between SEA and FEM as long as there is at least one local mode in the frequency band. This conclusion is same to that found in the isolated L junction.

In Fig.5 (b), (e), (h) and (j), ASEA provides significantly better estimates than SEA for non-adjacent subsystems at high frequencies. This is due to ASEA taking the propagation losses into account. For systems of coupled plates modelled with ASEA, tunnelling mechanisms tend to result in higher energy level difference for the most distant subsystems hence SEA tends to underestimate the energy. However with this rectangular beam framework, SEA overestimates the energy for the most distant beams, either B model or BL model. This phenomenon also occurred with Heron's linear array of six-rods in which only longitudinal vibration was considered. In addition, Fig.5 (i) shows that compared to SEA, ASEA provides better agreement with FEM for adjacent coupled subsystems.

In conclusion, for the rectangular beam framework, ASEA is needed to provide accurate predictions at high-frequencies because SEA does not account for propagation losses.
5. Conclusions

SEA and ASEA models considering bending only and bending and longitudinal wave motion have been investigated on an L-junction and a rectangular beam framework. The energy level differences calculated from SEA and ASEA are compared with FEM in one-third octave bands between 10 Hz to 20 kHz. The results show that both SEA and ASEA agree well with FEM for physically connected subsystems as long as there is at least one mode in the frequency band. However, SEA overestimates the energies for subsystems which are not directly connected to the source subsystem. The existence of indirect coupling to more distant receiver subsystems is essentially a tunnelling mechanism and would normally cause SEA to underestimate the receiver subsystem energy. However with the beam systems considered in this paper there is an overestimate of receiver subsystem energy by SEA which is mainly due to losses incurred during wave propagation across the beams. ASEA successfully improves the predictions by accounting for these propagation losses in the unavailable modal energy that is not accounted for in SEA.

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REFERENCES