FORCED VIBRATION OF THE PARTICLE-DAMPING BEAM BASED ON MULTIPHASE FLOW THEORY OF GAS-PARTICLE

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In this paper, we focus our attention on the particle damping effect of the cantilever beam under the forced vibration, which has potential application to variety of systems. In this work, the nonlinear damping effect of the particle damper is interpreted as an equivalent viscous damping based on multiphase flow theory (MFT) of gas-particle. It is worth noting that the co-simulation of the COMSOL Multiphysics live link for MATLAB is conducted using this developed model creatively. The dynamic response of the structure with the particle damper can be predicted in a finite element model of a structure. And experimental verifications are performed. Good correlations are obtained between the analytical results and the experimental data as show that the theoretical work in this paper is valid. Meanwhile, the influence of the position, packing ratio of the damper on damping effect of particle damping are studied. Tungsten carbide particles are embedded in the enclosure attached to the beam. The particle damping is found to be remarkably effective. Although it is nonlinear, a strong rate of energy dissipation is achieved within a broadband frequency range. The results show that the particle damping effect is heavily dependent on the vibration amplitude of the structure and packing ratio.

1. Introduction

High cycle fatigue is a serious problem in engineering machinery. Results showed that the high cycle fatigue crack may be the determinable factor for metal cracking. Metal cracking due to high cycle fatigue increase maintenance and inspection costs and has even resulted in the accident. Current design trends toward higher engine performance will result in higher aerodynamic loads which will increase the high cycle fatigue problem. High cycle fatigue is attributed to the high vibratory stresses that occur during resonant response. Adding damping is an attractive approach to reducing vibratory stresses [1].

Currently in engineering systems, widely used damping devices include viscoelastic materials, frictional devices, fluid dampers, tuned absorbers and isolators, etc [2]. Because the constitutive relations of viscoelastic materials vary with the frequency and temperature, much work is still needed to be studied further. Frictional dampers dissipate energy via sliding friction across the interface between two solid bodies. However, their performance is oftentimes a function of the sliding interface conditions which may be changed with the time. Fluid dampers are commonly used as passive energy dissipation devices for seismic protection of structures. However, the increase in temperature may be of concern due to the potential for heat-induced damage to the damper seals [3].
Tuned absorbers and isolators are only effective in a small frequency range and are sensitive to operating condition changes [4].

One attractive alternative is particle damping. Particle damping is a derivative of single-mass impact damper which can perform well even in severe environments where traditional passive damping methods such as the use of viscoelastic materials are ineffective. Granular particle damping is a promising technique of providing damping with granular particles placed in an enclosure attached to or embedded in the holes drilled in the vibrating structure[5, 6], which has been thoroughly studied over the years with a large volume of books and papers in the published literature. Friend et al., many authors modeled a bed of particles as a single particle [7-11], estimating the performance of the particles damper based upon this equivalent particle. Liu et al.[12] who estimated the damping contribution of the particles damper as an equivalent linear viscous damping. Fowler et al.[7, 13], Chen et al.[5], and Saluenea et al.[14]respectively developed different models, which all get involved in the use of the particle dynamics. There have been considerable researches in the area of particle damping, and analytical models have been developed. However, applications in the literature have been based on heuristic evaluations of particle damping. Despite the increasing use of particle damping technology [15, 16] the modeling of the particle damper remains difficult due to a number of problems.

One of the principal reasons in using the particle damper is that the particle damping phenomena are the remarkable nonlinear behavior making them complicated to design [17]. The design problem is be closely related with the large number of parameters, such as the arrangement locations of the particle dampers, volumetric packing ratios, the geometry of the enclosure, the shape and material of particles, the level of displacement and acceleration of the primary structure[18].

The particle damping is an attractive alternative for engines, specifically engine blades (non-rotating as well as rotating). These structures are subject to high levels of harmonic excitation. The nonlinear effects averaged out during harmonic excitation will be a concern. In this paper, a novelty approach based on multiphase flow theory (MFT) of gas-particle is developed with greatly reduced the complexity of the analysis and computational cost, and capturing the physics nature of granular damping. It is worth noting that the co-simulation of the COMSOL Multiphysics live link for MATLAB is conducted using this developed model creatively. The dynamic response of the structure with the particle damper can be predicted in a finite element model of a structure. A cantilever beam under forced vibration is chosen as the test specimen to validity this theory for prospecting particle damping performance.

2. Model development

The energy dissipation mechanisms for particle damping include friction and collision between the particles and the cavity wall and inter-particles. Bai et al. [19] pointed out that although the interaction between the particle and cavity wall is very important for energy transfer, it does not play a very important role for energy dissipation by the use of discrete element method (DEM). Therefore, we will only consider the interaction between the particles, while ignoring the interaction between the particles and the cavity wall from the view of energy dissipation.

Recently, some researchers have performed limited studies to mathematically evaluate the dissipative properties of granular materials using the multiphase flow theory of gas-particle approach are obtained [20]. Wu et al. [20] explored a theoretical model based on multiphase flow theory of gas-particle to evaluate the granular particle damping characteristics. A cantilever particle-damping beam is equivalent to a single-degree-of-freedom (SDOF) system by focusing on the motion at the free end of the beam. Fang and Tang [4] further validated the multiphase flow theory of gas-particle approach based on previous work of Wu et al. [20], and carry out detailed studies under various forced excitation levels, granular packing ratios, and enclosure dimensions, where the damping effect due to different energy dissipation mechanisms is quantitatively analyzed. However
the particle damping system was also considered a single degree of freedom system. An improved analytical model using the multiphase flow theory is developed to describe the particle damping characteristics based on previous work [20] by Wu et al. [21, 22]. The equivalent viscous damping coefficient due to inter-particle collisions and friction as the following forms:

\[
c_{eq} = c_1 |\dot{x}|^{1/2} + c_2 |\dot{x}| - c_3 |\ddot{x}|^{2/3} + c_{11} |\dot{x}| + c_{21} |\dddot{x}| - c_{31} |\dddot{x}|^6
\]

where \(c_1\), \(c_2\) and \(c_3\) are the coefficient related to collision, \(c_{11}\), \(c_{21}\) and \(c_{31}\) are the coefficient related to friction. The derivation process of the formulas and the description of parameters can be found in our previous work [21, 22].

The beam is modelled by finite element method using discrete Kirchhoff quadrilateral element. The damping contribution of particle damper is modelled as a spring mass system (see Fig.1); however the system does not exhibit any stiffness.

\[
\begin{align*}
\text{(a)} & \quad \text{Particle damper} \\
\text{(b)} & \quad \text{model}
\end{align*}
\]

Figure 1. (a) Particle damper and (b) model.

The damping contribution of the particle damper is modelled by the equivalent viscous damping coefficient. Consider the intrinsic structure damping and particle damping, the motion of the global system is governed by

\[
[M] \cdot \{\ddot{X}\} + [C] \cdot \{\dot{X}\} + [K] \cdot \{X\} = \{f\}
\]

where \(\{X\}\) is the nodal displacement of the beam, \(\{f\}\) is the external force applied to the system. \([K]\) and \([M]\) represent, respectively, the stiffness and mass matrix of both the beam and the particle damper

\[
[M] = [M_p] + [M_{eq}]
\]

where \([M_p]\) is the mass matrix of the beam and \([M_{eq}]\) is the additional mass matrix caused by the presence of the particle damper where \([M_{eq}]\) represents the mass of the particle damper with the particles. \([C]\) which represents the damping matrix of the global system is given by

\[
[C] = [C_o] + [C_{eq}]
\]

where \([C_o]\) is the proportional damping matrix of the beam and \([C_{eq}]\) represents the additional damping matrix caused by the particle damper.

One can analysis the responses and the damping characteristics of structures with granular particles in a finite element model of a structure. The implementation of this modelling was performed in the COMSOL Multiphysics environment.

3. Numerical simulation and experimental validation

The beam dimensions are: Young's modulus 66 GPa, density 2828 kg/m3, length 0.38 m, width 0.02 m and height 0.006 m. The mass of the enclosure is 14.52 g and its interior diameter and height are 16 mm and 20 mm, respectively. The particle is made of tungsten powder whose density is 17000 kg/m3, and the mean diameter of particles is 0.3 mm. The restitution coefficient of particles is 0.6 on the basis of testing. The kinetic friction coefficients between the individual particle and between the particles and the wall of the cavity are 0.3 and 0.2 respectively, from experimental re-
To validate the finite element model, the first three natural frequencies of the system formed by the beam without particle damper are compared with those of the experiment. FEA of the beam is performed using COMSOL. Results from experimental impact testing of the structure indicate that the first three fundamental mode of the structure are 57.49Hz, 359.85 Hz and 1005.97 Hz. FEA indicate that this first three fundamental mode are 58Hz, 354 Hz and 914Hz. These values differ slightly from the experimentally determined, the relative error of FEA values to the experiment values are respectively 0.88%, 1.6% and 9.1%.

The first three natural frequencies of the system formed by the beam without the particle damper are compared with those of the experiment. The variation of the third natural frequency exceeds 5%, the connection between vibration exciter and beam adding a constraint to the beam may lead to the deviation in calculating the high natural frequencies. We only consider the first two natural frequencies of the system in the simulation.

![Figure 2. Schematic of the experimental apparatus.](image)

A schematic of the test set-up is shown in Fig.2. The experimental model consisted of the primary structure (cantilever beam) and an aluminium enclosure containing tungsten particles. We choose a beam for this study the reason is that it is an infinite DOF system as opposed to the single DOF systems usually studied in the literature [1, 5, 7, 14, 17, 23-25]. This would allow us to investigate the broadband frequency effect of particle damping. The enclosure that was partially filled with tungsten particles was attached to the beam which was itself attached to a Smart Shaker (MT K2004E01) itself with an amplifier providing an excitation force. The force and acceleration signals were measured with the force sensor (Dytran1051V4) having a mass of 28 g and acceleration transducer with a mass of 0.6 g (Dytran 3133B1). A Dynamic Signal Analyzer (M+P SO Analyzer) was used to collect and process the data.

The developed experimental method for identification the damping of the particle damper consisted in measuring the evolutions of both force and acceleration of the system versus the frequency of excitation. In order to characterize the damping characteristics, a sine-sweep excitation was used with a small frequency step. The frequency response functions (FRF) acceleration /force of the beam are successively measured at the end of the beam.

![Figure 3. FRFs of the beam without particle damper.](image)

Figure 3 shows a simulation of the FRFs (acceleration/force) of the undamped beam (without effects of particle damper). It is noted that the simulation results in COMSOL differ slightly from...
the experimental results. That is to say, the actual intrinsic structural damping in the experimental beam is not exactly the damping considered in the COMSOL model.

Figure 4 shows a comparison of numerical and experimental responses of the system with particle damper ($\alpha_{mp} = 40\%$). The particle damper is exerted at the L, 0.78L and 0.78L of the beam length, respectively (see Fig.5). There are also differences at the peak amplitudes in the vicinity of the natural frequencies between the numerical and experimental responses for the system with particle damper. These differences stem from the hypothesis considered when modelling the system. Nevertheless, the analysis of simulation results of the structure with particle damper shows the ability of the model developed in this work to predict the dynamic behaviour of the structure taking into account the effect of damping by particles.

![Figure 4. FRFs of the beam with particle damper: (a) at the L; (b) at the 0.78L; (c) at the 0.78L.](image)

### 4. Experiment research

The experimental process is organized in two parts:

In the first part, the solid beam without particle damper is first made in order to characterize the modal behavior of the primary structure. Then the measurement is repeated with the enclosure containing 11.85g of tungsten particles with the diameter 0.3mm. The enclosure mass is 14.52g. Last this test is made with a fixed added mass block equal to the mass of the enclosure and particles. The mass of the added mass block is 26.37g. The excitation level is kept the same as the one used in the above tests. Fig.2 shows the experimental apparatus according to the considerations mentioned above. The exert positions of added mass block and the particle are made respectively at the L, 1.78L and 0.47L of the beam length (see Fig 5).

![Figure 5. Emplacements of the particle damper on the beam.](image)

The second part, in order to reveal the particle damper locations mass packing ratios and on the dynamic response of the beam, an experimental investigation for a beam treated by the particle damper is investigated. The particle damper arrangement sees Fig.4. The exert position of the particle damper is respectively on the L, 1.78L and 0.47L of the beam length respectively, and the exciting position is on the 0.4L of the beam length. The mass packing ratio of the particle is respectively $\alpha_{mp} = 40\%$, $\alpha_{mp} = 80\%$ and $\alpha_{mp} = 95\%$. The mass packing ratio $\alpha_{mp}$, which is defined as the actual packing mass of particles to the maximum permissive packing mass of particles in a cavity, is also introduced to describe the packing condition of the damper.
A magnetic shaker was used to provide a harmonic excitation force with varying excitation frequency. The signals of both acceleration and force of the excited structure were measured, respectively, using the acceleration transducer and force transducer. The excitation level is kept the same, the frequency response functions (FRFs) acceleration/force of the beam are successively measured at the end of the beam.

In the first part test, the frequency responses of the beam under the broadband random excitation as measured at the point1 (see the Fig. 5). The examinations of these FRFs (without particle damper and with added mass blocks and with particle damper) show the effectiveness of the particle dampers in reducing the vibratory levels of the structure over a wide frequency band (see the Fig. 6). The effect of the particle damper is visible on each one of the three first modes of the beam. It is found that the presence of particles causes an increase of modal damping which can reach quite high levels without significant changes of the natural frequencies and mode shapes compared with this case the beam attached the added mass only. The results show that the particle damping is remarkably effective and that strong attenuations are achieved within a broad frequency range for achieving high damping effect from the use of a minimal quantity of particles. These results reveals that a high values of the damping capability were reached, showing the efficiency of this passive process.

![Figure 6. FRFs of the beam: (a) at the L; (b) at the 0.78L; (c) at the 0.47L.](image)

In the second part test, Fig.7 shows the FRFs (acceleration/force) measured for the different mass packing ratios of the particle damper. In general, with the increasing of the mass packing ratio, the damping effect will be more obvious. It is noticed that in Fig.7, when comparing the modal damping of the system for the three configurations (see the Fig.5); the Fig.7 (a) provides greater modal damping for all modes. In the Fig.7 (b), the damping is nearly unchanged for the second mode. This is due to the particle damper position which coincides with the nodal lines of the second mode. This result confirms that the particle damper efficiency depends on the mode shapes of the beam. In addition, the level of the damping increases when the displacement amplitude increases.

![Figure 7. FRFs of the beam with different mass packing ration: (a) at the L; (b) at the 0.78L; (c) at the 0.47L.](image)
5. Concluding Remarks

In this article, a numerical modelling and analysis technique have been developed for the particles damping structure. In this work, an equivalent viscous damping based on MFT is used to characterize the damping of the particle damper. It is worth noting that the co-simulation of the COMSOL Multiphysics live link for MATLAB is conducted using this developed model creatively. The dynamic response of the structure with the particle damper can be predicted in a finite element model of a structure. Good correlations are obtained between the analytical results and the experimental data as show that the theoretical work in this paper is valid.

Meanwhile, the damping capacity of a continuous particle damping beam is experimentally studied. The experimental verifications prove that: one of the results is that the particle damping is remarkably effective and that strong attenuations are achieved within a broad frequency range. It would facilitate the development of application techniques for achieving high damping effect from the use of a minimal quantity of particles; the second is that total filled particle mass and the particle damper arrangement position appear to have a fairly significant effect on damping for the particle damping structure. As well the particle damper which is attached to the structure in a region of high vibration levels can significantly reduce the vibration of the host structures. As might be expected, changes in the total filled particle mass can lead to a fairly significant shift in the frequency of peak response. It would facilitate the development of application techniques for achieving high damping effect from the use of a minimal quantity of particles.

This method provides an effective instruction to the implementation of particle damping in practice and offers the possibility to analyze the more complex particle-damping system with lower computational cost than DEM and it can be lay a theoretical foundation for the vibration and acoustic radiation response prediction problem for particle damping composite structures.

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REFERENCES


