A SIMPLIFIED MODEL FOR VEHICLE BODY PANEL VIBRATION AND SOUND RADIATION AND INTERIOR BOOMING CONTROL

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The paper analyzes the structural modes and sound radiation of a simply-supported rectangular panel by theory and CAE technology. The result shows that the 1st order vibration can be modeled as a point source radiator. The first mode of the panel is simplified as a Single-Degree-Of Freedom (SDOF) system in which the transfer function characteristics of velocity are analyzed. The relations among the panel’s vibration, sound radiation and SDOF model are established. The paper concludes that the SDOF model is a good substitute to present the panel’s first modal vibration and sound radiation. The mechanism of vehicle interior booming is provided and the control method by tuning vehicle body panel’s mass is proposed. A vehicle interior booming is tested, identified, and finally reduced by tuning the panel’s mass distribution that comes from the SDOF analysis. The analysis results have a good consistent with the test results. The simplified model provides a guideline for vehicle interior booming control.

1. Introduction

The customers’ requirement on riding comfort is increasing. NVH (Noise, Vibration and Harshness) is one the most important indicators of riding comfort. OEMs around world spend huge money on NVH research and development\textsuperscript{1,2}.

There are many vibration sources for a vehicle, such as engine, road, wind and others. A vehicle body consists of many panels, such as dash panel, roof, floor, door, liftgate and hood, etc. The body panels are thin plates similar to speaker membranes. The modal frequencies for the body plates are usually low, from several decades Hz to several hundred Hz. When vehicles are driven on coarse roads, the body plate modes are easily excited which will radiate sound to customers’ compartments. The radiation sound is sometimes resonant with vehicle cavity acoustic modes, which will make the noise worse. The low frequency noise inside compartments is called interior booming noise which makes people very uncomfortable. One of vehicle body NVH research topics around world is to control the body design to eliminate or reduce the interior booming caused by the body panel structures.

Many scholars have studied plate vibration and shell radiation noise, and some of them focused on their research on vehicle body panel vibration and sound radiation. Frank Fahy\textsuperscript{3} did a lot of fundamental research on plate vibration and sound radiation, such as reciprocity method between sound sources and the receivers. Emiel Tijs\textsuperscript{4} provided a fast identification method for plate sound
radiation based on particle vibration. Oliver Wolff\textsuperscript{5,6} did binaural panel noise contribution analysis which was an alternative to the conventional window method.

Some of vehicle body panels are similar to the simply-supported plate, such as dash panel, liftgate and floor, etc. The paper focuses research on the simply-supported plate and its structural vibration and sound radiation. In far field, sound intensities analyzed by the plate vibration and point source are the same, thus the vibration analysis of the plate can be simplified as a SDOF model. The SDOF’s velocity transfer function (VTF) is obtained and influences of mass, stiffness and damping on VTF are analyzed. A vehicle with interior booming caused by the liftgate vibration is provided, and the booming noise is reduced by adding mass on the plate, which proves the theoretical analysis results.

2. Panel Vibration and Sound Radiation

2.1 Vibration of simply-supported rectangular plate

Among rectangular plate bending vibration problems, the case with simply-supported boundary condition is the simplest one and is the only one that analytical solution can be obtained. According to thin plate vibration theory, deflection equation for a simply-supported rectangular panel in an infinite baffle, shown in Figure 1, can be expressed as follows\textsuperscript{7},

\[
w_{pq}(x, y, t) = A_{pq} \sin\left(\frac{p\pi x}{a}\right) \sin\left(\frac{q\pi y}{b}\right) e^{i\omega t} \quad p, q = 1, 2, 3\ldots
\]

where \(w_{pq}(x, y)\) denotes the deflection of the vibrating plate; \(A_{pq}\) is a constant relating to the boundary condition, materials and excitation; \(a\) and \(b\) represent the length and width of the plate; \(p\) and \(q\) denote plate mode number in \(x\) and \(y\) directions, respectively; \(\omega\) is the angular frequency. Physically, the number \(p\) and \(q\) represent the number of half sine-waves in \(x\) and \(y\) directions of plate in the corresponding mode shapes.

![Figure 1 A rectangular panel in an infinite baffle](image)

The normal vibration velocity distribution becomes,

\[
v_{pq}(x, y, t) = j\omega A_{pq} \sin\left(\frac{p\pi x}{a}\right) \sin\left(\frac{q\pi y}{b}\right) e^{i\omega t} \quad p, q = 1, 2, 3\ldots
\]

and the eigen frequencies of \((p, q)\) order are expressed as follows,

\[
\omega_{pq} = \sqrt{\left(\frac{p\pi}{a}\right)^2 + \left(\frac{q\pi}{b}\right)^2} \quad p, q = 1, 2, 3\ldots
\]

Where, \(D_0\) is bending stiffness of thin plate, \(D_0 = \frac{Eh^3}{12(1-\mu^2)}\); \(h\) is the panel thickness; \(\rho\) is the density of plate; \(E\) is denotes Youngs modulus; and \(\mu\) represents Poisson ratio.

Most vehicle body panels are similar to rectangular plates, such as ceiling, floor and dash panel, etc. The boundary conditions for these body plates are complicated, however, some cases can be simplified as simply-supported plate. Take a steel simply-supported plate as an example to calculate its modes and modal frequencies. Table 1 provides geometrical and material properties of
the steel plate. Table 2 lists the first 8 modal frequencies based on CAE simulation and theoretical calculation. The simulation and theoretical results are very close, however, the errors between two methods increase with the modal number increase due to reduction of the steel plate rigidity during CAE meshing. Figure 2 shows the first 8 modal shapes by CAE analysis.

**Table 1** Properties of the steel plate

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length/m</td>
<td>$a$</td>
<td>0.4</td>
</tr>
<tr>
<td>Width/m</td>
<td>$b$</td>
<td>0.3</td>
</tr>
<tr>
<td>Thickness/mm</td>
<td>$h$</td>
<td>1</td>
</tr>
<tr>
<td>Density/kg/m$^3$</td>
<td>$\rho$</td>
<td>7850</td>
</tr>
<tr>
<td>Youngs Modulus/GPa</td>
<td>$E$</td>
<td>210</td>
</tr>
<tr>
<td>Poisson ratio/-</td>
<td>$\mu$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

**Table 2** First 10 eigenfrequencies of the rectangular simply supported plate

<table>
<thead>
<tr>
<th>modes</th>
<th>Eigenfreq./Hz theory</th>
<th>Eigenfreq. /Hz numerical</th>
<th>Rel. error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 1)</td>
<td>42.7</td>
<td>42.6</td>
<td>0.23</td>
</tr>
<tr>
<td>(2, 1)</td>
<td>88.8</td>
<td>88.5</td>
<td>0.34</td>
</tr>
<tr>
<td>(1, 2)</td>
<td>124.6</td>
<td>124.4</td>
<td>0.16</td>
</tr>
<tr>
<td>(3, 1)</td>
<td>165.6</td>
<td>165.1</td>
<td>0.30</td>
</tr>
<tr>
<td>(2, 2)</td>
<td>170.7</td>
<td>169.8</td>
<td>0.52</td>
</tr>
<tr>
<td>(3, 2)</td>
<td>247.6</td>
<td>245.6</td>
<td>0.80</td>
</tr>
<tr>
<td>(1, 3)</td>
<td>261.2</td>
<td>260.7</td>
<td>0.19</td>
</tr>
<tr>
<td>(4, 1)</td>
<td>273.2</td>
<td>_272.3</td>
<td>0.33</td>
</tr>
</tbody>
</table>

![Figure 2](image-url)  
*Figure 2* First 10 modal shapes of the simply-supported rectangular plate

### 2.2 Sound radiation of simply-supported rectangular plate

Figure 3 shows the sound radiation model of simply-supported rectangular panel in an infinite baffle. The acoustic pressure can be written in terms of surface velocity using the Rayleigh integral $^8$ $^9$

$$P(\vec{r}, t) = \frac{j \omega \rho_0}{2\pi} \int_{S} \frac{v_n(\vec{r}_s)}{|\vec{R}|} e^{-j\omega t} dS$$

(4)

where $v_n(\vec{r}_s)$ denotes the normal vibration velocity distribution of plate; $\vec{r}$ is the position vector of observation point; $\vec{r}_s$ in the position vector of the surface position having normal velocity $v_n(\vec{r}_s)$ and $\vec{R}$ is the magnitude of vector $|\vec{r} - \vec{r}_s|$, i.e. $R = |\vec{r} - \vec{r}_s|$. 
Figure 3 Sound radiation model of simply supported rectangular panel in infinite baffle

Substitute equation (2) into equation (4), the acoustic pressure for the simply-supported rectangular panel is obtained and expressed as follows,

\[ P(x', y', z', t) = -\frac{\omega^2 \rho_0 A_{pq}}{2\pi} e^{i\omega t} \int_0^\infty \int_0 \sin(p\pi x/a) \sin(q\pi y/b) e^{-j\omega t} d\frac{x}{a} d\frac{y}{b} \]  

(5)

As already stated, the integral does not admit an analytic solution for arbitrary observer points \((x', y', z')\). But in the case of far field, where \(R\) is much greater than the source size as defined by the panel dimension \(a\) and \(b\), i.e. \(R \gg a, b\).

\[ R \approx r - x \sin \theta \cos \phi - y \sin \theta \sin \phi \]  

(6)

The pressure intensity is given by the real part of the expression of multiplying acoustic pressure by particle velocity in the location of observation point.

\[ I(r, \theta, \phi, \omega) = \frac{1}{2} \text{Re}\left\{ \overline{P}(r, \theta, \phi, \omega) \cdot \overline{v}(r, \theta, \phi, \omega) \right\} \]  

(7)

where the superscript * indicates the complex conjugate and \(\overline{v}(r, \theta, \phi, \omega)\) represents the medium vibration velocity. In the far field, the latter is equal to \(\overline{P}(r, \theta, \phi, \omega)/\rho_0 c\).

For harmonic vibration, the sound intensity caused by vibration of simply-supported rectangular plate in an unlimited baffle can be expressed as follows,

\[ \overline{I}(r, \theta, \phi, \omega) = 2\rho_0 c^2 A_{pq}^2 \left( \frac{kab}{\pi^3 \rho q} \right)^2 \left\{ \frac{\cos \left(\frac{\alpha}{2}\right) \cos \left(\frac{\beta}{2}\right)}{\left(\frac{\alpha}{p\pi}\right)^2 - 1} \frac{\sin \left(\frac{\alpha}{2}\right) \sin \left(\frac{\beta}{2}\right)}{\left(\frac{\beta}{q\pi}\right)^2 - 1} \right\}^2 \]  

(8)

where \(\alpha\) and \(\beta\) are defined as \(\alpha = k a \sin \theta \cos \phi\) and \(\beta = k b \sin \theta \sin \phi\), respectively. The \(\cos(\alpha/2)\) is used when \(p\) is an odd integer and the \(\sin(\alpha/2)\) is used when \(p\) is an even integer. Similarly, the \(\cos(\beta/2)\) is used when \(q\) is an odd integer while the \(\sin(\beta/2)\) is used when \(q\) is an even integer.

When the radiation acoustic wave frequency is such that the acoustic wavelength greatly exceeds both the structural trace wavelength, i.e. \(\alpha \ll p\pi\) and \(\beta \ll q\pi\), the maximum sound intensity in far field can be expressed as follows,

\[ \overline{I}_{\text{max}} \approx 2\rho_0 c^2 A_{pq}^2 \left( \frac{kab}{\pi^3 \rho q} \right)^2 \]  

(9)

The modal radiation acoustic power is obtained by integrating the far-field acoustic intensity over a hemisphere enclosing the sound source,

\[ \overline{W}_{pq}(\omega) = 2\rho_0 c^2 A_{pq}^2 \left( \frac{kab}{\pi^3 \rho q} \right)^2 \int_0^{2\pi} \int_0^\infty \sin \theta d\theta \sin \phi \left\{ \frac{\cos \left(\frac{\alpha}{2}\right) \cos \left(\frac{\beta}{2}\right)}{\left(\frac{\alpha}{p\pi}\right)^2 - 1} \frac{\sin \left(\frac{\alpha}{2}\right) \sin \left(\frac{\beta}{2}\right)}{\left(\frac{\beta}{q\pi}\right)^2 - 1} \right\}^2 \]  

(10)

The radiation efficiency is defined\(^{10,11}\),
\[ \sigma_{pq} = \frac{\overline{W_{pq}}}{\rho_{c}c_{ab}\langle v_{n}^2 \rangle_{pq}} \] (11)

where \( \langle v_{n}^2 \rangle \) denotes the average mean-square velocity and is expressed as,

\[ \langle v_{n}^2 \rangle_{pq} = \frac{1}{8} \omega^2 |A_{pq}|^2 \] (12)

Substitute equation (10) and (12) into (11), the radiation efficiency for the simply-supported plate is,

\[ \sigma_{pq} = \frac{16k^2 ab}{(\pi^2 pq)^2} \int_{0}^{\pi} \int_{0}^{\pi} \cos \left( \frac{\alpha}{2} \right) \cos \left( \frac{\beta}{2} \right) \sin \theta \, d\theta \, d\phi \] (13)

At very low frequency, wave length is very long, i.e. \( \alpha \ll p\pi \) and \( \beta \ll q\pi \), and both \( p \) and \( q \) are odd integers, the sound radiation efficiency is,

\[ \sigma_{pq} \approx \frac{16k^2 ab}{(\pi^2 pq)^2} \int_{0}^{\pi/2} \cos^2 \left( \frac{\alpha}{2} \right) \cos^2 \left( \frac{\beta}{2} \right) \sin \theta \, d\theta \, d\phi = \frac{32}{\pi^3} \frac{1}{p^2q^2} \left( \frac{b}{a} \right) \left( \frac{ka}{\pi} \right)^2 \] (14)

### 2.3 Sound radiation of point source radiator

Considering a single modal cell of area \((ab/pq)\) in the absence of any other surface motion, as shown in Figure 4, since the dimensions of the cell are assumed to be smaller than an acoustic wavelength, the single cell may be modeled by a point source of volume velocity given by \(^{12}\),

\[ \mathbf{d} \dot{\mathbf{Q}} = \mathbf{v}_{n} \, dS \] (15)

Figure 4 Sound radiation model of a single cell of area \((a/p, b/q)\)

The volume velocity for \( dS \) is,

\[ \dot{Q} = 2j \omega A_{pq} \int_{0}^{a/p} \int_{0}^{\pi/p} \sin(p\pi x/a) \sin(q\pi y/b) \, dx \, dy = j\omega A_{pq} \frac{8ab}{pq\pi^2} \] (16)

The pressure at point source

\[ P(r, t) = j\omega \rho_{0} \frac{\dot{Q}}{4\pi r} \exp[j(\omega t - kr)] \] (17)

The radiation sound pressure by \( dS \) is,

\[ P(r, t) = \omega^2 \rho_{0} \frac{1}{4\pi r} A_{pq} \frac{8ab}{pq\pi^2} \exp[j(\omega t - kr)] \] (18)

The far-field intensity radiated by element \( dS \) is,

\[ \bar{T}(r, \theta, \phi) = \frac{|P(r, t)|^2}{2\rho_{c}c} = 2\rho_{0}c\omega^4 A_{pq}^2 \left( \frac{ka}{\pi r pq} \right)^2 \] (19)
Compared equations (19) and (9), the maximum sound intensities radiated by the point source \( \delta S \) and by the plate have the same expressions. Thus, in far field, sound radiation by 1\(^{st}\) order vibration of the plate can be simplified by a point source, which provides theoretical foundation to simplify the 1\(^{st}\) order vibration of a plate as a SDOF model.

### 3. Single Degree of Freedom Model

#### 3.1 SDOF model simplified from a simply-supported plate

The body panels such as closure, roof, floor and so on are similar to rectangular plates and the first order vibration is the most effective radiator when \( ka, kb \ll 1 \) and can be simplified as a lumped model, i.e. SDOF model as shown in Figure 5.

According to energy equivalent principle, the kinetic energies between the SDOF and the plate are equal, and as the same as the potential energies. Thus, the plate is simplified as an equivalent SDOF model, and equivalent mass and stiffness are,

\[
m_{e,(1,1)} = 0.25m_p 
\]

\[
k_{e,(1,1)} = \frac{1}{4} abD_0 \left[ \left( \frac{\pi}{a} \right)^2 + \left( \frac{\pi}{b} \right)^2 \right]^2 
\]

where, \( m_{e,(1,1)} \) and \( k_{e,(1,1)} \) are the equivalent mass and stiffness for the plates’ (1,1) mode; \( m_p \) is the plate mass.

If the damping consumed for the plate vibration is obtained, the corresponding equivalent damping can be calculated as well.

![Figure 5 The schematic of panel simplified into SDOF model](image)

#### 3.2 Transfer function characteristic of single degree system

The vibration differential equation of SDOF system is

\[
m z \ddot{z} + c \dot{z} + k z = f(t)
\]

Fourier transform operator is acted on Eq. (22) with assuming that the initial displacement and velocity are zero. The velocity transfer function (VTF) is obtained

\[
H_v(\omega) = \frac{j\omega}{k - m\omega^2} + j\omega c
\]

and the VTF amplitude is

\[
|H_v(\omega)| = \frac{\omega}{\sqrt{(k - m\omega^2)^2 + (\omega c)^2}}
\]

Considering the first order mode (1, 1) vibration of the steel panel with dimension of 0.3m×04m×1mm, the equivalent mass and stiffness are \( m_e = 0.2355 \) kg and \( k_e = 16938.4 \) N/m, respectively. Assume the damping is 10 N·s/m. Figure 6 shows the logarithm amplitude of VFT of SDOF model, which indicates that the eigenfrequency decreases with mass, increases with stiffness and is invariant with damping. The VFT amplitude is invariable with mass and stiffness, and decreases with and damping.
Figure 6  Velocity transfer function (VTF) of SDOF model
(a) VTF varying with mass; (b) VTF varying with stiffness; (c) VTF varying with damping

4. Case Study: Interior Booming Caused by Body Panel Vibration

When a vehicle was running on a coarse road at speed of 60 km/h, a serious booming noise was heard and passengers felt very uncomfortable. Interior sound was recorded and there is a big peak at 46.6Hz and its value is 59 dB(A), as shown in Figure 7.

Figure 7 Interior noise with booming and without booming after masses are added on the lifgate

According to “source–path–receive” NVH analysis model, all possible sources and paths were analyzed. The possible sources are the airborne and structural borne excitation sources including engine, intake, exhaust and road, and the possible paths are different body panels\textsuperscript{14,15}. Figure 8 shows the possible sources and transfer paths.

Figure 8 The possible sources and transfer paths for the interior booming

After lots of testing and analysis, the booming noise is identified from road excitation transferred through the structural borne paths. The body panels’ structural modes and body cavity acoustic modes are analyzed. The information related to the 46 Hz booming noise is listed in table 3.

Table 3 The information related to the 46Hz booming noise

<table>
<thead>
<tr>
<th>Components</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior booming noise</td>
<td>46.5</td>
</tr>
<tr>
<td>Road excitation</td>
<td>46.2</td>
</tr>
<tr>
<td>First cavity mode</td>
<td>46.8</td>
</tr>
<tr>
<td>Lifigate first structural mode</td>
<td>47</td>
</tr>
</tbody>
</table>

The lifigate is excited by the coarse road which the excitation frequency (46.2 Hz) is close to the lifigate first structural modal frequency (47 Hz). The structural mode is further resonant with the first cavity mode (46.8Hz). Thus a huge booming noise is produced inside the compartment.

Based on pervious theoretical analysis, the lifigate plate is simplified to SDOF model and the velocity transfer function is analyzed. Eventually, two masses with 1kg for each are added on the
plate, shown in Figure 9, and booming peak at 46.6 Hz is reduced to 52.5 dB(A) from 59 dB(A), shown in Figure 7. Subjectively, passengers felt much better than the case before modification.

![Figure 9 Two masses are added on the liftgate](image)

5. Conclusions

The paper analyzes vibration and sound radiation of a simply-supported plate and SDOF model and also applies the theory on a vehicle booming noise analysis. Some conclusions can be obtained as follows,

1) Some vehicle body panels can be simplified as a simply-supported plate where the first mode possesses the highest sound radiation efficiency. For sound radiation analysis in far field, the plate’s first mode vibration can be replaced by a SDOF model.

2) According to energy equivalent principle, equivalent mass and stiffness are obtained for the SDOF system, which helps for tuning the plate’s mass, stiffness and damping in order to suppress the plate vibration and sound radiation.

3) After the vehicle booming noise is identified, the theoretical analysis is applied on it. Two masses are added on the liftgate and interior booming noise is significantly reduced.

REFERENCES

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