In this paper, we apply dynamic mode decomposition (DMD) on time accurate simulations of the pressure distribution on a realistic full-scale nose landing gear configuration in order to identify noise generating structures on landing gear surfaces. The simulated pressure data is obtained from DES simulations using the commercial software STAR-CCM+ by CD-adapco. The dynamics of the surface pressure on a tyre are discussed and the DMD modes are computed from instantaneous pressure snapshots. The far-field noise is determined via the Ffowcs Williams-Hawkings analogy, where a given frequency band source term can be reconstructed by choosing an appropriate number of DMD modes.

1. Introduction

The noise of deployed landing gears is a significant contributor to airframe noise and is currently an active research topic. The landing gear geometry is a complex assembly, composed mainly of bluff bodies spanning a wide range of scales. Consequently, the noise generated is broadband in nature and is affected by interactions between the wakes of various components.

Early schemes to predict landing gear noise relied on semi-empirical techniques. For instance, Fink [1] devised an empirical method by fitting equations to a large dataset for 2 and 4 wheel gears. This encouraged the creation of a standard on aircraft noise prediction developed at NASA, the Aircraft Noise Prediction Program (ANOPP) [2].

More sophisticated semi-empirical methods have been devised to address some issues of the early schemes, such as the underprediction of high frequencies. Guo [3] introduced a method that separates the noise sources into three groups – low, medium and high frequencies – and relies on statistical descriptions of the pressure fluctuations. Lopes et al. [4] composed a framework, the Landing Gear Model and Acoustic Prediction (LGMAP) code, which is capable of predicting the noise of individual components. Within this framework, the landing gear is viewed as an assembly of fundamental acoustic elements.

Though very promising, semi-empirical methods have a limited reliability in predicting the impact of add-on noise reduction devices or new unconventional designs. Therefore, there is a need for reliable aeroacoustic simulations of landing gears. In particular, hybrid approaches combining computational fluid dynamics (CFD) with acoustic analogies have received sustained attention.

Regarding the flow field computations, DNS is out of the question for complex geometries at high Reynolds number. Nevertheless, the prediction of the noise sources relies on the resolution of turbulent phenomena in the landing gear region. Detached-Eddy Simulation (DES) studies of simplified...
Landing gears have had some success, by achieving reasonable comparisons with experimental noise measurements. Vuillot et al. [5] used Delayed-DES combined with the Ffowcs Williams-Hawkings (FWH) acoustic analogy to predict the far-field noise of a standard simplified Gulfstream gear set by the Benchmark for Airframe Noise Computations. They obtained encouraging agreement with experimental far-field noise data, even with a relatively coarse grid of 12 million cells. Wang et al. [6] performed an Improved Delayed-DES (IDDES) of the Boeing four-wheel Rudimentary Landing Gear and obtained agreements within 2dB of acoustic measurements.

The present paper follows the approach mentioned above of combining DES with FWH to predict the far-field noise of a realistic two-wheel nose landing gear. The main objective is to investigate a model reduction method with a view to reducing the cost of the noise propagation and gaining some insight about the noise generating structures. This method is examined by applying Dynamic Mode Decomposition (DMD) to surface pressure snapshots on one of the tyres before using the pressure data as input in the acoustic analogy.

2. Problem formulation

2.1 Landing gear geometry

This work forms part of a European Clean Sky project called Advanced Low noise Landing (main and nosE) Gear for Regional Aircraft (ALLEGRA). The landing gear geometry was set by the topic manager, Alenia Aermacchi.

The geometry and computational domain considered are illustrated in figure 1. The diameter of the wheels, $D_{\text{wheel}} = 0.58$ meters, is used as a reference length scale. The streamwise extent of the domain is $L_x/D_{\text{wheel}} = 30$, the height is $L_z/D_{\text{wheel}} = 10$ and the width is $L_y/D_{\text{wheel}} = 18$. Note that the geometry has a symmetry plane $y = 0$. The model geometry includes the main gear assembly body as well as other features which are likely to impact the emitted noise: the lower part of the fuselage, the doors, the retraction actuators and the bay cavity.

![Figure 1: (a) Perspective view of the computational domain and (b) detailed view of the gear assembly.](image)

2.2 Flow computations

The commercial flow solver STAR-CCM+, version 8.02, was used to perform the computational fluid dynamics simulations. IDDES of the compressible flow equations were carried out. The turbulence model is a modified version of the $k-\omega$ shear stress transport formulation which emulates an LES subgrid-scale model in separated regions.

The flow governing equations are approximated via a cell-centered finite-volume method. The convective flux is discretized with a second-order hybrid upwind/bounded-central scheme while the
diffusive flux is evaluated with a centered scheme. The discretization approach results in a linear algebraic system of equations which is solved iteratively. The temporal discretization is second order.

A uniform velocity field \( \mathbf{V}_\infty = (40, 0, 0) \text{ms}^{-1} \) is imposed as a boundary condition on the inlet plane, corresponding to a freestream Mach number of 0.12. On the outlet plane, a Dirichlet condition imposes atmospheric pressure. A no-slip condition is applied on the top boundary, because this boundary corresponds to the tunnel floor in the wind tunnel experiments to be conducted within ALLEGRA. Finally, the sides and bottom boundaries are assumed to be far enough from the body to be modeled safely with a symmetry boundary condition.

The mesh is unstructured and predominantly composed of hexahedral cells, with a so-called prism layer near solid walls. The resolution is \( y^+ \approx 1 \) around the walls of the gear assembly, whilst it is higher on the upper wall, on the fuselage skin and on the surfaces of the bay. A mesh of 12 million cells is used throughout for the results presented here. It must be noted that this mesh is relatively coarse, especially in the far-wake region. But it has been observed that reasonable noise predictions can be obtained with similar resolutions (see, e.g., Vuillot et al. [5]). In addition, the emphasis in this paper is on the model reduction method. Comparison with a finer mesh is under way and will be discussed in a future study.

### 2.3 Acoustic waves propagation

The DES simulations are capable of computing the generation of sound and its propagation within the near-field of the landing gear. However, the direct approach of computing far-field noise with the full Navier-Stokes equations is not tractable in most cases. To propagate the sound to far-field receivers, acoustic analogies are obtained by rewriting the compressible Navier-Stokes equation in the form of a wave equation for a quiescent medium. Source terms appear on the right-hand side of the equation that are responsible for the sound generation.

In the present paper, we consider the Ffowcs Williams-Hawkins [7] analogy. Formulation 1C [8] was selected as it takes a simplified form in the case of a stationary body in a wind tunnel. One key assumption in FWH is that the mean flow is uniform in the propagation region. This is of course not true here, but previous studies on landing gear acoustics using FWH have shown that acceptable results can be obtained nonetheless (e.g., [5], [6]).

FWH takes as input a time-series of pressure fluctuations on a source surface and outputs the resulting noise at far-field observers. Formulation 1C for a wind-tunnel configuration can be written as

\[
p'(x, t) = \int_{f=0} C(x - y, M_0) A(y, \tau_e) dS
\]  

where \( p' \) is the fluctuating pressure at time \( t \) and at an observer located at \( x \), \( C \) is a time-independent function of the source-observer distance and the Mach number, and \( A \) is a function of the pressure and its time derivative at points \( y \) on the source surface \( S \) (at retarded time \( \tau_e \)) [8]. The process involves computing the integral over the source surface defined by \( f = 0 \) for every time step of the series. This becomes expensive for large datasets.

The main objective of the present paper is to obtain insight into the fluctuating pressure field on the source surface and to reduce the computational cost of the noise propagation procedure. For this purpose, we perform a DMD of the source surface fluctuations before computing FWH.

We obtain \( N+1 \) snapshots \( s_n \) from the DES, where \( s_n = A(y, t_n) \). The aim is then to find a set of \( N \) vectors onto which to decompose the flow so that

\[
s_n \approx \sum_{j=1}^N \alpha_j(t_n) \phi_j(y).
\]
In the context of DMD, it is assumed that the time-dependence of the modes can be expressed as

\[ \alpha_j(t_n) = (\lambda e^{i\omega_j \Delta t})^n = \mu_j^n, \]  

(3)

where \( \Delta t = t_n/n \) is the time step step between the snapshots. That is, if \( \lambda = 1 \), \( \alpha_j \) is a periodic function of time with frequency \( \omega_j/(2\pi) \).

Let \( \Phi = [\phi_1 \phi_2 ... \phi_N] \) and \( \mu = [\mu_1^T \mu_2^T ... \mu_N^T]^T \). Then, by (2) and (3),

\[ s_{n+1} = \Phi \mu^{n+1} = \Phi \cdot \text{diag}(\mu) \cdot \mu^n \]

(4)

and, if \( \Phi \) is invertible,

\[ s_{n+1} = \Phi \cdot \text{diag}(\mu) \cdot \Phi^{-1} \Phi \cdot \mu^n = Ks_n \]

(5)

In other words, by finding the eigenvalues and eigenvectors of \( K \) it is possible to express respectively the time and space dependent parts of the decomposition written in (2). Although \( K \) is not known, it can be computed from the snapshots \( s_n, n = 1, ..., N + 1 \), following one of the methods explained in [9] and developed in [10].

When using this approximation to compute FWH, the time iterations are replaced by iterations over the modes. Therefore, if a number of modes \( m \ll N \) is sufficient to obtain a satisfactory approximation of \( A \), the far-field noise can be obtained at a lower cost. The expense of computing the DMD is offset if the noise is to be propagated to a large number of observers.

### 3. Results

In this section, we present and discuss the main results of this study. First, a very brief description of the computed flow field is given. Since the chief interest of the present paper is the study of the model reduction applied to FWH, the focus is put on the surface pressure which will be fed as an input to the acoustic analogy. Only impermeable surfaces on the landing gear assembly were considered in this work so far. To remain concise, only the surface pressure on one of the tyres is presented here. But similar results were observed with the surface pressure on the door and the oleo strut.

After discussing the surface pressure, the DMD is applied to snapshots of the surface pressure on the gear tyre. The procedure and the resulting DMD modes are discussed. The pressure dynamics are reconstructed with a specified number of modes. Finally, FWH is performed using both the full tyre pressure snapshots and the DMD-reconstructed signals. The far-field noise at 5 observers is examined.

#### 3.1 Flow field and surface pressure

Figure 2 shows the time-averaged velocity magnitude in the symmetry plane. Part of the fuse-lage is hidden to show the flow inside the bay. The mean velocity is low in most of the bay, except near the opening where the main strut and the folding stay are located. As expected, an acceleration zone over the wheels and a large wake behind the gear assembly can be observed.

Figure 3 shows contours of time-averaged and rms surface pressure on the gear tyre. A stagnation point with high pressure on the upstream side is evidenced by Figure 3a. The rms fluctuations are small on the upstream side, suggesting that the near-wall flow in this region is nearly steady. The largest fluctuations occur on the downstream side, where the flow is separated. The fluctuations are small compared to the mean, reaching only about 0.5% of one atmosphere. Interestingly, there is a
A small zone of low mean pressure in the upper part of the downstream edge. This zone is associated with high rms fluctuations.

The distribution of the rms pressure fluctuations along the tyre circumference is plotted in Figure 4. The angle convention is such that $\theta = 0$ corresponds to the furthest downstream point of the tyre and $\theta$ is positive on the fuselage side. Each curve corresponds to the circumferential distribution for one value of $y$; i.e. in a given vertical plane. In figure 4a, the lines which are more red indicate a $y$-value closer to the inner edge of the tyre, while in figure 4b the lines become more blue as the $y$-value goes from the symmetry plane of the wheel towards its outer side. In the wheel symmetry plane, a peak $P_{rms}$ is visible close to $\theta = 0$. The wake appears to widen towards the inner side. On the outer side, the main peak is attenuated and a secondary peak emerges around $\theta = -\pi/4$. 
3.2 Modal decomposition

The DMD modes of the tyre surface pressure are computed from 1720 snapshots with a constant sampling time $T_s = 2.5 \cdot 10^{-4}$ seconds. The surface mesh on the tyre is composed of roughly 56000 cells. The objective is to reconstruct the dynamics of the surface pressure with only a small number of modes.

A special note should be made here on the ordering of the modes. The scaling and ordering of the modes should be defined so as to select the modes which contribute most to the dynamics of the system. The modes can be sorted by energy, as in proper orthogonal decomposition. In this paper, the modes were sorted in ascending order of their associated frequency $\omega_j$. Surprisingly, this order provides a better fit to reconstruct the surface pressure snapshots in this case. This arises here because of modes with a high $||\phi_j||$ but a growth rate $|\mu_j|$ smaller than 1, which leads to damping of these modes over time. Using $|\mu_j|$ as a weighting factor was also considered (scaling the modes by $||\phi_j|| \cdot |\mu_j|$) but provided a worse match than the criterion based on $\omega_j$. The underlying cause of these small growth rates may signal the fact that the time period of the snapshots used is not long enough. Further investigation is under way to elucidate this.

Figure 5: Contours of $|\phi_j|$ for the six DMD modes with largest $||\phi_j||$ (among the first 50 modes). Each mode is associated with a frequency $\omega_j = 2\pi f_j$ (for modes 0, 36, 18, 9, 12, 45 respectively $f_j = 0, 84.8, 43.5, 21.7, 28.2, 105$ Hz.

As a result of the sorting by frequency, the reconstructed signal behaves as a low-pass filter of the full surface pressure dynamics. A range of frequency of interest can then be selected, and a good approximation can be constructed within this range. For instance, it was found that the band from 0 to 105Hz, which contains several coherent structures, can be represented with 50 modes. Figure 5 plots contours of the 6 DMD modes with largest $||\phi_j||$ among these first 50 modes.

These modes are thus associated with frequencies in the range 0 to 105Hz. As expected, mode 0 is associated with a zero frequency and corresponds to the mean pressure. The other modes exhibit some rather complex structures. Small regions of alternate high and low amplitude can be observed in modes 9, 18 and 36, clustered around the edge between the tyre and the rim.

Figure 6 shows the surface pressure signal at a single point with maximum x-coordinate (the
most downstream point). The time-series is compared with a reconstruction of the signal using the first 50 modes. A good agreement can be observed on the time domain plot. The power spectral density illustrates the low-pass behaviour of the reconstructed signals since only low frequencies have been selected.

Figure 6: Surface pressure at the point of maximum x-coordinate in the time domain (left) and as a power spectral density in the frequency domain (right). The full signal from DES is compared with the reconstructed signal using the first 50 DMD modes.

### 3.3 Far-field noise

Finally, we investigate the far-field noise computed with the FWH analogy. Both the full surface pressure snapshots and the DMD model reconstructed with 50 modes are fed to a FWH solver. The noise is computed at 5 far-field observers located in the symmetry plane of the tyre, all at a distance $r = 10$ meters from the centroid of the tyre. The observers, numbered from 1 to 5, are placed at uniformly spaced polar angles $[-\pi, -3\pi/2, -\pi/2, -\pi/4, 0]$ respectively, again following the polar angle convention introduced above.

Figure 7: Power spectral density of pressure fluctuations at 5 far-field observers. The noise obtained with the full snapshots (solid lines) and with the first 50 DMD modes (symbols) are compared. Different colours correspond to different observers.

Figure 7 shows the power spectral density of pressure fluctuations at the 5 observers. Solid lines correspond to the full signal. The first striking observation is that the noise is omnidirectional,
especially in the low frequencies. Several peaks in the spectrum can be distinguished at low frequencies. Interestingly, these correspond to some of the modes shown in figure 5. For example, the peak around $f = 105$ Hz corresponds to mode 5, whilst the hump around $f = 45$ Hz corresponds to mode 2. The first large peak is in fact associated with the mode of eighth largest norm (not shown). The modes identified are indeed noise-generating structures. Further work will focus on understanding their mechanisms.

4. Conclusions

We have discussed the use of DMD to obtain some insight into the noise sources on an impermeable surface and reduce the cost of propagating the noise to the far-field via the FWH acoustic analogy. As a first approach, this combination was simply applied to the surface pressure on the tyre of a realistic nose landing gear geometry.

It was found that sorting the modes by frequency $\omega_j$ yields better results than selecting the modes with highest $||\phi_j||$ in our case. This is caused by the presence of transient energetic modes with a small growth rate, which are damped out over time. This phenomenon is being investigated further. Nonetheless, our approach allows one to reconstruct the surface pressure dynamics over a specified frequency band and hence to predict the far-field over this range accurately, and at a reduced cost. Isolating modes with a high norm, we observed that their spatial structure is rather complex. Small alternating regions of positive and negative amplitude can be seen along the tyre-rim interface. Furthermore, it was shown that these isolated modes are responsible for peaks in the noise spectrum. In future work, this procedure will be extended to impermeable surfaces enclosing the entire landing gear assembly.

REFERENCES